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ANALYSIS OF THE SPRUCE BUDWORM MODEL USING THE HEUN METHOD AND THIRD-ORDER RUNGE-KUTTA

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Abstract. *This study discusses the analysis of the Spruce Budworm model using numerical methods, namely the Heun method and the Third Order Runge-Kutta method. The purpose of this study is to determine the numerical results of the Heun method and the Third Order Runge-Kutta method on the cypress caterpillar model and to determine the comparison of errors from the two methods, namely the Heun method and the Third Order Runge-Kutta method in analyzing the Spruce Budworm model. The results of the study using the Heun method for the initial conditions* $B(t_0) = 2, S(t_0) = 10\ cm, E(t_0) = 2\ cm,$ *at* $t = 5$ *years, for* $h = 0{,}05$, *the result obtained is* $B \approx 3$, $S = 14{,}9058\ cm$ *and* $E = 1{,}0047\ cm$. *For the calculation result of the Spruce Budworm model using the third-order Runge-Kutta method, the result obtained is* $B \approx 3$, $S = 14{,}9057\ cm$, *and* $E = 1{,}0046\ cm$. *while the error of Caterpillar Density* (B) *with the third-order Runge-Kutta method is bigger than the Heun method, while the error of Branch Surface Area* (S) *and the error of Reserve Food* (E) *error using the Heun method bigger than using the third-order Runge-Kutta method.*

Keywords: *Heun, Runge-Kutta, Spruce Budworm Model.*

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1. INTRODUCTION

Mathematics is the language of science, it is possible to describe the field of ecology. Through a mathematical description of an ecological system, it is possible to understand the basic principles of a system and determine its implications [1]. Many events that occur in the field of science can be modeled into mathematical form either in the form of ordinary differential equations or in the form of partial differential equations [2]. Solving differential equations is generally classified into initial value and boundary value problems, depending on the conditions specified at the end point of the domain [3]. The discussion of ordinary differential equations has been a major concern in the history of applied mathematics. Differential equations are widely used in everyday life [4] Differential equations can describe almost any event that changes. Many mathematicians have studied the nature of these differential equations which can be described with precise and concise mathematical expressions [5]. To determine the solution of the differential equation, several analytical methods can be used. However, in certain situations where analytical solutions cannot be used for the problem at hand, which usually occurs in non-linear differential equation problems, numerical methods are needed to obtain approximate solutions [6]. Numerical methods are used to obtain solutions to complex differential equations. By using the help of computer programming, a numerical method is a method that can be used to solve complex problems quickly [7]. The result of the numerical analysis is the result with the approximate value of the exact solution. Error analysis is an important part of numerical analysis because calculation error is a measure of the efficiency of a method [8].

The differential equation of the spruce caterpillar model can be determined by using a numerical method. The spruce caterpillar is an epidemic that has a negative impact on forests, so a lot of research has focused on improving various models and simulations in solving the Spruce Budworm problem [9].The spruce caterpillar (Choristomeura fumiferana) is a very dangerous plague in evergreen forests in North America. Spruce caterpillars will begin to attack the shoots of spruce plants during the larval stage. Adult cypress caterpillars will kill plants within four to five years from the time they first attack the photosynthetic tissue in spruce plants which are very important for cypress growth [10]. The methods that will be used to solve the Spruce Budworm model numerically are the Heun and Runge-Kutta methods. The Heun method is an improved Euler method because the Heun method is a revision of the Euler method, wherein the Heun method the predictor is searched using the Euler method (predictor) and then corrected using a corrector [11]. The advantage of the Runge-Kutta method is that in its calculations there is no need to calculate derivatives and get accurate results, but in its calculations, the higher the order used, the more iterations used [12]. In the use of the Runge-Kutta method of order 3, three total values of a function are required at each step interval [13].

The main objective of this research is to find more accurate results in the numerical solution of ordinary differential equations. The errors that often occur are cutting errors and rounding errors [14]. Withholding errors in numerical analysis arise when estimates are used to estimate some quantity. Rounding errors stem from the fact that computers can only represent a finite number of significant figures. Thus, the numbers cannot be represented accurately in the computer's memory. The accuracy of the solution will depend on how small the size of the steps taken is the value of h [15]. A numerical method is said to be good if the numerical solution is close to the exact solution. In a previous study conducted by Murad Hossen, he compared the error in the Heun method and the Runge-Kutta method. In the results of this study, it was concluded that the Runge-Kutta method gave more accurate results than using the Heun method [16]. In this study, we will also compare two numerical methods to solve an equation, namely the Heun method and the Third Order Runge-Kutta method but with a different model, namely the spruce caterpillar model. From the results of this study, it will be known the best numerical method from the Heun method and the Third Order Runge-Kutta method which has a high level of accuracy or produces a smaller error for completing the Spruce Budworm model.

