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THE ROLE OF COST OF LOAN IN BANKING LOAN DYNAMICS: BIFURCATION AND CHAOS ANALYSIS

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Abstract. *The gradient adjustment process is used to create a dynamic model of banking loan. The sign of the loan's marginal profit determines how much money will be loaned in the future. In this research, using bifurcation theory, we investigate the cost of loan in the dynamics of a bank's loan. The results of the analysis indicate that the stability of the loan equilibrium might be impacted by the cost of loan. Loan equilibrium may become unstable through transcritical bifurcation if the cost of the loan is sufficiently high. The loan equilibrium may become unstable through flip bifurcation and path to chaos, however, if the cost of loan is too low. If the cost of loan lies between the bifurcation values, the loan equilibrium is stable. The numerical simulations back up these conclusions. Additionally, we display the Lyapunov exponent graph, which shows the presence of chaos, and the chaotic loan graph, which is sensitive to the initial condition.*

Keywords: *chaos, flip bifurcation, loan cost, transcritical bifurcation,.*

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1. INTRODUCTION

All expenses incurred to support the bank's operations are included in bank operating costs. They play a big part in the bank's ability to preserve its finances, thus it is important to keep them in good shape. Like other financial or business entities, banks prefer to have low costs, but these costs should not be so low that they engage in risk-taking behavior when conducting business. On the other side, excessive costs can prevent operations from being carried out successfully due to a significant decline in profit. The more emancipated banks are from various business-related restraints, the better they can manage their operations to minimize costs and increase profit [1]. Without the right considerations and guidelines, running costs could increase and have a negative effect on the company's financial situation. One tactic used by banks to cut operational expenses is to further lower interest rates and focus on raising low-cost funds, like those from current accounts and savings accounts. It is vital to develop efficiency measures through expanding the use of technology in corporate processes in order to keep operating costs under control.

Deposits and loans make up the majority of a bank's finances. Numerous scholars used the assumption that the bank's operating costs were a linear or nonlinear function of deposits and loans [2], [3], [4], [5], [6]. Since competent loan screening and monitoring require higher expenditures, banks with lower incidences of bad loans may have operational costs beyond the optimum minimum and be viewed as being inefficient [7]. Bank performance in terms of cost efficiency is greater the more free the loan market is, and there is a strong correlation between loan market openness and bank cost effectiveness [8]. Based on the costs associated with risk-taking, the regulatory enforcement will force the penalized banks to act more responsibly by providing less expensive loans to less risky companies [9]. According to the stickiness level theory, banking costs for non-financial businesses increase as a result of banking rivalry since it increases competitive pressure and makes loans more accessible [10]. It would be easy to increase costs while sales were up but difficult to reduce spending when sales were down when a corporation, like a bank, was up against competitors [11].

Studies have been done to examine the connections between the cost of banking and other financial aspects. [12] focused on the influence of market discipline by analyzing the relationship between the cost of deposits and bank capital buffers. [13] offered proof that carbon risk affects bank lending rates via two essential channels: a company's profitability and earnings volatility. Debt maturity dispersion is crucial for the borrowers in lowering rollover risk, which in turn lowers loan costs [14]. Changes in borrowers' credit ratings may alter the risk weights on bank loans under regulations requiring ratings-contingent capital, which may have an immediate impact on the capital requirements of lending banks and the cost of the bank's financial intermediation [15]. Paper [16] examined the effects of banking laws on the ethical behavior of commercial banks with a focus on the regulatory ramifications of the banking cost structure. According to the study on shadow banking, [17] the ability of banks to save expenses is improved by using shadow banks to help find ways around the rigorous limitations on their operations and product choices. Regarding financial instability, [18] demonstrated that regardless of the type of borrower, financial instability lowers the cost of bank debt.

