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TIME SERIES MODELING OF NATURAL GAS FUTURE PRICE WITH FUZZY TIME SERIES CHEN, LEE AND TSAUR

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Abstract. *Investment is the process of investing money or capital for profit or material results. The investor carefully calculates the investment object to minimize losses and maximize profits. One of the essential investment objects is the futures price of natural gas considered a commodity that plays a vital role in the world economy. The movement of natural gas futures prices can be modeled using a time series model. The data in the time series model is believed to have a particular pattern to model the data in the future. The natural gas futures price is modeled into a time series method by using the fuzzy time series (FTS) approach of the FTS Chen, Lee and Tsaur. Model accuracy is calculated using the criteria of Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE). The three FTS methods have good performance of accuracy for this time series data, where FTS Tsaur as fuzzy times series approach with average based method shows the best results with the smallest error rate to the data of natural gas future price.*

Keywords: *FTS Chen, FTS Lee, FTS Tsaur, model accuracy, natural future gas price.*

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1. INTRODUCTION

Investment is the process of investing money or capital in a company or project to make a profit [1]. Careful arrangements of capital investments are needed so that investors minimize losses and maximize profits. One form of careful arrangement is that investors can model the investment object. One of important investment objects in high demand is natural gas futures (NGF), it is considering that natural gas as one of the commodity that plays an important role in the world economy.

The natural gas futures price modeling method as time series data can be expressed in the fuzzy time series (FTS) model because FTS has the advantage that fuzzy logic is flexible and simple to understand, and it can model complex nonlinear functions [2]. The best modeling results for NGF prices with the FTS approach can be used as a reference for investment.

The beginning of FTS was introduced by Song and Chissom, who elaborated fuzzy logic into the FTS model [3]. The FTS model is a time series data modeling that uses fuzzy principles as its basis. The FTS model uses previous data patterns to predict future data [4]. In 1996, the FTS method was developed by Chen using simple arithmetic operations to perform modeling of enrollment at the university of Alabama [5]. FTS Chen's concept has been used in various modeling of gold price data and other time series data [6]-[7]. Subsequently, in 2008, a specific application FTS Chen was applied to stock prices [8], and it has been used in various more practical models [9]-[10].

Meanwhile, FTS Lee has also been introduced, which is used to forecast short-term models on static and non-static data patterns. This concept has also been used in forecasting the exchange rate model of farmers in the livestock sub-sector [11] and forecasting the price of gold [12].

Furthermore, Tsaur in 2012 proposed the FTS Markov chain, which is also known as FTS Tsaur, where Tsaur combines the FTS method with the Markov chain in his research on the analysis of the accuracy of forecasting the Taiwan currency exchange rate against the US dollar [13]. The FTS Tsaur application has been used in various models [14]-[16]. The application of FTS is also including the case of bitcoin price forecasting with the FTS Chen, FTS Segmented Chen, and FTS Markov chain models [17], also predicting air pollution index by the FTS Markov chain [18]. In addition to these three FTS models, in 2020, Gao and Duru proposed parsimonious FTS modeling as another simple concept from FTS [19], but the model of FTS Chen, Lee and Tsaur still show superior accuracy.

Furthermore, it is very important to model natural gas futures prices into FTS Chen, Lee and Tsaur. This is done because these three FTS models represent the basic concept (FTS Chen) and the new concept (FTS Lee and Tsaur), all three models still use interval division using the average-based method. The process of dividing this interval is very detailed so that it still produces a good value for the accuracy of the model.

2. RESEARCH METHODS

2.1 Data Collection and Presentation

The research begins with collecting NGF price data (USD/MMBtu). The time series data is shown in Figure 1. The data was taken from January 2017 to December 2021 on the official website of www.investing.com [20]. From the data plot, it can be seen that the pattern of time series data with a monthly period, the data fluctuates and occasionally shows an up and down trend, so time series data like this can be modeled using the FTS approach.