2. RESEARCH METHODS

The type of research used in this research is library research. This research procedure includes: Determining the solution of the Spruce Budworm model using the Heun method, determining the completion of the Spruce Budworm model using the Runge-Kutta method of order Three, and Comparing the error from the completion of the Spruce Budworm model using the Heun method and the third-order Runge-Kutta method; and the last is input the results of the completion of the Spruce Budworm Model based on the smallest error value of the Heun method and the third-order Runge-Kutta method.

The steps of the Heun method in completing the Spruce Budworm model are: Determine the time interval to be used, namely $t = 0$ to $t = 5$, the value of $h = 0.05$, and the value of $n = 100$. Next, calculate the value of the predictor using the Euler method and then correct it using the corrector using the Heun method, while the formula used is as follows:

Predictor

$$\begin{aligned} B_{i+1}^{(p)} &= B_i + f(B_i; S_i; E_i; t_i)\, h \\ S_{i+1}^{(p)} &= S_i + g(B_i; S_i; E_i; t_i) h \\ E_{i+1}^{(p)} &= E_i + j(B_i; S_i; E_i; t_i)\, h \end{aligned} \tag{1}$$

corrector

$$\boldsymbol{B_{i+1} = B_i + \left[f(B_i; S_i; E_i; t_i) + f\left(B_{i+1}^{(p)}; S_{i+1}^{(p)}; E_{i+1}^{(p)}\right)\right]\frac{h}{2}}$$
$$\boldsymbol{S_{i+1} = S_i + \left[g(B_i; S_i; E_i; t_i) + g\left(B_{i+1}^{(p)}; S_{i+1}^{(p)}; E_{i+1}^{(p)}\right)\right]\frac{h}{2}}$$
$$\boldsymbol{E_{i+1} = E_i + \left[j(B_i; S_i; E_i; t_i) + j\left(B_{i+1}^{(p)}; S_{i+1}^{(p)}; E_{i+1}^{(p)}\right)\right]\frac{h}{2}}$$

The steps of the third-order Runge-Kutta method in completing the cypress caterpillar model are: Determining the time interval to be used, namely $t = 0$ to $t = 5$, $h = 0.05$, and $n = 100$. Next, calculate the value of the three evaluation functions of the Runge-Kutta formula of order Three, namely the variable $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{a_3}$, $\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}$, $\boldsymbol{c_1}$, $\boldsymbol{c_2}$ and $\boldsymbol{c_3}$. The formula used is as follows:

$$\begin{aligned} a_1 &= f(B_i; S_i; E_i; t_i)\, h \\ b_1 &= g(B_i; S_i; E_i; t_i)\, h \\ c_1 &= j(B_i; S_i; E_i; t_i)\, h \\ a_2 &= f\left(B_i + \tfrac{1}{2}a_1; S_i + \tfrac{1}{2}b_1; E_i + \tfrac{1}{2}c_1; t_i + \tfrac{1}{2}h\right) h\, h \\ b_2 &= g\left(B_i + \tfrac{1}{2}a_1; S_i + \tfrac{1}{2}b_1; E_i + \tfrac{1}{2}c_1; t_i + \tfrac{1}{2}h\right) h \\ c_2 &= j\left(B_i + \tfrac{1}{2}a_1; S_i + \tfrac{1}{2}b_1; E_i + \tfrac{1}{2}m_1; c_i + \tfrac{1}{2}h\right) h \\ a_3 &= f(B_i - a_1 + 2a_2; S_i - b_1 + 2b_2; E_i - c_1 + 2c_2; t_i + h)\, h \\ b_3 &= g(B_i - a_1 + 2a_2; S_i - b_1 + 2b_2; E_i - c_1 + 2c_2; t_i + h)\, h \\ c_3 &= j(B_i - a_1 + 2a_2; S_i - b_1 + 2b_2; E_i - c_1 + 2c_2; t_i + h)\, h \end{aligned} \tag{2}$$

The values of B_{i+1}, S_{i+1} and E_{i+1} are obtained using the values a_1, a_2, a_3, b_1, b_2, b_3 , c_1 , c_2 and c_3, i.e.

$$\begin{aligned} B_{i+1} &= B_i + \frac{(a_1+4a_2+a_3)}{6} \\ S_{i+1} &= S_i + \frac{(b_1+4b_2+b_3)}{6} \\ E_{i+1} &= E_i + \frac{(c_1+4c_2+c_3)}{6} \end{aligned} \tag{3}$$

3. RESULTS AND DISCUSSION

The spruce budworm model is [9]:

$$f(x) = \frac{dB}{dt} = r_B B\left(1 - \frac{B}{K_B S}\right) - \beta\frac{B^2}{(\alpha S)^2 + B^2}$$

$$g(x) = \frac{dS}{dt} = r_S S\left(1 - \frac{S}{K_S \frac{E}{K_E}}\right) \tag{4}$$
$$j(x) = \frac{dE}{dt} = r_E E\left(1 - \frac{E}{K_E}\right) - p\frac{B}{S}$$

Indicators value and initial value were used based on [17].