In this study, we build a dynamic model of banking loan that includes the operational cost of the bank in terms of loan cost. The gradient adjustment process, on which the model is built, calculates how much money will be lent in the future based on the sign of the loan's marginal profit. Numerous works [19], [20], [21], [5], [6] investigated dynamic models based on the gradient adjustment process to assess various elements of oligopoly markets in banking and economics, and they employed bifurcation theory to look at the stability criteria of the models. On this study, the influence of loan costs in loan dynamics is also examined using the bifurcation theory. The analysis's findings show that the stability of the loan equilibrium is susceptible to the loan's cost. The loan equilibrium may lose stability due to transcritical bifurcation and become zero if the cost of the loan is too high. In the meanwhile, if the cost of the loan is sufficiently low, the loan equilibrium may flip bifurcate and lose stability, leading to chaotic behavior.

2. RESEARCH METHOD

2.1 Dynamic Model

To model the loan dynamics of a bank, we use [22], [23], but with a simpler balance sheet structure than in [21]. Consider a bank's balance sheet item: deposit ($\boldsymbol{D}$), equity ($\boldsymbol{E}$), and loan ($\boldsymbol{L}$). The capital rule from the central bank will set lower boundaries for the bank's equity. In practice, as shown in [22], the banking data demonstrates that the equity to loan ratio can be taken to remain constant. Therefore, we are able to write $\boldsymbol{E/L = \kappa}$, for some $\boldsymbol{0 < \kappa < 1}$. The deposit serves as the balancing variable in this approach. Consequently, we have $\boldsymbol{D = L - E = (1-\kappa)L}$.

According to the gradient adjustment process, loan dynamics are modeled [24], [21]. The bank's amount of loan in the next period ($\boldsymbol{L_{t+1}}$) is determined by the loan's marginal profit ($\boldsymbol{\partial\pi_t/\partial L_t}$). The model is provided below

$$\boldsymbol{L_{t+1} = L_t + \alpha_L L_t \frac{\partial \pi_t}{\partial L_t}} \tag{1}$$

where $\boldsymbol{\alpha_L}$ is referred to as the adjustment speed parameter, $\boldsymbol{\alpha_L > 0}$.

Calculating the interest on the loan ($\boldsymbol{r_L L}$), deducting the cost of the deposit ($\boldsymbol{r_D D}$) and equity ($\boldsymbol{r_E E}$), as well as the bank's costs ($\boldsymbol{C}$), yields the profit for the bank ($\boldsymbol{\pi}$). The assumptions of the Monti-Klein model [25], [26] are used to define the interest rates for loans and deposits $\boldsymbol{r_L = a_L - b_L L}$ and $\boldsymbol{r_D = a_D + b_D D}$, where $\boldsymbol{a_L, b_L, a_D, b_D > 0}$. It is assumed that the equity expense $\boldsymbol{r_E}$ is a constant. The cost of deposit and the cost of loan are included in the bank's operating costs, as shown below

$$\boldsymbol{C = c_D D + c_L L}$$

where $\boldsymbol{0 < c_D, c_L < 1}$. The primary subject of analysis in this paper is the cost of loan parameter $\boldsymbol{c_L}$ which is also referred as the marginal cost of loan.

The profit at time $\boldsymbol{t}$ is determined using previous information

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{r_L L_t - r_D D_t - r_E E_t - C_t}\\ &= \boldsymbol{(a_L - b_L L_t)L_t - (a_D + b_D(1-\kappa)L_t)(1-\kappa)L_t - r_E \kappa L_t - (c_D(1-\kappa)L_t + c_L L_t)}\\ &= \boldsymbol{(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)])L_t - \left[b_L + b_D(1-\kappa)^2\right]L_t^2}\end{aligned}$$

Then there is the loan's marginal profit

$$\boldsymbol{\frac{\partial \pi_t}{\partial L_t} = a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)] - 2\left[b_L + b_D(1-\kappa)^2\right]L_t} \tag{2}$$

When (2) is put in place of (1), we obtain

$$\boldsymbol{L_{t+1} = L_t + \alpha_L L_t\left(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)] - 2\left[b_L + b_D(1-\kappa)^2\right]L_t\right)} \tag{3}$$

2.2 Stability Analysis

By putting $L_{t+1} = L_t$, the equilibrium of the loan in model (3) can be derived. As a result, there exist two points of equilibrium, $L^*_{(1)} = 0$ and

$$L^*_{(2)} = \frac{a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]}{2[b_L + b_D(1-\kappa)^2]}$$

The requirement for equilibrium $L^*_{(2)}$ is positive is given by $a_L > r_E\kappa + c_L + (a_D + c_D)(1-\kappa)$.

