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ANALYSIS OF BANKING DEPOSIT COST IN THE DYNAMICS OF LOAN: BIFURCATION AND CHAOS PERSPECTIVES

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Abstract. *A dynamic model of banking loans based on the gradient adjustment process is presented. The amount of loan that will be channeled in the future depends on the sign of marginal profit of loan. In this paper, we study the deposit cost in the dynamics of a bank's loan using the bifurcation theory. The analysis shows that the deposit cost can affect the stability of loan equilibrium. If the deposit cost is too high, then the loan equilibrium can lose its stability through transcritical bifurcation. Meanwhile, if the deposit cost is too low, then the loan equilibrium may lose its stability via flip bifurcation and road to chaos. The loan equilibrium is stable if the deposit cost is in between the bifurcation values. These findings are confirmed by the numerical simulations. In addition, we present the graph of Lyapunov exponent to see the existence of chaos and the graph of chaotic loan that is sensitive to the initial condition.*

Keywords: *chaos, deposit cost, flip bifurcation, transcritical bifurcation.*

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1. INTRODUCTION

Bank operating costs include all costs incurred to fund the bank's operations. They have a significant role in the bank's financial preservation and they must be maintained so that the bank's finances remain healthy. Like other financial or company institutions, banks will love to have minimum costs, but too low costs can make them have risk-seeking behavior in running the company operations. On the other hand, too high costs can cause the activities cannot run effectively because of a big reduction in the profit. Banks as the financial institution will manage their operations more effectively to cut costs and maximize profit the more liberated they are from various restrictions in conducting business [1]. Without proper considerations and policies, the operating costs may swell, negatively impacting the financial condition. To further reduce interest rates and concentrate on raising low-cost funds, such as current accounts and savings accounts, is one strategy used by banks to reduce operational costs. In order to keep operational costs under control, it is necessary to develop efficiency measures by increasing the use of technology in business processes.

The main financial aspects of banks are deposits and loans. Many researchers assumed the bank's operating costs are formed as a linear or nonlinear function of deposit and loan [2], [3], [4], [5], [6]. Banks with a lower incidence of bad loans may have operational costs beyond the optimal minimum and may thus be branded as inefficient since good loan screening and monitoring incur greater expenditures [7]. The openness of the loan market and bank cost effectiveness are closely related, particularly, the performance of the banks is better in terms of cost efficiency the more free the loan market [8]. Based on the consequence of risk-taking, the regulatory enforcement's impact on loan costs will make the penalized banks act more responsibly by offering cheaper loans to less hazardous businesses [9]. In the view of stickiness level, banking cost among non-financial companies rises as a result of banking competition by escalating competitive pressure and enhancing loan accessibility [10]. When a company, like a bank, is up against the competition, it would be simple to raise costs when sales were up, but tough to reduce spending when sales were down [11].

A number of studies have been conducted to address the relationships between banking costs and other financial factors. By specifically examining the connection between the cost of deposits and bank capital buffers, [12] examined the impact of market discipline. [13] provided evidence that the cost of bank loans is influenced by carbon risk through two fundamental channels: a company's profitability and earnings volatility. For the borrowers, debt maturity dispersion is essential in reducing rollover risk, which in turn lowers loan costs [14]. Under ratings-contingent capital regulation, changes in borrowers' credit ratings may change the risk weights on bank loans, which might have an immediate effect on the capital needs of lending banks and the cost of financial intermediation of the bank [15]. Examining the impact of banking regulations on commercial banks' ideal conduct, [16] paid particular attention to the regulatory implications of the banking cost structure. In the study on shadow banking, [17] argued that shadow banks assist in identifying ways to get over the stringent restrictions on bank operations and product selection, enhancing banks' capacity to reduce costs. Related to financial instability, [18] showed that financial instability reduces the cost of bank debt, irrespective of the borrower type.

