MULTIPLE STRATEGIES AS OPTIMAL CONTROL OF A COVID-19 DISEASE WITH QUARANTINE AND USING HEALTH MASKS

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Abstract.

This research develops an optimal control as an effort to push down the widely of Covid-19 with a mathematical model. Where, the problem of optimal control is conducted by adding three control variables, i.e an effort to avoid direct contact between the susceptible populations without masks and the infected populations without masks, and the thoughtfulness of a mask-wearing policy. The primary goal of optimal control is to minimize the infected populations without and with masks, and minimize the cost of weight control. Furthermore, if it applies Pontryagin's principle and discovers the Hamiltonian function, then optimal control conditions for the COVID-19 approximation are determined. Finally, as an addition to the model analysis results, numerical simulations are conducted to represent the solutions behavior of each subpopulation before and after the control was designed.

Keywords: *Optimal Control, Covid-19, Quarantine Population, Health Mask Strategic, Pontryagin's Principle, Numerical Simulations.*

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1. INTRODUCTION

The worldwide population was shocked toward the end of 2019 by a malignant disease infection caused by a viral genome, specifically Coronavirus Disease 2019 (COVID-19) [1], [2]. Previously, the Coronavirus was responsible for more serious outbreaks, such as the Middle East Respiratory Syndrome Coronavirus (MERS-CoV) in 2012 and the Severe Acute Respiratory Syndrome Coronavirus (SARS-CoV) in 2003 [3]. COVID-19 is a pulmonary syndrome infection characterized by the SARS-CoV-2 Coronavirus [4]. In general, the contagion of this virus occurs through droplets on a person or objects between one and two meters, by coughing and sneezing [5]. In reference to World Health Organization (WHO), the COVID-19 epidemic was first detected on December 31, 2019, in Wuhan, China [6]. Furthermore, the World Health Organization realized on March 11, 2020, that COVID-19 had infected nearly 118,000 humans worldwide, spreading widely through 114 countries. As a result, the WHO has declared COVID-19 a worldwide hazard, assuming that it has spread globally [7]. Whereas, on March 2, 2020, Indonesia verified the first occurrence of COVID-19, involving two Indonesian people who positive test again for Coronavirus, it has been transmitted by a Japanese peoples [8]. Unfortunately, the spread of the coronavirus in Indonesia remained until the end of 2021. Following that, the coronavirus spread started to slow in the first quarter of 2022. However, the spread of COVID-19 infectious began to rise in May 2022.

Considering the state of Indonesia, regulating the spread of the coronavirus is crucial. Strategies to control the spread of COVID-19 in Indonesia are being made through the implementation of a health protocol policy, as known Using Masks, Washing Hands, and Maintaining Distance, Staying Away from the Crowds, and Limiting Mobility and Interaction). In reference to Wuhan, COVID-19 prevention initiatives, the Republic of Indonesia's government has implemented an isolation policy or a quarantine for infected individuals [8]–[11]. With the goal of avoiding contact between the infected populations and the community at large. On the other hand, quarantine policies are also carried out by countries around the world in the non-essential sector, while the critical sector continues to function as before [12]. Assuming that the COVID-19 spread can be controlled, it has been shown that the quarantine strategy resulted in a decrease in infected cases. Moreover, based on the phenomenon of the state government policies in handling COVID-19. In addition, the epidemiologist attempt to fund studies using a mathematical model approach [13]–[17].

The *SIR* compartment mathematical model was presented by Kermack and McKendrick in 1927 as the first epidemic model [18]. Further, it become the foundational model in research on the transmission of COVID-19, and the mathematical concept involved in Kusumo et al. [19], Husniah et al. [20], Kurniawan et al. [7], Sihotang et al. [21], Fatmawati et al. [22], Husniah et al. [23], and Nuraini et al. [24]. Then, a quarantine policy and isolation are used to design a mathematical model by Manaqib et al. [8], Ali et al. [9], Batista et al. [10], Musafir et al. [11], and Khan et al. [12], to investigate the spread of the corona disease. Supplementary information is a systematical model of COVID-19 with a vaccination effect in a compartment [11], [25]. The mathematical model was especially redeveloped by incorporating the use of masks in an effort to minimize the COVID-19 epidemic [8].

As far as we know from above, some control needs to be performed by applying the optimum control problem. The optimal control problem is applied in controlling the spread of Malaria disease [26], Tuberculosis and HIV [27], [28], and Cervical Cancer Model [29]. Additionally, the implementation of optimal control was also carried out in controlling Cholera [30], Hepatitis B [31], [32], and Measle [17], [33], [34]. Based on previous research, which applied the optimal control problem as a disease control tool. In this study, the model was organized as a procedure to prevent the escalation of COVID-19 by adding control variables in research model by Manaqib et al. [8]. The sections of this work are arranged as follows. The development of the research methods and the model are resolved in Section 2, followed by results and discussion in Section 3. Part 3, is divided into three sections, and section 3.1, describes the reconstruction of the Covid-19 model with control variables. The terms and conditions of optimal control are exhausted in Section 3.2. While, part 3.3, discusses a numerical simulation and its interpretation. Finally, in Section 4, we make some closing observations.