Consider rewriting the model (3) as $L_{t+1} = f(L_t)$. A one-dimensional map represents the model. The equilibrium of the map is stable if $|f'(L^*)| < 1$, as stated in [27]. The following theorem gives the stability of the equilibriums $L^*_{(1)}$ and $L^*_{(2)}$. The first derivative of f is given by

$$f'(L_t) = 1 + \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) - 4\alpha_L[b_L + b_D(1-\kappa)^2]L_t$$

Theorem 1

The loan equilibium $L^*_{(1)}$ is unstable. The loan equilibium $L^*_{(2)}$ is stable when $c_L > a_L - [2/\alpha_L + r_E\kappa + (a_D + c_D)(1-\kappa)]$.

Proof. The fact that

$$f'\left(L^*_{(1)}\right) = 1 + \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) > 1$$

then $L^*_{(1)}$ is unstable. For the next case, we have

$$\begin{aligned} f'\left(L^*_{(2)}\right) &= 1 + \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) - 2\alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) \\ &= 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) < 1 \end{aligned}$$

and the following inequality

$$f'\left(L^*_{(2)}\right) = 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) > -1$$

holds when $c_L > a_L - [2/\alpha_L + r_E\kappa + (a_D + c_D)(1-\kappa)]$. Therefore $L^*_{(2)}$ is stable. □

2.3 Transcritical and Flip Bifurcation

For a one-dimensional map, we adhere to the Jury stability conditions in [28]. When $f'\left(L^*_{(2)}\right) = 1$, the equilibrium $L^*_{(2)}$ will become unstable by transcritical bifurcation. When $f'\left(L^*_{(2)}\right) = -1$, the equilibrium $L^*_{(2)}$ becomes unstable due to flip bifurcation. The paper's main objective is to examine the cost of loan parameter c_L. The parameter c_L will therefore serve as the bifurcation parameter. The following theorem can be obtained using straightforward computations.

Theorem 2
The loan equilibrium $L^*_{(2)}$ may lose its stability via transcritical bifurcation when $c_L = c_L^T$, where

$$c_L^T = a_L - [r_E\kappa + (a_D + c_D)(1-\kappa)]$$

and $L^*_{(2)}$ may lose its stability via flip bifurcation when $c_L = c_L^F$, where

$$c_L^F = a_L - \left[\frac{2}{\alpha_L} + r_E\kappa + (a_D + c_D)(1-\kappa)\right]$$

Proof. The transcritical bifurcation value can be directly obtained by solving

$$f'\left(L^*_{(2)}\right) = 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) = 1$$

for parameter c_L and then we get $c_L^T = a_L - [r_E\kappa + (a_D + c_D)(1-\kappa)]$. Similarly, the flip bifurcation value is obtained by solving

$$f'\left(L^*_{(2)}\right) = 1 - \alpha_L(a_L - [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]) = -1$$

for parameter c_L. The solution is $c_L = a_L - [2/\alpha_L + r_E\kappa + (a_D + c_D)(1-\kappa)]$. □

We can see that $c_L^F < c_L^T$. In order for the loan equilibrium to remain stable, the parameter of cost of loan must lie between the flip and transcritical bifurcation values. Theorem 2 provides the following statement regarding the flip bifurcation values that are inferred by the requirement of the adjustment speed parameter.