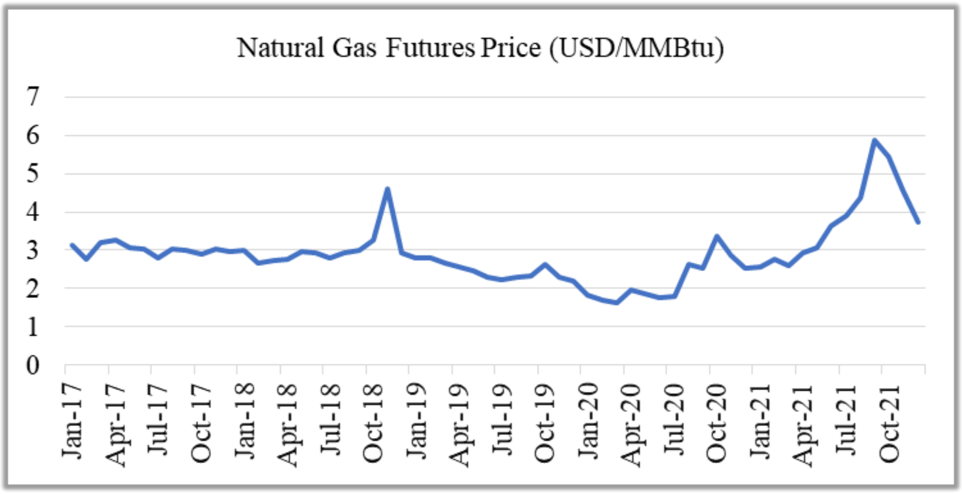

Figure 1. Natural Gas Futures Price Data Plot.

2.2 Data Processing

The process of modeling the NGF price data with the FTS Chen, Lee and Tsaur approaches is divided into several steps, steps 1-4 are the initial steps used for the three FTS methods. Step 5 and so on are processed based on the respective FTS algorithm with the following description:

1) Determine the universal set (U) of concrete data through the process,

$$U = [D_{min} - D_1; D_{max} + D_2] \tag{1}$$

where D_1 and D_2 are any positive numbers.

2) Calculating the number of fuzzy set intervals with an average based through the process:

a. Determine the length of the interval U,

$$R = [D_{max} + D_2 - D_{min} - D_1]. \tag{2}$$

b. Calculate the value of the lag absolute and the average value of the lag absolute,

$$lag\ absolute = |(D_{t+1}) - D_t|, \tag{3}$$

$$mean\ lag\ absolute = \frac{\sum_{t=1}^{N-1}|(D_{t+1}) - D_t|}{N-1}, \tag{4}$$

where D_t is the data at the *t-th* condition and N represents the number of data.

c. Determine the base interval by using the formula,

$$K = mean/2, \tag{5}$$

where the basis interval based on the following Table 1,

Table 1. The Base Interval.

Range	Base
0.1 – 1	0.1
1.1 – 10	1
11 – 100	10
101 – 1000	100
1001 – 10000	1000

d. Determine the number of fuzzy sets through the process,

$$n = R/K. \tag{6}$$

e. Finding the median of the fuzzy set,

$$m_i = (lower\ limit\ u_i + upper\ limit\ u_i)/2. \tag{7}$$

3) Formulating the position level of the fuzzy set against A_i and performing *fuzzyfication* on concrete data, namely the process of converting the input data characters, which were originally numeric, into linguistic using the level of position stored in the fuzzy knowledge base. The number of linguistic variables obtained may be independent because there are no standard rules. The formulation of the fuzzy set on A_i is by position level. The position level of the u_i fuzzy set can be simplified as follows such as in Table 2 with position level in the formulation,

$$\mu_{A_i}(u_i) = \begin{cases} 1 & \text{if } i = i \\ 0.5 & \text{if } i = i-1 \text{ or } i = i+1 \\ 0 & \text{for other } i \end{cases} \tag{8}$$

where $1 \leq i \leq n$ and n is the number of fuzzy sets.

Table 2. Fuzzy Set Position Level.

$\mu_{A_i}(u_i)$	A_1	A_2	A_3	...	$A_{(n-2)}$	$A_{(n-1)}$	A_n
A_1	1	0.5	0	...	0	0	0
A_2	0.5	1	0.5	...	0	0	0
A_3	0	0.5	1	...	0	0	0
...	...	...	...	...	...	...	...
$A_{(n-2)}$	0	0	0	...	1	0.5	0
$A_{(n-1)}$	0	0	0	...	0.5	1	0.5
A_n	0	0	0	...	0	0.5	1

4) Develop a fuzzy logical relationship (FLR) based on concrete data. This step defines the fuzzy logic relation, namely $A_i \rightarrow A_j$. The set A_i is current condition, where $D_{(t-1)}$ and A_j the next condition at time *t*. FLR connects the relationship between the linguistic values determined based on the previously obtained *fuzzyfication* table.

5) Develop a fuzzy logical relationship group (FLRG). FLRG is carried out through a fuzzy categorization process that has similar current conditions and is then collected into one group in the following condition. Each FLR is aggregated to form an interconnected FLRG.

   - FTS Chen

     For example, there are four fuzzy logical relationships (FLR) where 3 of them are equal $A_1 \rightarrow A_2$, $A_1 \rightarrow A_2$, $A_1 \rightarrow A_2$, dan $A_1 \rightarrow A_3$, forming FLRG $A_1 \rightarrow A_2, A_3$ because according to Chen $A_1 \rightarrow A_2$, $A_1 \rightarrow A_2$, $A_1 \rightarrow A_2$, it does not affect the model results. so one FLRG is enough.