In this paper, we construct a dynamic model of a banking loan consisting of the bank's operational cost in the case of deposit cost. We continue the works of authors in [19], where they analyze the cost of loan's effects on the banking loan dynamics. The model is based on the gradient adjustment process, where the sign of the loan's marginal profit determines how much money will be loaned in the future. Many papers studied dynamic models based on the gradient adjustment process to analyze various aspects that appeared in oligopoly markets in economics and banking [20], [21], [22], [5], [6], and they used bifurcation theory to examine the stability conditions of the models. In this research, we also use bifurcation theory to investigate the role of the deposit cost in loan dynamics. The results of the analysis demonstrate that loan equilibrium stability can be impacted by the deposit cost. When the deposit cost is too large, the loan equilibrium might lose its stability via transcritical bifurcation and becoming zero. In the meantime, if the deposit cost is sufficiently low, the loan equilibrium may lose its stability via flip bifurcation and can produce chaotic behavior.

2. RESEARCH METHODS

We follow [23], [24], [19], to model the loan dynamics of a bank but with a simpler balance sheet structure as in [22]. Suppose a bank has a balance sheet component: deposit ($\boldsymbol{D}$), equity ($\boldsymbol{E}$), and loan ($\boldsymbol{L}$). The bank's equity will be bounded below by the capital regulation from the central bank. In the reality, as presented in [23], the banking data shows that the ratio of equity respect to loan can be assumed as constant. In other words, we can write

$$\frac{E}{L} = \kappa$$

for some $0 < \kappa < 1$. In this model, the deposit acts as the balancing variable. Therefore, we have

$$D = L - E = (1 - \kappa)L$$

Suppose the period in the model is a discrete time $t, t = 0, 1, 2, \cdots$ The dynamics of loans are modeled following the gradient adjustment process [25], [22]. The amount of loan channeled by the bank in the next period (L_{t+1}) depends on the marginal profit of loan ($\partial \pi_t / \partial L_t$). The model is given below

$$L_{t+1} = L_t + \alpha_L L_t \frac{\partial \pi_t}{\partial L_t} \tag{1}$$

where α_L is called the speed of adjustment parameter, $\alpha_L > 0$.

The bank's profit (π) is obtained by calculating the interest of the loan ($r_L L$) minus the expense of deposit ($r_D D$) and equity ($r_E E$) and the bank's costs (C). Following the assumptions of the Monti-Klein model [26], [27], the interest rates of loans and deposits are defined by

$$r_L = a_L - b_L L$$

and

$$r_D = a_D + b_D D$$

where $a_L, b_L, a_D, b_D > 0$. The expense of equity r_E is assumed to be a constant. The bank's costs contain the cost of the deposit and the cost of the loan as written below

$$C = c_D D + c_L L$$

where $0 < c_D, c_L < 1$. The deposit cost parameter c_D, which is also called as the marginal cost of deposit, is the main topic of analysis in this paper.

The profit at time t is calculated by

$$\begin{aligned}
\pi_t &= r_L L_t - r_D D_t - r_E E_t - C_t \\
&= (a_L - b_L L_t)L_t - (a_D + b_D D_t)D_t - r_E \kappa L_t - (c_D D_t + c_L L_t) \\
&= (a_L - b_L L_t)L_t - (a_D + b_D(1-\kappa)L_t)(1-\kappa)L_t - r_E \kappa L_t - (c_D(1-\kappa)L_t + c_L L_t) \\
&= (a_L - [r_E \kappa + c_L + (a_D + c_D)(1-\kappa)])L_t - \left[b_L + b_D(1-\kappa)^2\right]L_t^2
\end{aligned}$$

and then we have the marginal profit of the loan

$$\frac{\partial \pi_t}{\partial L_t} = a_L - [r_E \kappa + c_L + (a_D + c_D)(1-\kappa)] - 2\left[b_L + b_D(1-\kappa)^2\right]L_t \tag{2}$$

By substituting (2) into (1), we get

$$L_{t+1} = L_t + \alpha_L L_t\left(a_L - [r_E \kappa + c_L + (a_D + c_D)(1-\kappa)] - 2\left[b_L + b_D(1-\kappa)^2\right]L_t\right) \tag{3}$$

2.1 Equilibrium Analysis

The equilibrium of the loan in the model (3) can be obtained by setting $L_{t+1} = L_t$. The result is there exist two equilibrium points $L^*_{(1)} = 0$ and

$$L^*_{(2)} = \frac{a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]}{2[b_L + b_D(1-\kappa)^2]}$$