Remark 3
The flip bifurcation value $c_L^F = a_L - [2/\alpha_L + r_E\kappa + (a_D + c_D)(1-\kappa)]$ exists, which must be in between 0 and 1, if

$$\frac{2}{a_L - [r_E\kappa + (a_D + c_D)(1-\kappa)]} < \alpha_L < \frac{2}{a_L - [1 + r_E\kappa + (a_D + c_D)(1-\kappa)]}$$

Proof. The following inequality

$$0 < a_L - \left[\frac{2}{\alpha_L} + r_E\kappa + (a_D + c_D)(1-\kappa)\right] < 1$$

is solved for parameter α_L and it will directly proving the theorem. □

3. RESULTS AND DISCUSSION

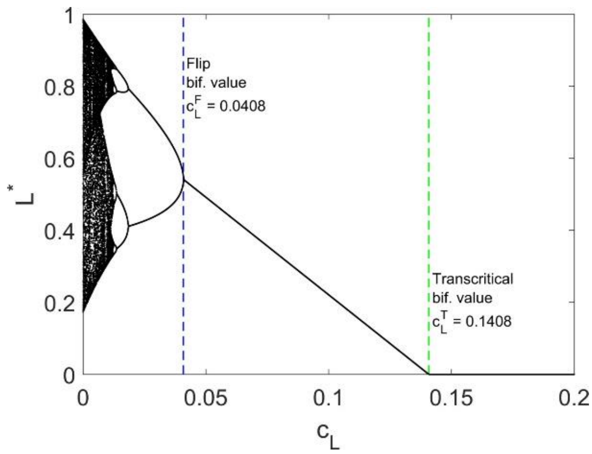
3.1 Simulations

We run several numerical simulations to both depict and validate the results from the previous section. The simulations make use of the values for the parameters listed in Table 1. Although the parameters' values were chosen purely for simulation purposes, they nonetheless satisfy the equilibrium's positive condition $a_L > [r_E\kappa + c_L + (a_D + c_D)(1-\kappa)]$. We can determine the bifurcation values of deposit and loan costs from Table 1, which is $c_L^F = 0.0408$ and $c_L^T = 0.1408$.

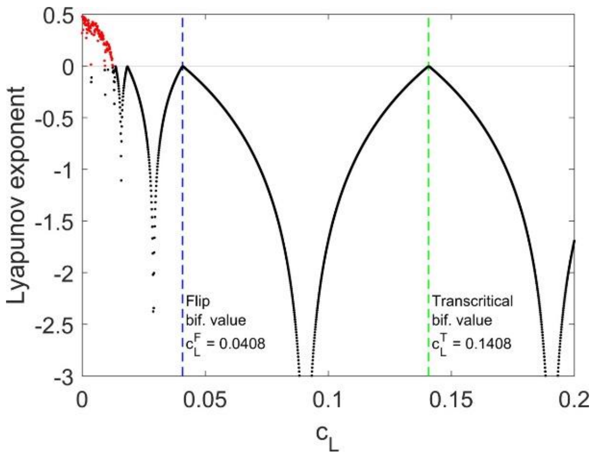
Table 1. Parameters' value for the simulations

Parameter	Value
a_L	0.2
b_L	0.05
a_D	0.01
b_D	0.05
r_E	0.05
κ	0.08
c_D	0.05
c_L	0.05
α_L	20
L_0	0.5

Bifurcation diagrams can be used to analyze how changes in a parameter affect a map's dynamics and can provide rich interpretations, such as whether or not the map is stable and when it may result in chaotic behavior. We provide a bifurcation diagram for the cost of loan parameter c_L in Figure 1a. The figure shows that when the cost of loan parameter exceeds the transcritical bifurcation value or $c_L > c_L^T$, the loan equilibrium is zero. When the cost of loan parameter is between the transcritical and flip bifurcation values or $c_L^F < c_L < c_L^T$, we obtain a stable and positive loan equilibrium. While the parameter c_L is decreasing, the loan equilibrium is rising at this point. The loan equilibrium causes period-doubling (2-period, 4-period, 8-period, etc.) starting at $c_L < c_L^F$, and when c_L is very small we get chaos. Figure 1b shows the graph of the Lyapunov exponent related to Figure 1a. If the graph of the Lyapunov exponent does not exceed zero, we depict it with black dots, and if it does, we plot it with red dots. The dynamics of loans become chaotic when the Lyapunov exponent is positive.