   - FTS Lee and Tsaur

     For example, there are three fuzzy logical relationships (FLR) where 2 of them are equal $A_1 \rightarrow A_2$, $A_1 \rightarrow A_2$, dan $A_1 \rightarrow A_3$, forming FLRG $A_1 \rightarrow A_2, A_2, A_3$. Because Lee and Tsaur's rule describes $A_1 \rightarrow A_2$, $A_1 \rightarrow A_2$ is calculated more than once because it can affect the model results.

6) Transition probability matrix

$$P_{ij} = \frac{M_{ij}}{M_i}, i,j = 1,2,3,\dots,n \tag{9}$$

P_{ij} : the probability of changing from condition A_i to A_j on one step.

M_{ij}: change time from state A_i to A_j on one step.

M_i : the amount of data from condition A_i follows process number 5 above.

The following rule of probability transition under these conditions, a transition probability matrix with an element in real support for a dimension of $n \times n$ is obtained, presented by the formula:

$$R = \left(\begin{bmatrix} P_{11} & \cdots & P_{1n} \\ \vdots & \ddots & \vdots \\ P_{n1} & \cdots & P_{nn} \end{bmatrix} \right) \tag{10}$$

7) *Defuzzyfication* is converting the fuzzy output data obtained from the fuzzy logic arrangement into the output data with a firm value (numeric) as the model value using the appropriate position value at the time of *fuzzyfication*.

- FTS Chen

1. If the FLRG of A_i changes to the empty set $A_i \rightarrow \emptyset$, then the F_t model has the value m_i, which is the median of u_i with the formula,

$$F_t = m_i. \tag{11}$$

2. If the FLRG of A_i changes from one to one ($A_i \rightarrow A_k$ with $P_{ij} = 0\ and\ P_{ik} = 1, j \neq k$) then the F_t model has the value m_k, which is the median of u_i,

$$F_t = m_k. \tag{12}$$

3. If the FLRG of A_i changes from one to many ($A_i \rightarrow A_1, A_2, \ldots, A_p,\ i = 1, 2, \ldots, p$), then the model F_t has the value,

$$F_t = \frac{m_{1(t-1)} + m_{2(t-1)} + \ldots + m_{p(t-1)}}{p}. \tag{13}$$

- FTS Lee

The calculation of the model FTS Lee

1. If the FLRG of A_i changes to the empty set $A_i \rightarrow \emptyset$, then the F_t model has the value m_i, which is the median of u_i with the formula,

$$F_t = m_i. \tag{14}$$

2. If the FLRG of A_i changes from one to one ($A_i \rightarrow A_k$ with $P_{ij} = 0\ and\ P_{ik} = 1, j \neq k$) then the F_t model has the value m_k, which is the median of u_i,

$$F_t = m_k P_{ik} = m_k. \tag{15}$$

3. If the FLRG of A_i changes from one to many ($A_i \rightarrow A_1, A_2, \ldots, A_n,\ i = 1,2, \ldots, n$), then the model F_t has the value,

$$F_t = m_1 P_{i1} + m_2 P_{i2} + \ldots + m_{(i-1)} P_{i(i-1)} + m_i P_{ii} + \ldots + m_n P_{in}, \tag{16}$$

where $m_1, m_2, \ldots, m_{i-1}, m_{i+1}, \ldots, m_n$: median of $u_1, u_2, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n$.

- FTS Tsaur

The calculation of the model value in FTS Tsaur is divided into three processes: the initial results, the synchronization, and the final result of the model.

In the first process, the initial results of the model are calculated based on the probability matrix of changes in R, with the following conditions,

1. If the FLRG of A_i changes to the empty set $A_i \rightarrow \emptyset$, then the F_t model has the value m_i, which is the median of u_i with the formula,

$$F_t = m_i. \tag{17}$$

2. If the FLRG of A_i changes from one to one ($A_i \rightarrow A_k$ with $\mathrm{P_{ij}} = 0\ and\ \mathrm{P_{ik}} = 1, j \neq k$) then the F_t model has the value m_k, which is the median of u_i.

$$F_t = m_k P_{ik} = m_k. \tag{18}$$

3. If the FLRG of A_i changes from one to many ($A_i \rightarrow A_1, A_2, \ldots, A_n,\ i = 1, 2, \ldots, n$) and the data group X_{t-1} when t-1 is in A_i condition, then the model $\mathrm{F_t}$ has the value,

$$F_t = m_1 P_{i1} + \ldots + m_{(i-1)} P_{i(i-1)} + X_{t-1} P_{ii} + m_{(i+1)} P_{i(i+1)} + \ldots + m_n P_{in}, \tag{19}$$

where X_{t-1} substitutes m_j in order to get data from condition A_j when t-1.