The positivity condition of the equilibrium $L^*_{(2)}$ is

$$a_L > [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]$$

Suppose the model (3) is rewritten as $L_{t+1} = f(L_t)$. The model is a one-dimensional map. As mentioned in [28], the equilibrium of the map is stable if $|f'(L^*)| < 1$. The stability of the equilibriums $L^*_{(1)}$ and $L^*_{(2)}$ are given in the following theorems. Before that, the first derivative of f is given by

$$f'(L_t) = 1 + \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) - 4\alpha_L[b_L + b_D(1-\kappa)^2]L_t$$

Theorem 1

The equilibrium of the loan $L^*_{(1)}$ is unstable.

Proof. We see that

$$f'\left(L^*_{(1)}\right) = 1 + \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) > 1$$

Thus $L^*_{(1)}$ is unstable. □

Theorem 2

The equilibrium of loan $L^*_{(2)}$ is stable if $c_D > \frac{1}{1-\kappa}\left(a_L - \left[\frac{2}{\alpha_L} + r_E\kappa + c_L + a_D(1-\kappa)\right]\right)$.

Proof. We have

$$\begin{aligned} f'\left(L^*_{(2)}\right) &= 1 + \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) - 2\alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) \\ &= 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) < 1 \end{aligned}$$

On the other hand, we obtain

$$f'\left(L^*_{(2)}\right) = 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) > -1$$

requiring $c_D > (a_L - [2/\alpha_L + r_E\kappa + c_L + a_D(1-\kappa)])/(1-\kappa)$. Therefore $L^*_{(2)}$ is stable. □

2.2 Bifurcation Analysis

We follow the Jury stability criterion in [29] for a one-dimensional map. The equilibrium $L^*_{(2)}$ will lose its stability through transcritical bifurcation when $f'\left(L^*_{(2)}\right) = 1$. On the other hand, $L^*_{(2)}$ will lose its stability through flip bifurcation when $f'\left(L^*_{(2)}\right) = -1$. The focus of this paper is to analyze the parameter of deposit cost c_D. Therefore, the parameter c_D will act as the bifurcation parameter. By doing simple calculations, we have the following theorems.

Theorem 3

The equilibrium of the loan $L^*_{(2)}$ may lose its stability through transcritical bifurcation when $c_D = c_D^T$, where

$$c_D^T = \frac{a_L - [r_E\kappa + c_L + a_D(1-\kappa)]}{1-\kappa}$$

Proof. The proof is directly obtained by solving the equation

$$f'(L^*_{(2)}) = 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) = 1$$

for parameter c_D. The solution is $c_D = (a_L - [r_E\kappa + c_L + a_D(1-\kappa)])/(1-\kappa)$. □

Theorem 4

The equilibrium of the loan $L^*_{(2)}$ may lose its stability through flip bifurcation when $c_D = c_D^F$, where

$$c_D^F = \frac{1}{1-\kappa}\left(a_L - \left[\frac{2}{\alpha_L} + r_E\kappa + c_L + a_D(1-\kappa)\right]\right)$$

Proof. Similar to the proof of Theorem 3, the proof of this theorem is directly obtained by solving the equation

$$f'(L^*_{(2)}) = 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) = -1$$

for parameter c_D. The solution is $c_D = (a_L - [2/\alpha_L + r_E\kappa + c_L + a_D(1-\kappa)])/(1-\kappa)$. □

Here, we can observe that $c_D^F < c_D^T$. In other words, the parameter of deposit cost must be in between the flip and transcritical bifurcation values so that the equilibrium of the loan can stay stable. From Theorem 4, we have the following remark about the existence of the flip bifurcation values that are implied by the condition of the speed of adjustment parameter.