(a)



(b)

Figure 1. (a) Bifurcation diagram of the cost of loan parameter c_L and (b) the respective Lyapunov exponent

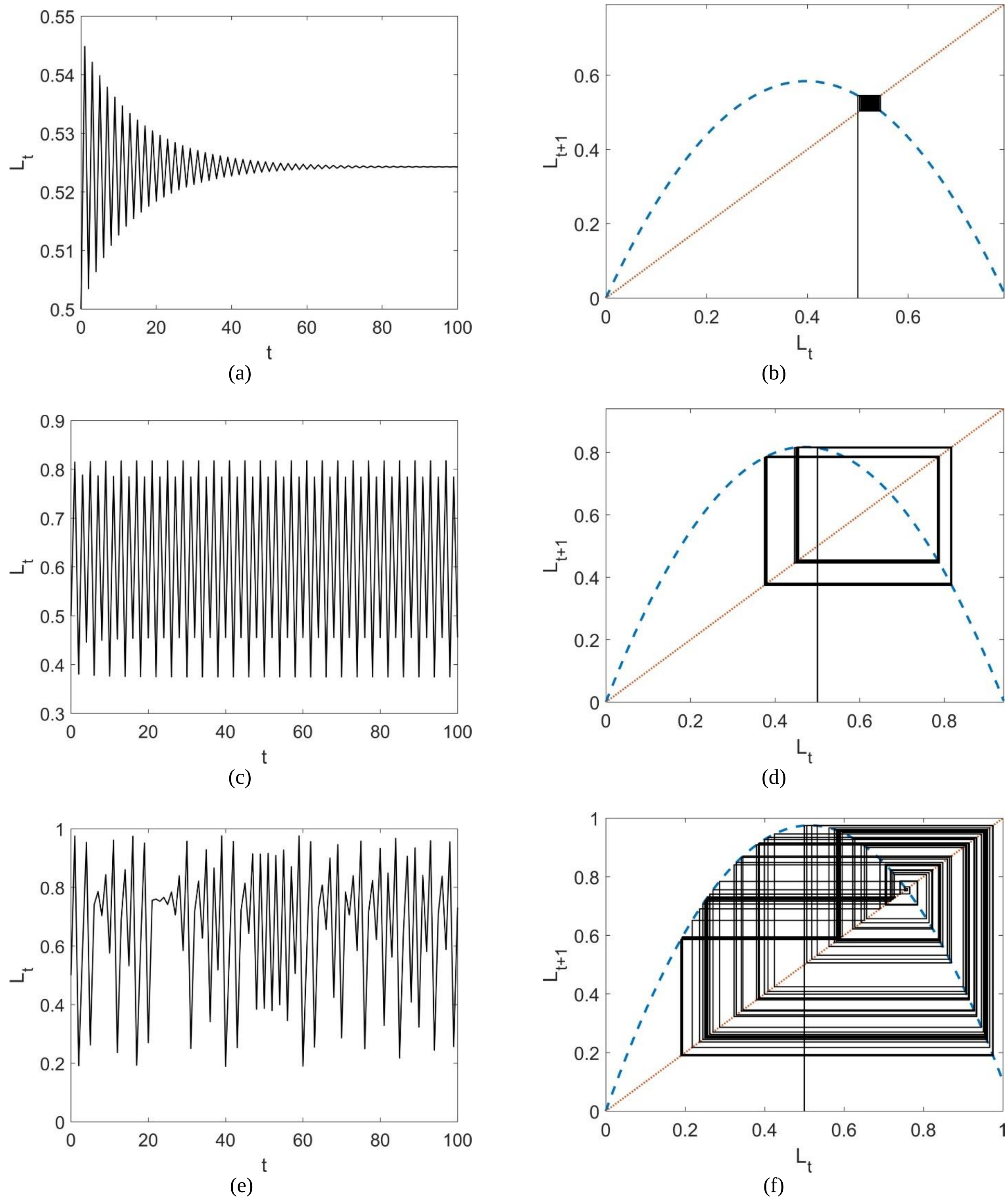


Figure 2. Graphs of loan L_t versus time t and the corresponding cobweb diagrams. Panel (a)-(b) show the dynamics of loan when reaching the stable equilibrium using value $c_L = 0.044$. Panel (c)-(d) show the dynamics of loan when having the 4-periodic orbits using value $c_L = 0.017$. Panel (e)-(f) show the dynamics of loan when chaos happens using value $c_L = 0.001$

The behavior of a map when its dynamics are unstable, such as when they become periodic or even chaotic, is always fascinating to observe. The easiest visualization is of the map's trajectory by plotting the graph L_t versus time t. There, we can watch the trajectory's dynamics as it progresses toward equilibrium in each time. The cobweb diagram is another method for analyzing qualitative behavior of the map's dynamics. Figure 2 presents both graphs of loan versus time and the respected cobweb diagram. Figure 2a shows the dynamics of loan when reaching a stable equilibrium. This simulation takes value $c_L = 0.044$. The cobweb diagram of this case is given in Figure 2b. The black paths converge to the intersection of the blue dashedline and the red dotted-line which indicating the paths converging to equilibrium. In Figure 2c-d, we present the dynamics of loan when having 4-periodic cycles in direct trajectory and cobweb diagram, respectively. The last one, Figure 2e-f depict the dynamics of loan when chaos happens. We can see that the loan's trajectory fluctuates with no patterns. The cobweb diagram also produces similar result.