The second process, calculation of model synchronization, aims to correct modeling errors caused by the bias of the transition probability matrix. Model synchronization rules (D_t),

1. If condition A_i when t-1 as $F_{t-1} = A_i$, there is a forward movement to condition A_{i+s} for t, where $1 \leq s \leq n-1$, then D_t synchronization,

$$D_t = (\frac{l}{2})s, (1 \leq s \leq n-1), \quad (20)$$

   where s is many leap-forward changes.

2. If condition A_i when t-1 as $F_{t-1} = A_i$, there is a backward movement to condition A_{i+s} for t, where $1 \leq s \leq n-1$, then D_t synchronization,

$$D_t = -(\frac{l}{2})v, (1 \leq v \leq i), \quad (21)$$

   where v is many leap backward changes.

The third process, the final result of the model, is obtained through the process of adding the initial results of the model and synchronizing the model. The general form of the final model (FM_t) is,

$$FM_t = F_t + D_t, \quad (22)$$

FM_t : the final result of the model at time t.

2.3 Model Accuracy

To determine the level of accuracy. it is calculated the error measurement. The best model is the model with the smallest error obtained by comparing the variation of the difference between the X_t observation data and the estimated Ft data at data time t. Among the methods of calculating the error value that is often used are [17].

Root Mean Squared Error (RMSE). RMSE is an alternative method to measure the accuracy of the model,

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(X_t - F_t)^2} \quad (23)$$

Mean Absolute Error (MAE). To measure the accuracy of the modeling by averaging the absolute value of the modeling error,

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|X_t - F_t| \quad (24)$$

Mean Absolute Percentage Error (MAPE). MAPE is the average of all percentage errors between the original data and the modeling results,

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{X_t - F_t}{X_t}\right| \; x\ 100\% \quad (25)$$

The following are the MAPE assessment categories,

a) If the MAPE rate is < 10%. the criteria for the model are very good.
b) If the MAPE rate is 10% to 20%. then the model criteria are good
c) If the MAPE rate is 20% to 50%. then the model criteria are quite good
d) If the MAPE rate is >50% then the model criteria are not good

3. RESULTS AND DISCUSSION

3.1 Initial Steps of Calculation

The initial step of modeling natural gas futures prices with the FTS approach in this subsection begins with the process of calculating the initial four steps as described in Subsection 2.2 as follows:

1) Determine the universal set (U) of concrete data through the process,

$$U = \left[\mathrm{D}_{\min} - D_1\,;\ \mathrm{D}_{\max} + D_2\right] = [1.640 - 0\,; 5.867 + 0.003] = [1.640\,; 5.870].$$

2) Calculating the number of fuzzy set intervals,

   a. Determine the length of the interval U.

$$\text{where } R = \left[D_{max} + \mathrm{D}_2 - D_{min} - \mathrm{D}_1\right] = [5.870 - 1.640] = 4.230.$$

   b. Calculate the value of the lag absolute and the average value of the lag absolute, the price and the lag absolute in the Table 3.

Table 3. Price and Lag Absolute of Natural Gas Future Price.

Month	Price	Lag absolute
Jan-17	3.117	0.607
Feb-17	2.774	0.343
Mar-17	3.190	0.416
⋮	⋮	⋮
Oct-21	5.426	0.441
Nov-21	4.567	0.021
Dec-21	3.370	1.197

$$mean = \frac{\sum_{t=1}^{N-1}\left|\left(D_{t+1}\right) - D_t\right|}{N-1} = \frac{16.199}{59} = 0.275.$$

   c. Determine the base interval $K = \frac{mean}{2} = \frac{0.275}{2} = 0.137 \approx 0.1$ (following Table 1).

   d. Determine the number of fuzzy sets $n = \frac{R}{K} = \frac{4.230}{0.1} = 42.300 \approx 43$ (rounding up).

   e. Determine the median of fuzzy sets $m_i = \frac{(lower\ limit\ u_i + upper\ limit\ u_i)}{2}$ as in the Table 4.

Table 4. Natural Gas Futures Price Data Interval With Average Based.

Interval	Lower limit	Upper limit	Median
U_1	1.640	1.740	1.690
U_2	1.740	1.840	1.790
U_3	1.840	1.940	1.890
⋮	⋮	⋮	⋮
U_{41}	5.640	5.740	5.690
U_{42}	5.740	5.840	5.790
U_{43}	5.840	5.940	5.890

3) *Fuzzyfication*. *Fuzzyfication* is carried out according to the stages in accordance with Table 2.