Remark 5

The flip bifurcation value $c_D^F = (a_L - [2/\alpha_L + r_E\kappa + c_L + a_D(1-\kappa)])/(1-\kappa)$ *exists* ($0 < c_D^F < 1$) *if*

$$\frac{2}{a_L - [r_E\kappa + c_L + a_D(1-\kappa)]} < \alpha_L < \frac{2}{a_L - [r_E\kappa + c_L + (a_D + 1)(1-\kappa)]}$$

Proof. The proof is directly obtained by solving the inequality

$$0 < \frac{1}{1-\kappa}\left(a_L - \left[\frac{2}{\alpha_L} + r_E\kappa + c_L + a_D(1-\kappa)\right]\right) < 1$$

for parameter α_L.

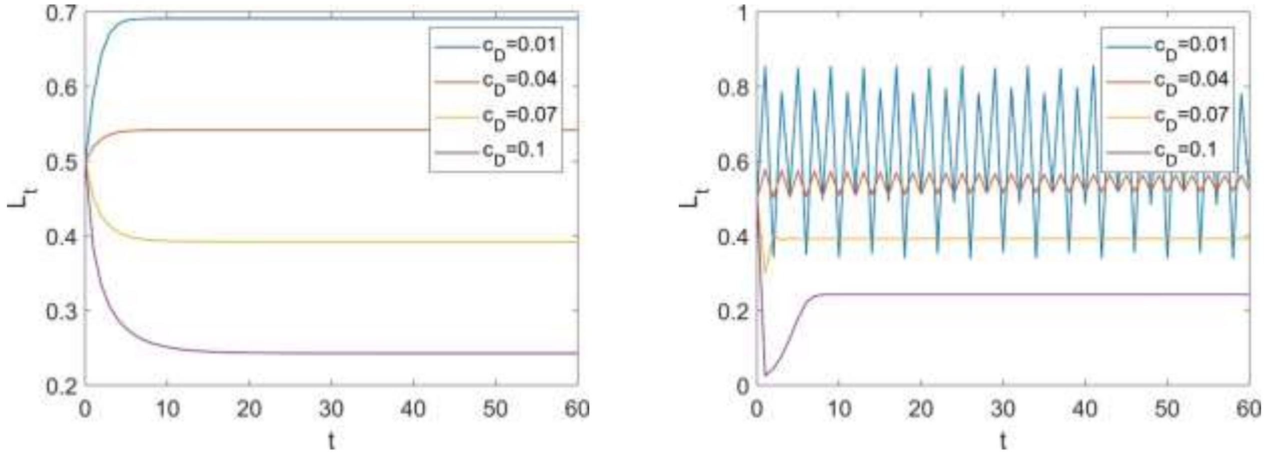
3. RESULTS AND DISCUSSION

We perform some numerical simulations to visualize and also to confirm the findings in the previous section. The simulations use parameters' value as presented in Table 1. The reason for choosing those parameters' values for the need of simulations only, but they still fulfill the positivity condition of equilibrium i.e. $a_L > [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]$. From Table 1, we can calculate the bifurcation values of deposit and loan costs, that is $c_D^F = 0.04$ and $c_D^T = 0.1487$.

Table 1. Parameters' value used in simulations

Parameter	Value
a_L	0.2
b_L	0.05
a_D	0.01
b_D	0.05
r_E	0.05
κ	0.08
c_D	0.05
c_L	0.05
α_L	5 and 20
L_0	0.5