2. RESEARCH METHODS

In this work, we adopt a transmission epidemic COVID-19 model by Manaqib et al. [8]. The model is divided into the following subpopulation, i.e. susceptible without a mask (S_1), susceptible with mask (S_2),

infected without a mask (I_1), infected with a mask (I_2), quarantine (Q), and the recovery subpopulations (R). This compartment can be characterized by using the mathematical model of a nonlinear differential equation. According to the compartment model used in [8], the variable R does not affect to other populations. Therefore, the model reduction can be carried out as follows:

$$\frac{dS_1(t)}{dt} = \mu + \eta_1 S_2(t) - (\mu + \eta_1)S_1(t) - \beta S_1(t)I_1(t)$$

$$\frac{dS_2(t)}{dt} = \eta_1 S_1(t) - (\mu + \eta_2)S_2(t)$$

$$\frac{dI_1(t)}{dt} = \beta S_1(t)I_1(t) + \eta_2 I_2(t) - (\eta_1 + \mu + \gamma + \alpha)I_1(t) \tag{1}$$

$$\frac{dI_2(t)}{dt} = \eta_1 I_1(t) - (\eta_2 + \mu + \gamma + \alpha)I_2(t)$$

$$\frac{dQ(t)}{dt} = \alpha\big(I_1(t) + I_2(t)\big) - (\mu + \gamma + \theta)Q(t)$$

Based on the equation system (1), a model modification was performed by considering three control variables, namely $z_1(t)$ as an effort to control direct contact between susceptible populations without masks and infected populations without masks. The variable $z_2(t)$, represents an effort to prevent COVID-19 by requiring infected populations to use masks, and the control $z_3(t)$ represents a responsibility for susceptible groups to constantly wear masks. Furthermore, the following methods may be used to deal with the problem of optimal control of the COVID-19 spread model:

1. Develop a Covid-19 model with variable control $\mathbf{z_1(t)}$, $\mathbf{z_2(t)}$, and $\mathbf{z_3(t)}$.
2. Determine the objective function to minimize the infected populations without masks and infected populations with masks.
3. By using the Pontryagin Minimum Principle to examine the best solution for optimal control.
4. Define the state systems, adjoint systems, and stationary conditions.
5. Using Matlab software, to describe the behavior solution an optimal control problems.

3. RESULTS AND DISCUSSION

3.1. Mathematical Model

In this segment, we consider a mathematical problem of covid-19 with variable control, which is organized as follows:

$$\frac{dS_1(t)}{dt} = \mu + \eta_1 S_2(t) - (\mu + \eta_1)S_1(t) - \big(1 - z_1(t)\big)\beta S_1(t)I_1(t) - z_2(t)S_1(t)$$

$$\frac{dS_2(t)}{dt} = \eta_1 S_1(t) + z_2 S_1(t) - (\mu + \eta_2)S_2(t)$$

$$\frac{dI_1(t)}{dt} = \big(1 - z_1(t)\big)\beta S_1(t)I_1(t) + \eta_2 I_2(t) - (\eta_1 + \mu + \gamma + \alpha)I_1(t) - z_3(t)I_1(t) \tag{2}$$

$$\frac{dI_2(t)}{dt} = \eta_1 I_1(t) - (\eta_2 + \mu + \gamma + \alpha)I_2(t) + z_3(t)I_1(t)$$

$$\frac{dQ(t)}{dt} = \alpha\big(I_1(t) + I_2(t)\big) - (\mu + \gamma + \theta)Q(t),$$

where $z_1(t)$ is an effort to control direct contact between susceptible and infected without masks. The variable $z_2(t)$ reflects an attempt to restrict disease by requiring infected subpopulations to wear masks, while control $z_3(t)$ as a policy to always use masks.

3.2. The Optimal Control Conditions

The aim of optimal control is to carry out the best value condition for the model (2). In this part, we construct an optimal control condition to identify the minimum of the objective function, such that

$$J\big(z_1(t), z_2(t), z_3(t)\big) = \int_{t_0}^{t_f} \Big(AI_1(t) + BI_2(t) + Cz_1^2(t) + Dz_2^2(t) + Ez_3^2(t)\Big)\,dt. \tag{3}$$