4) Create fuzzy logical relationship (FLR) concrete data. Table 5 below is the result of the FLR by following the description in Subsection 2.2 in Point 4 above.

Table 5. Fuzzy Logical Relationship (FLR).

Month	Price	*Fuzzyfication*	FLR	Month	Price	*Fuzzyfication*	FLR
Jan-17	3.117	A_{15}	$A_{15} \rightarrow A_{12}$	Jul-19	2.233	A_6	$A_6 \rightarrow A_7$
Feb-17	2.774	A_{12}	$A_{12} \rightarrow A_{16}$	Aug-19	2.285	A_7	$A_7 \rightarrow A_7$
Mar-17	3.19	A_{16}	$A_{16} \rightarrow A_{17}$	Sep-19	2.330	A_7	$A_7 \rightarrow A_{10}$
Apr-17	3.276	A_{17}	$A_{17} \rightarrow A_{15}$	Oct-19	2.633	A_{10}	$A_{10} \rightarrow A_7$
May-17	3.071	A_{15}	$A_{15} \rightarrow A_{14}$	Nov-19	2.281	A_7	$A_7 \rightarrow A_6$
Jun-17	3.035	A_{14}	$A_{14} \rightarrow A_{12}$	Dec-19	2.189	A_6	$A_6 \rightarrow A_3$
Jul-17	2.794	A_{12}	$A_{12} \rightarrow A_{15}$	Jan-20	1.841	A_3	$A_3 \rightarrow A_1$
Aug-17	3.040	A_{15}	$A_{15} \rightarrow A_{14}$	Feb-20	1.684	A_1	$A_1 \rightarrow A_1$
Sep-17	3.007	A_{14}	$A_{14} \rightarrow A_{13}$	Mar-20	1.640	A_1	$A_1 \rightarrow A_4$
Oct-17	2.896	A_{13}	$A_{13} \rightarrow A_{14}$	Apr-20	1.949	A_4	$A_4 \rightarrow A_3$
Nov-17	3.025	A_{14}	$A_{14} \rightarrow A_{14}$	May-20	1.849	A_3	$A_3 \rightarrow A_2$
Dec-17	2.953	A_{14}	$A_{14} \rightarrow A_{14}$	Jun-20	1.751	A_2	$A_2 \rightarrow A_2$
Jan-18	2.995	A_{14}	$A_{14} \rightarrow A_{11}$	Jul-20	1.799	A_2	$A_2 \rightarrow A_{10}$
Feb-18	2.667	A_{11}	$A_{11} \rightarrow A_{11}$	Aug-20	2.630	A_{10}	$A_{10} \rightarrow A_9$
Mar-18	2.733	A_{11}	$A_{11} \rightarrow A_{12}$	Sep-20	2.527	A_9	$A_9 \rightarrow A_{18}$
Apr-18	2.763	A_{12}	$A_{12} \rightarrow A_{14}$	Oct-20	3.354	A_{18}	$A_{18} \rightarrow A_{13}$
May-18	2.952	A_{14}	$A_{14} \rightarrow A_{13}$	Nov-20	2.882	A_{13}	$A_{13} \rightarrow A_9$
Jun-18	2.924	A_{13}	$A_{13} \rightarrow A_{12}$	Dec-20	2.539	A_9	$A_9 \rightarrow A_{10}$
Jul-18	2.782	A_{12}	$A_{12} \rightarrow A_{13}$	Jan-21	2.564	A_{10}	$A_{10} \rightarrow A_{12}$
Aug-18	2.916	A_{13}	$A_{13} \rightarrow A_{14}$	Feb-21	2.771	A_{12}	$A_{12} \rightarrow A_{10}$
Sep-18	3.008	A_{14}	$A_{14} \rightarrow A_{17}$	Mar-21	2.608	A_{10}	$A_{10} \rightarrow A_{13}$
Oct-18	3.261	A_{17}	$A_{17} \rightarrow A_{30}$	Apr-21	2.931	A_{13}	$A_{13} \rightarrow A_{15}$
Nov-18	4.612	A_{30}	$A_{30} \rightarrow A_{14}$	May-21	3.055	A_{15}	$A_{15} \rightarrow A_{21}$
Dec-18	2.940	A_{14}	$A_{14} \rightarrow A_{12}$	Jun-21	3.650	A_{21}	$A_{21} \rightarrow A_{23}$
Jan-19	2.814	A_{12}	$A_{12} \rightarrow A_{12}$	Jul-21	3.914	A_{23}	$A_{23} \rightarrow A_{28}$
Feb-19	2.812	A_{12}	$A_{12} \rightarrow A_{11}$	Aug-21	4.377	A_{28}	$A_{28} \rightarrow A_{43}$
Mar-19	2.662	A_{11}	$A_{11} \rightarrow A_{10}$	Sep-21	5.867	A_{43}	$A_{43} \rightarrow A_{38}$
Apr-19	2.575	A_{10}	$A_{10} \rightarrow A_9$	Oct-21	5.426	A_{38}	$A_{38} \rightarrow A_{30}$
May-19	2.454	A_9	$A_9 \rightarrow A_7$	Nov-21	4.567	A_{30}	$A_{30} \rightarrow A_{21}$
Jun-19	2.308	A_7	$A_7 \rightarrow A_6$	Dec-21	3.370	A_{21}	$A_{21} \rightarrow$ -