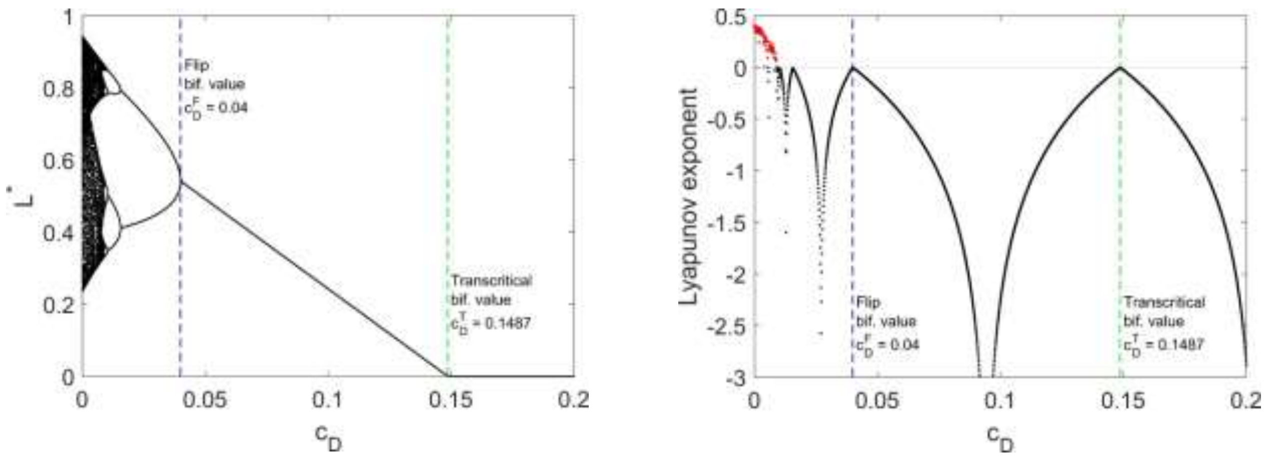
First, we want to see the trajectory of the loan L_t versus time in accordance with changes in the deposit cost parameter c_D. The graphs are presented in Figure 1. Figure 1a shows loan trajectories that all are convergent. This can be obtained by using small value of $\alpha_L = 5$. The lower value of c_D makes the loan equilibrium become larger. Meanwhile, in Figure 1b, when the value of α_L is quite bigger, the trajectory of the loan fluctuates as the parameter c_D goes lower.



(a) (b)

Figure 1. Graphs of the Loan L_t versus Time t for Various Values of the Deposit Cost Parameter c_D. Panel, (a). is for the Case $\alpha_L = 5$ and Panel, (b). is for the Case $\alpha_L = 20$.

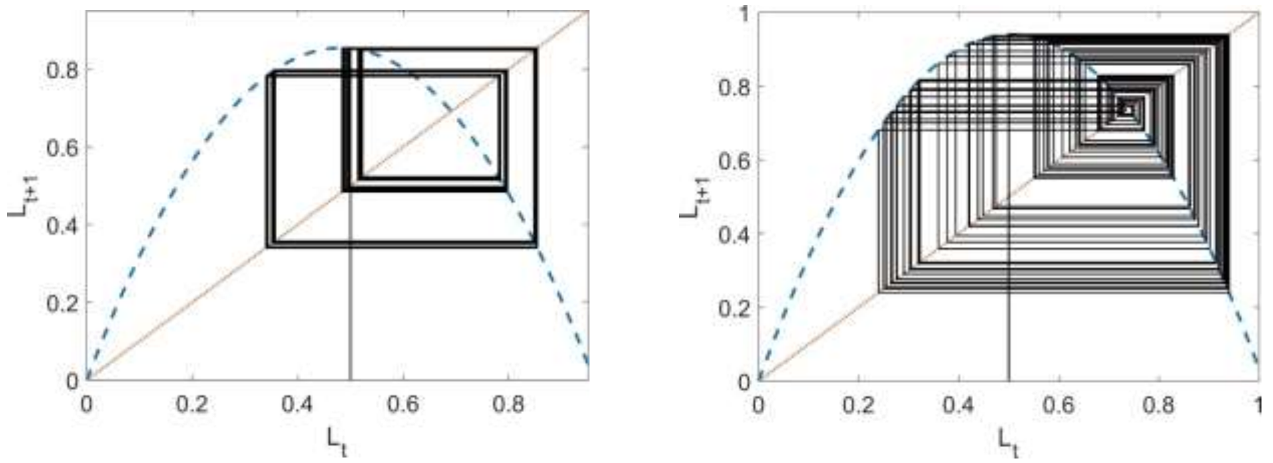
The use of a bifurcation diagram for analyzing the effect of changes in a parameter on the dynamics of a map can give rich interpretations, for example when the map is stable or not and when it can cause chaotic behavior. In Figure 2a, we present a bifurcation diagram of the deposit cost parameter c_D. From Figure 2a we can see that the loan equilibrium is zero when the deposit cost parameter is greater than the transcritical bifurcation value or $c_D > c_D^T$, and we have a stable and positive loan equilibrium when the deposit cost parameter is in between the transcritical and flip bifurcation values or $c_D^F < c_D < c_D^T$. At this interval, the loan equilibrium increases while the parameter c_D decreases. Starting from $c_D < c_D^F$, the loan equilibrium produces period-doubling (2-period, 4-period, and so on), and this leads to chaos. In Figure 2b, we depict the graph of Lyapunov exponent respected to Figure 2a. We plot the graph of Lyapunov exponent with black dots if the graph does not exceed zero, and with red dots when the graph is greater than zero. The positive Lyapunov exponent means the dynamics of the loan become chaotic.