The next simulation is to observe the behavior of chaotic loan with given slightly different initial values. Figure 3 illustrates an example of the loan map's chaotic behavior for the case where the value $c_L = 0.001$. Two graphs of the loan map with somewhat different initial values are shown in the figure. Initial values for the blue and red graphs are $L_0 = 0.5$ and $L_0 = 0.50001$, respectively. In the illustration, it is clear that the blue and red graphs first resemble one another before they diverge and create independent pathways.

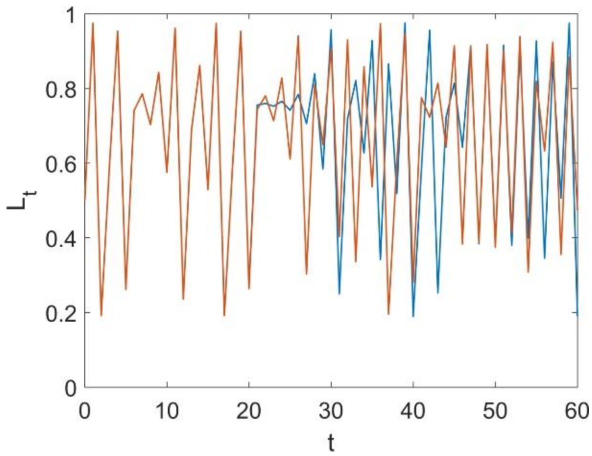


Figure 3. Sensitivity dependence of the chaotic loan dynamics on the initial condition for the cost of loan parameter $c_L = 0.001$

4. CONCLUSIONS

Costs incurred by a bank that are based on the size of the loan are operational issues that must be handled efficiently and effectively. Such a problem is objectively evaluated in this research to see how it affects the dynamics of loans. Higher loan equilibrium is produced by lower loan costs. The findings of this study demonstrate that the cost of a loan should not be too high or cheap. If the interest rate is too high, the loan will eventually disappear. Meanwhile, a loan with an excessively low interest rate may become unstable and produce mayhem. This study uses a very straightforward model of a single bank with a simple balance sheet structure. In order to handle specific banking policies, such as macroprudential policy, which focuses on limiting the growth of banking loans, the model can be generated into a more general model with more balance sheet components [29], [30]. The approach can also be used to banking data, as shown in [31], [32], and other future publications.

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