3.2 FTS Chen Model Calculation

5) Determining fuzzy logic relationship group FTS Chen. FLRG FTS Chen is described based on Subsection 2.2 in Point 5 so that the following Table 6 is obtained,

Table 6. Fuzzy Logical Relationship Group (FLRG) FTS Chen.

Fuzzyfikasi	FLRG
A_1	$\rightarrow A_1, A_4$
A_2	$\rightarrow A_2, A_{10}$
A_3	$\rightarrow A_1, A_2$
$\vdots$	$\vdots$
A_{41}	$\rightarrow \emptyset$
A_{42}	$\rightarrow \emptyset$
A_{43}	$\rightarrow A_{38}$

6) Transition probability matrix. By using the formula in Subsection 2.2 at Point 6, it is obtained in Table 7, which is the FTS Chen score matrix table.

Table 7. **Standardized FTS Chen Score Matrix.**

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	...	A_{43}
A_1	0.5	0	0	0.5	0	0	0	...	0
A_2	0	0.5	0	0	0	0	0	...	0
A_3	0.5	0.5	0	0	0	0	0	...	0
A_4	0	0	1	0	0	0	0	...	0
A_5	0	0	0	0	0	0	0	...	0
A_6	0	0	0.5	0	0	0	0.5	...	0
A_7	0	0	0	0	0	0.333	0.333	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A_{43}	0	0	0	0	0	0	0	...	0

7) *Defuzzyfication.* By using the FTS Chen *defuzzyfication* rule in Subsection 2.2 in Point 7 above, the results of FTS Chen modeling are shown in Table 8.

Table 8. **Chen's FTS Model Results.**

Month	FTS Chen	Month	FTS Chen	Month	FTS Chen	Month	FTS Chen	Month	FTS Chen
Jan-17		Jan-18	2.930	Jan-19	2.930	Jan-20	2.090	Jan-21	2.757
Feb-17	3.157	Feb-18	2.930	Feb-19	2.890	Feb-20	1.740	Feb-21	2.615
Mar-17	2.890	Mar-18	2.690	Mar-19	2.890	Mar-20	1.840	Mar-21	2.890
Apr-17	3.290	Apr-18	2.690	Apr-19	2.690	Apr-20	1.840	Apr-21	2.615
May-17	3.840	May-18	2.890	May-19	2.615	May-20	1.890	May-21	2.840
Jun-17	3.157	Jun-18	2.930	Jun-19	2.757	Jun-20	1.740	Jun-21	3.157
Jul-17	2.930	Jul-18	2.840	Jul-19	2.357	Jul-20	2.190	Jul-21	3.890
Aug-17	2.890	Aug-18	2.890	Aug-19	2.090	Aug-20	2.190	Aug-21	4.39
Sep-17	3.157	Sep-18	2.840	Sep-19	2.357	Sep-20	2.615	Sep-21	5.890
Oct-17	2.930	Oct-18	2.930	Oct-19	2.357	Oct-20	2.757	Oct-21	5.390
Nov-17	2.840	Nov-18	3.840	Nov-19	2.615	Nov-20	2.890	Nov-21	4.590
Dec-17	2.930	Dec-18	3.340	Dec-19	2.357	Dec-20	2.840	Dec-21	3.340

3.3 FTS Lee Model Calculation

5) Determining fuzzy logic relationship group FTS Lee. FLRG FTS Lee is described based on Subsection 2.2 in Point 5 so that the following Table 9 is obtained.

Table 9. **Fuzzy Logical Relationship Group (FLRG) FTS Lee.**

Fuzzyfikasi	FLRG
A_1	$\rightarrow A_1\,(1), A_4\,(1)$
A_2	$\rightarrow A_2\,(1), A_{10}\,(1)$
A_3	$\rightarrow A_1\,(1), A_2\,(1)$
⋮	⋮
A_{41}	$\rightarrow Ø$
A_{42}	$\rightarrow Ø$
A_{43}	$\rightarrow A_{38}\,(1)$

The difference between Lee's FLRG table above and Chen's FLRG table lies in its score. For the FTS Lee, FLRG $A_7 \rightarrow A_6$, $A_7 \rightarrow A$ is rated with a score of 2. while in FTS Chen it is considered 1. The FLRG FTS Lee process is the same as FLRG FTS Tsaur, so use the same table.