(a) (b)

Figure 2. (a) Bifurcation Diagram of the Deposit Cost Parameter c_D and (b) the Respective Lyapunov Exponent. The Simulations Use $\alpha_L = 20$.

It is always fascinating to see how a map behaves when its dynamics become unstable, such as when they become periodic or even chaotic. The map's trajectory is the most straightforward visualization. There, we can observe the dynamics of the trajectory in real-time as it moves toward equilibrium. The cobweb diagram of the map is another technique for investigating qualitative behavior. The plot (L_t, L_{t+1}) associated cobweb diagram for the case of $c_D = 0.01$ is given in Figure 3b. The black trajectory line only crosses the dashed-blue curve eight times when t is big, as can be seen in Figure 3b. To put it another way, the map is moving toward an 8-period cycle. Figure 3b shows the cobweb diagram for the case of $c_D = 0.0005$. The black trajectory line frequently crosses the dashed-blue curve in many places when t is big. Or, to put it another way, the map is chaos.



(a) (b)

Figure 3. Cobweb Diagram of (L_t, L_{t+1}) when the Deposit Cost Parameter (a) $c_D = 0.01$ and (b) $c_D = 0.0005$. The Simulations Use $\alpha_L = 20$.

An example of the chaotic behavior of the loan map is given in Figure 3 for the parameter $c_D = 0.0005$. In the figure, we plot two graphs of the loan map with slightly different initial values. The blue graph uses initial value $L_0 = 0.5$, and the red graph uses initial value $L_0 = 0.50001$. In the figure, it can be seen that both blue and red graphs are similar at the beginning period of time and then they separate and produce distinctive paths.

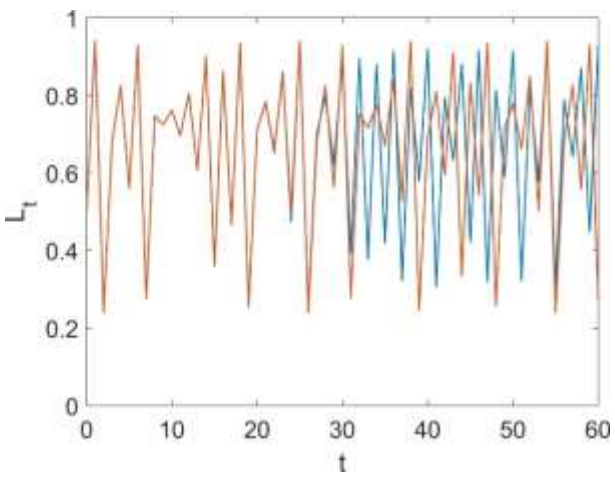


Figure 4. Sensitivity dependence of the chaotic loan dynamics on the initial condition for deposit cost parameter $c_D = 0.0005$. The simulation uses $\alpha_L = 20$

4. CONCLUSIONS

The number of a bank's costs that depends on the amount of deposit always be an operational problem that needs to be managed effectively and optimally. Such problem is assessed in this paper with the objective to see their effects on the dynamics of the loan. Smaller deposit cost produces higher loan equilibrium. The results in this paper show that the deposit cost must be not too high or too low. Too high a deposit cost will cause the loan to vanish in the future. Meanwhile, too low deposit cost may cause the loan to be unstable and lead to chaos. The model in this paper is a very simple model of one bank that has a simple balance sheet structure. Thus, the model can be generated into a more general model which consists of more balance sheet components or to address some banking policies such as macroprudential policy that focuses on controlling the growth of banking loans [30], [31]. Other future works can also be done by implementing the model on banking data such as in [32], [33].

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