The constant parameter A, indicate the important value to reduce infected subpopulations without a mask, and the constant B represents the significance of lowering the infectious transmission of Covid-19 with a mask. While the parameters C, D, and E denote the effort required to apply the controls. By using the positive parameter A, B, C, D, and E, we derive the best control $z_1^*(t), z_2^*(t)$, and $z_3^*(t)$ such that:

$$J\big(z_1^*(t), z_2^*(t), z_3^*(t)\big) = \min\{J\big(z_1(t), z_2(t), z_3(t)\big), \text{with } z_1(t), z_2(t), z_3(t) \in U\}, \tag{4}$$

with regard to domain $U = \{(z_1(t), z_2(t), z_3(t)): 0 \le z_1(t) \le 1; \ 0 \le z_2(t) \le 1; \ 0 \le z_3(t) \le 1\}$. Pontryagin's Minimum Principle gives an optimal control condition. This principle converts (2) - (4), into a Hamiltonian function, as seen below:

$$\begin{aligned}
H = {}& AI_1(t) + BI_2(t) + Cz_1^2(t) + Dz_2^2(t) + Ez_3^2(t) + \lambda_{S_1}\Big(\mu + \eta_1 S_2(t) - (\mu + \\
& \eta_1)S_1(t) - \big(1 - z_1(t)\big)\beta S_1(t)I_1(t) - z_2(t)S_1(t)\Big) + \lambda_{S_2}\big(\eta_1 S_1(t) + z_2 S_1(t) - (\mu + \\
& \eta_2)S_2(t)\big) + \lambda_{I_1}\Big(\big(1 - z_1(t)\big)\beta S_1(t)I_1(t) + \eta_2 I_2(t) - (\eta_1 + \mu + \gamma + \alpha)I_1(t) - \\
& u_3(t)I_1(t)\Big) + \lambda_{I_2}\big(\eta_1 I_1(t) - (\eta_2 + \mu + \gamma + \alpha)I_2(t) + z_3(t)I_1(t)\big) + \lambda_Q\Big(\alpha\big(I_1(t) + \\
& I_2(t)\big) - (\mu + \gamma + \theta)Q(t)\Big),
\end{aligned} \tag{5}$$

where $\lambda_{S_1}, \lambda_{S_2}, \lambda_{I_1}, \lambda_{I_2}, \lambda_Q$ are the costate (adjoint) variables for an optimal control condition. Then, the theorem an optimal control is derived using Pontryagin's Minimum Principle, as stated below:

Theorem 1. If the variables control $z_1^*(t), z_2^*(t), z_3^*(t)$ exist, and the simultaneous solution of $S_1^*(t), S_2^*(t), I_1^*(t), I_2^*(t), Q^*(t)$ holds on the system (2), that minimizes $J\big(z_1(t), z_2(t), z_3(t)\big)$ in domain U. So, there are exist adjoint (costate) variables $\lambda_{S_1}, \lambda_{S_2}, \lambda_{I_1}, \lambda_{I_2}, \lambda_Q$, that obey the equations system,

$$\begin{aligned}
\frac{d\lambda_{S_1}}{dt} ={}& \lambda_{S_1}\Big(\mu + \eta_1 + z_2(t) + \big(1 - z_1(t)\big)\beta I_1(t)\Big) - \lambda_{S_2}\big(\eta_2 + z_2(t)\big) \\
& - \lambda_{I_1}\big(1 - z_1(t)\big)\beta I_1(t) \\
\frac{d\lambda_{S_2}}{dt} ={}& -\lambda_{S_1}\eta_2 + \lambda_{S_2}(\mu + \eta_2) \\
\frac{d\lambda_{I_1}}{dt} ={}& -A + \lambda_{S_1}\Big(\big(1 - z_1(t)\big)\beta S_1(t)\Big) \\
& - \lambda_{I_1}\Big(\big(1 - z_1(t)\big)\beta S_1(t) - (\mu + \eta_1 + \gamma + \alpha) - z_3(t)\Big) - \lambda_{I_2}\big(\eta_1 + z_3(t)\big) \\
& - \lambda_Q\alpha \\
\frac{d\lambda_{I_2}}{dt} ={}& -B - \lambda_{I_1}\eta_2 + \lambda_{I_2}(\mu + \eta_2 + \gamma + \alpha) - \lambda_Q\alpha \\
\frac{d\lambda_Q}{dt} ={}& \lambda_Q(\mu + \gamma + \theta).
\end{aligned} \tag{6}$$

Then incorporate the transversality condition $\lambda_{S_1}(t_f) = \lambda_{S_2}(t_f) = \lambda_{I_1}(t_f) = \lambda_{I_2}(t_f) = \lambda_Q(t_f) = 0$, such that an optimal control problem sets $z_1^*(t), z_2^*(t)$, and $u_3^*(t)$ are provided by

$$\begin{aligned}
z_1^*(t) &= \min\left\{\max\left(0, \frac{\big(\lambda_{I_1} - \lambda_{S_1}\big)\beta S_1(t)I_1(t)}{2C}\right), 1\right\} \\
z_2^*(t) &= \min\left\{\max\left(0, \frac{\big(\lambda_{S_1} - \lambda_{S_2}\big)S_1(t)}{2D}\right), 1\right\} \\
z_3^*(t) &= \min\left\{\max\left(0, \frac{\big(\lambda_{I_1} - \lambda_{I_2}\big)I_1(t)}{2E}\right), 1\right\}.
\end{aligned} \tag{7}$$