6) Transition probability matrix. By using the formula in Subsection 2.2 at Point 6, it is obtained in Table 10, which is the Lee FTS score matrix table. The difference in FLRG scoring results in a different standardized matrix on FTS Lee.

Table 10. **Standardized FTS Lee Score Matrix**

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	...	A_{43}
A_1	0.5	0	0	0.5	0	0	0	...	0
A_2	0	0.5	0	0	0	0	0	...	0
A_3	0.5	0.5	0	0	0	0	0	...	0
A_4	0	0	1	0	0	0	0	...	0
A_5	0	0	0	0	0	0	0	...	0
A_6	0	0	0.5	0	0	0	0.5	...	0
A_7	0	0	0	0	0	0.5	0.25	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A_{43}	0	0	0	0	0	0	0	...	0

7) *Defuzzyfication*. The results of *defuzzyfication* according to the description of Subsection 2.2 at Point 7 (FTS Lee) are in Table 11.

Table 11. **FTS Lee Model Results**

Month	FTS Lee	Month	FTS Lee	Month	FTS Lee	Month	FTS Lee
Jan-17		Apr-18	2.690	Jul-19	2.590	Oct-20	2.757
Feb-17	3.115	May-18	2.890	Aug-19	2.315	Nov-20	2.890
Mar-17	2.890	Jun-18	2.915	Sep-19	2.090	Dec-20	2.870
Apr-17	3.290	Jul-18	2.870	Oct-19	2.315	Jan-21	2.757
May-17	3.840	Aug-18	2.890	Nov-19	2.315	Feb-21	2.590
Jun-17	3.115	Sep-18	2.870	Dec-19	2.590	Mar-21	2.890
Jul-17	2.915	Oct-18	2.915	Jan-20	2.315	Apr-21	2.590
Aug-17	2.890	Nov-18	3.840	Feb-20	2.090	May-21	2.870
Sep-17	3.115	Dec-18	3.340	Mar-20	1.740	Jun-21	3.115
Oct-17	2.915	Jan-19	2.915	Apr-20	1.840	Jul-21	3.890
Nov-17	2.870	Feb-19	2.890	May-20	1.840	Aug-21	4.390
Dec-17	2.915	Mar-19	2.890	Jun-20	1.890	Sep-21	5.890
Jan-18	2.915	Apr-19	2.690	Jul-20	1.740	Oct-21	5.390
Feb-18	2.915	May-19	2.590	Aug-20	2.190	Nov-21	4.590
Mar-18	2.690	Jun-19	2.757	Sep-20	2.190	Dec-21	3.340

3.3 FTS Tsaur Model Calculation

Steps 5 and 6 FTS Tsaur are the same as FTS Lee. and the difference is in step 7.

7) *Defuzzyfication*. The results of *defuzzyfication* according to the description of Subsection 2.2 at Point 7 (FTS Tsaur) are in Table 12.

Table 12. **FTS Tsaur Model Results.**

Month	FTS Tsaur	Month	FTS Tsaur	Month	FTS Tsaur	Month	FTS Tsaur
Jan-17		Apr-18	2.754	Jul-19	2.270	Oct-20	3.207
Feb-17	2.965	May-18	2.986	Aug-19	2.140	Nov-20	2.640
Mar-17	3.088	Jun-18	2.856	Sep-19	2.314	Dec-20	2.670
Apr-17	3.340	Jul-18	2.820	Oct-19	2.475	Jan-21	2.807
May-17	3.740	Aug-18	2.939	Nov-19	2.440	Feb-21	2.690
Jun-17	3.065	Sep-18	2.920	Dec-19	2.263	Mar-21	2.787
Jul-17	2.826	Oct-18	3.070	Jan-20	1.940	Apr-21	2.740
Aug-17	3.041	Nov-18	4.490	Feb-20	1.640	May-21	2.970
Sep-17	3.065	Dec-18	2.540	Mar-20	1.837	Jun-21	3.415
Oct-17	2.870	Jan-19	2.803	Apr-20	1.965	Jul-21	3.990
Nov-17	2.920	Feb-19	2.893	May-20	1.840	Aug-21	4.640
Dec-17	2.924	Mar-19	2.843	Jun-20	1.690	Sep-21	6.640
Jan-18	2.906	Apr-19	2.631	Jul-20	2.171	Oct-21	5.140
Feb-18	2.766	May-19	2.540	Aug-20	2.595	Nov-21	4.190
Mar-18	2.682	Jun-19	2.657	Sep-20	2.540	Dec-21	2.890

Due to the *defuzzyfication* of FTS Tsaur using previous class data and synchronization models, the difference between natural gas futures prices and estimates with the FTS Tsaur approach is smaller than FTS Chen and FTS Lee.