Table 1. Indicators and value indicators

Indicator	Indicator Value
$\boldsymbol{r_B}$	1,52/year
$\boldsymbol{r_S}$	0,095/year
$\boldsymbol{r_E}$	0,92/year
$\boldsymbol{\alpha}$	1,11 Larvae/Branch
$\boldsymbol{\beta}$	118 Larvae/year
$\boldsymbol{K_B}$	355 Larvae/Branch
$\boldsymbol{K_S}$	70 Branch/acre
$\boldsymbol{K_E}$	1,0/year
$\boldsymbol{p}$	0,00195/*year*

Data source: (Subadar, 2013)

with an initials value,

variables	Initial values
$\boldsymbol{B(t_0)}$	2 tails
$\boldsymbol{S(t_0)}$	10 cm
$\boldsymbol{E(t_0)}$	2 cm
$\boldsymbol{t_0}$	0 year

Data source: (Subadar, 2013)

using the indicator values (table 1), equation (1) becomes,

$$f(x) = \frac{dB}{dt} = 1{,}52\, B\left(1 - \frac{B}{355\, S}\right) - 118\frac{B^2}{(1{,}11\, S)^2 + B^2}$$
$$g(x) = \frac{dS}{dt} = 0{,}095\, S\left(1 - \frac{S}{70\frac{E}{1{,}0}}\right) \tag{5}$$
$$j(x) = \frac{dE}{dt} = 0{,}92\, E\left(1 - \frac{E}{1{,}0}\right) - 0{,}00195\frac{B}{S}$$

3.1. The Heun Method for the Spruce Budworm Model

Based on equations (3a) and (3b), and by using the initial values in table 1 and table 2, so the solution of the spruce budworm model is obtained. Namely as follows:

Table 2. The Heun Method Solution on the Spruce Budworm Model

$\boldsymbol{i}$	$\boldsymbol{t_i}$	$\boldsymbol{B_i}$	$\boldsymbol{S_i}$	$\boldsymbol{E_i}$
0	0,00	2,0000	10,0000	2,0000
1	0,05	1,9689	10,0441	1,9141

2	0,10	1,9422	10,0883	1,8387
3	0,15	1,9193	10,1324	1,7720
⋮	⋮	⋮	⋮	⋮
100	**5,00**	**3,2839**	**14,9058**	**1,0047**

Table 3 provides information that there was an increase in the number of pine caterpillars from $t = 0$ years to $t = 5$ years by 3.2839 or in other words, the density of Spruce budworms was 3 heads. Meanwhile, the Branch Surface Area $S(t)$ also increased by 14.9058 cm. As for the Reserve Food $E(t)$ there was a decrease of up to 1.0047 cm from 2.0000 cm. These conditions are described in the following Figure 1.

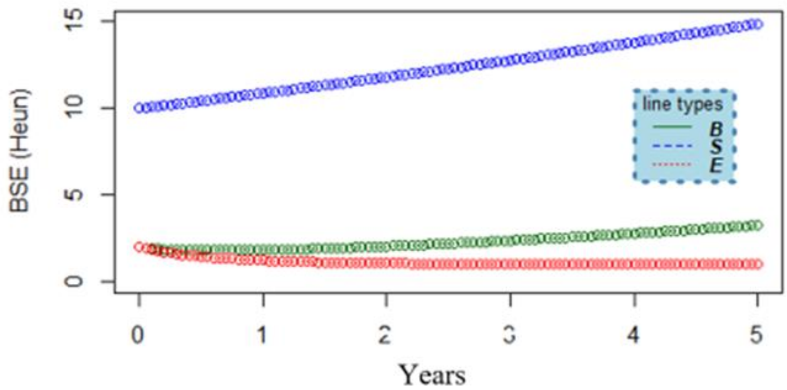

Figure 1. Graph of the Heun Method on the Spruce Budworms Model

3.2. The Third Order Runge-Kutta Method for the Spruce Budworm

Based on equations (4) and (5), as well as using the initial values in table 1 and table 2, the solution of the Spruce Budworm model is obtained. Namely as follows;