3.4 Plot Comparison

After the model is obtained. the plot comparison between natural gas futures price data with the three fuzzy approaches used, namely FTS Chen, FTS Lee and FTS Tsaur are presented in the following plot in Figure 2.

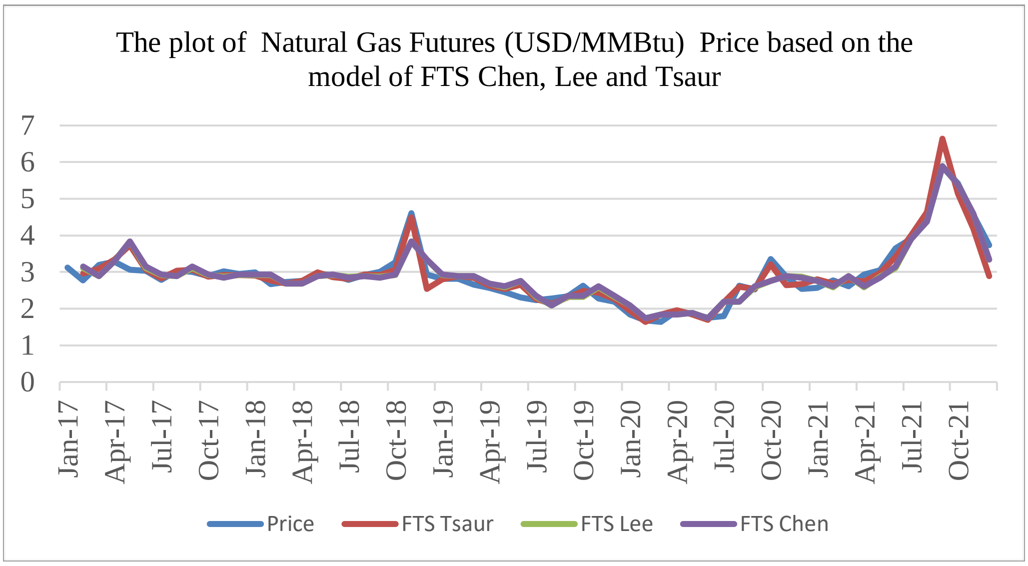

Figure 2. Comparison of Natural Gas Futures Price Plots using FTS Chen, Lee and Tsaur.

Based on the plot in Figure 2, in general, the data plot of the FTS Tsaur model with a minimum data of 1.640 and a maximum data of 6.640, it is almost close to the plot of natural gas futures prices (observation data) which has a minimum data of 1.640 and a maximum data of 5.876, although in some cases where there are data that are slightly away from the maximum value of the FTS Tsaur. The FTS Lee and FTS Chen models have the same minimum and maximum data, and they are 1.740 and 5.890 respectively. However, the FTS Lee plot is closer to the original data plot. This is confirmed by the RMSE, MAE and MAPE values calculated by the formula of (23), (24), (25). respectively. The results of the calculation of the RMSE, MAE and MAPE values are presented in Table 13.

Table 13. Model Accuracy Value by Using RMSE. MAE and MAPE.

	RMSE	MAE	MAPE
FTS Chen	0.264	0.194	7.030%
FTS Lee	0.264	0.191	6.885%
FTS Tsaur	0.229	0.150	5.021%

Based on the MAPE value for the model with the FTS Chen, Lee and Tsaur approach, the three models have very good accuracy because they have a value of <10%, but FTS Tsaur has the smallest error value of MAPE 5.021%, it means that the FTS Tsaur has the best model accuracy and suitability to the data, then it is followed by FTS Lee (6.885%) and FTS Chen (7.030%). This order also applies when compared using the RMSE and MAE accuracy values.

4. CONCLUSION

Natural gas futures prices are time series data with a monthly period so that they can be formed into the FTS model. After modeling natural gas futures prices with a fuzzy approach using the FTS Chen, Lee and Tsaur methods with interval division according to the average-based method, it is found that the three FTS have very good accuracy because their MAPE values are <10 %. In addition, based on the RMSE, MAE and MAPE accuracy values, the approach using the FTS Tsaur method is the best model because it has the smallest error value, followed by FTS Lee and FTS Chen. These results show that the FTS approach to time series data provides a model with very good accuracy, so this method can be used as an alternative in providing flexible and effective time series data estimates.

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