Table 3. The Heun Method Solution on the Spruce Budworm Model

i	t_i	B_i	S_i	E_i
0	0,00	2,0000	10,0000	2,0000
1	0,05	1,9688	10,0441	1,9139
2	0,10	1,9420	10,0882	1,8384
3	0,15	1,9191	10,1324	1,7716
⋮	⋮	⋮	⋮	⋮
100	5,00	3,2839	14,9057	1,0046

Table 4 provides information that there was an increase in the number of pine caterpillars from t=0 years to at t=5 years by 3.2839 or in other words, the density of caterpillars was 3 heads. Meanwhile, the Branch Surface Area S(t) also increased by 14.9057 cm. As for the Reserve Food E(t) there was a decrease of up to 1.0046 cm from 2.0000 cm. These conditions are described in the following figure.

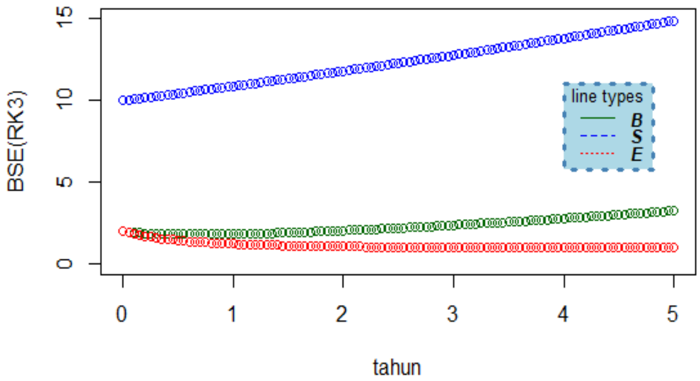

Figure 2. Graph of The Third Order Runge-Kutta Method on the Spruce Budworms Model

3.3. Error Analysis

Based on the calculation results of the Heun method and the Third Order Runge-Kutta method, the errors of the two methods can be compared. By using the approximation relative error formula, the following error is obtained:

Table 4. The Approximation Relative Error Formula

No	Error *B*		Error *S*		Error *E*	
	Heun	RK3	Heun	RK3	Heun	RK3
1	**0,01580653**	0,0158406830	**0,004392061**	0,004392601	**0,0448581146**	0,0449708145
2	**0,01375970**	0,0137860760	**0,004375682**	0,004376128	**0,0410180794**	0,0411007784
3	**0,01192061**	0,0119408444	**0,004359831**	0,004360198	**0,0376430268**	0,0377035363
4	**0,01026313**	0,0102784841	**0,004344489**	0,004344789	**0,0346565132**	0,0347004142
⋮	⋮	⋮	⋮	⋮	⋮	⋮
99	**0,007834610**	0,0078346892	0,003742199	**0,003742195**	0,0002520638	**0,0002514436**
100	**0,007830407**	0,0078304810	0,003738210	**0,003738206**	0,0002407711	**0,0002401746**

Based on table 5, the error value of the Runge-Kutta method of Order Three is greater than the error value of the Heun method for determining the density of cypress caterpillars (B) in iterations 1-10 and 15-100 iterations, so for the equation of Caterpillar Density (B), the method is more best used is Heun's method. While the error for the branch surface area or S in iterations 1-13, the third-order Runge-Kutta method error value is greater than the error value of the Heun method. In iterations 14 -100, the error value of the Heun method is greater than the error value of the third-order Runge-Kutta method, so for the S Branch Surface Area equation, it is said that the better method to use is the Third Order Runge-Kutta method. The error value for food reserves or E in iterations 1-10, the error value of the Runge-Kutta method of order Three is greater than the error value of the Heun method. Whereas in iterations 11 – 100, the error value of the Heun method is greater than the error of the Third Order Runge-Kutta method, so for the Equation of Reserve Food the better method to use is the Third Order Runge-Kutta method.

4. CONCLUSIONS

The conclusion of this study is a numerical solution using the Heun method on the Spruce Budworm model until now t=5 years, the density of caterpillars was 3 tails, branch surface area was 14.9058 cm and food reserves were 1.0047 cm. While the Runge-Kutta method of Order Three with t=5 years obtained a density of 3 caterpillars, a branch surface area of 14.9057 cm, and a food reserve of 1.0046 cm. The third-order Runge-Kutta method error value is greater than the Heun method, while the Branch Surface Area (S) and Food Reserve (E) errors with the Heun method are larger than the Third-order Runge-Kutta method.

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