Proof:

Due to the convexity condition of the integrand of $J(z_1(t), z_2(t), z(t))$ and correspond to the Lipschitz condition on the state system [35]. The existence of an optimal control problem can be conducted by working Pontryagin's Minimum Principle. Then, the adjoint variables are accomplished by differentiating the Hamiltonian function around the state variable, and the system can be immediately recognized as bellows,

$$\frac{d\lambda_{S_1}}{dt} = \lambda_{S_1}\left(\mu + \eta_1 + z_2(t) + (1 - z_1(t))\beta I_1(t)\right) - \lambda_{S_2}(\eta_2 + z_2(t)) - \lambda_{I_1}(1 - z_1(t))\beta I_1(t)$$

$$\frac{d\lambda_{S_2}}{dt} = -\lambda_{S_1}\eta_2 + \lambda_{S_2}(\mu + \eta_2)$$

$$\frac{d\lambda_{I_1}}{dt} = -A + \lambda_{S_1}\left((1 - z_1(t))\beta S_1(t)\right) - \lambda_{I_1}\left((1 - z_1(t))\beta S_1(t) - (\mu + \eta_1 + \gamma + \alpha) - z_3(t)\right)$$
$$- \lambda_{I_2}(\eta_1 + z_3(t)) - \lambda_Q\alpha$$

$$\frac{d\lambda_{I_2}}{dt} = -B - \lambda_{I_1}\eta_2 + \lambda_{I_2}(\mu + \eta_2 + \gamma + \alpha) - \lambda_Q\alpha$$

$$\frac{d\lambda_Q}{dt} = \lambda_Q(\mu + \gamma + \theta)$$

with the transversality condition $\lambda_{S_1}(t_f) = \lambda_{S_2}(t_f) = \lambda_{I_1}(t_f) = \lambda_{I_2}(t_f) = \lambda_Q(t_f) = 0$. Followed by section is an optimal control variable can be found by differentiating the Hamiltonian function around the control $z_1(t), z_2(t), z_3(t)$, and evaluating the result equal to zero, such that

$$\frac{\partial H}{\partial z_1(t)} = 2Cz_1(t) + \lambda_{S_1}(\beta S_1(t)I_1(t)) - \lambda_{I_1}(\beta S_1(t)I_1(t)) = 0$$

$$\frac{\partial H}{\partial z_2(t)} = 2Dz_2(t) - \lambda_{S_1}S_1 + \lambda_{S_2}S_1 = 0$$

$$\frac{\partial H}{\partial z_3(t)} = 2Ez_3(t) - \lambda_{I_1}I_1 + \lambda_{I_2}I_1 = 0.$$

Direct consequence, by solving an optimization problem provides

$$z_1^*(t) = \min\left\{\max\left(0, \frac{(\lambda_{I_1} - \lambda_{S_1})\beta S_1(t)I_1(t)}{2C}\right), 1\right\}$$

$$z_2^*(t) = \min\left\{\max\left(0, \frac{(\lambda_{S_1} - \lambda_{S_2})S_1(t)}{2D}\right), 1\right\}$$

$$z_3^*(t) = \min\left\{\max\left(0, \frac{(\lambda_{I_1} - \lambda_{I_2})I_1(t)}{2E}\right), 1\right\}.$$

3.3. Numerical Result

To support the analytical results of optimal control theorem before, we demonstrate a numerical simulation of a system (2) using the Matlab software. In this section, we apply the control weight into the simulation are $A = B = 1$, and $C = D = E = 0,5$. The initial condition is $S_1(0) = S_2(0) = I_1(0) = 1$, and $I_2(0) = Q(0) = 0$. According to Manaqib et al. [8], and the values in Table 1, we realize the basic reproduction number $R_0 = 1,1237$ for the system (1). By reason of the basic numbers $R_0 > 1$, and it is line with the spread of COVID-19 will persist in a subpopulation. Hence, the controlling process must be carried out by applying optimal control into model (2).

Table 1. Parameter Values

| Parameter	Explanation	Value	Sources
μ	The population's natural birth and death rates	0,0125	[8]
η_1	The rate of using the health mask	0,15	Assumed
η_2	The rate of without the health mask	0,65	Assumed
γ	The death rate causes the infectious	0,025	[8]
α	Quarantine rate	0,084	[8]
β	The contact rate for the susceptible population without a mask and the infected population without a mask	0,2	[8]
θ	Recovery rate population	0,255	Assumed
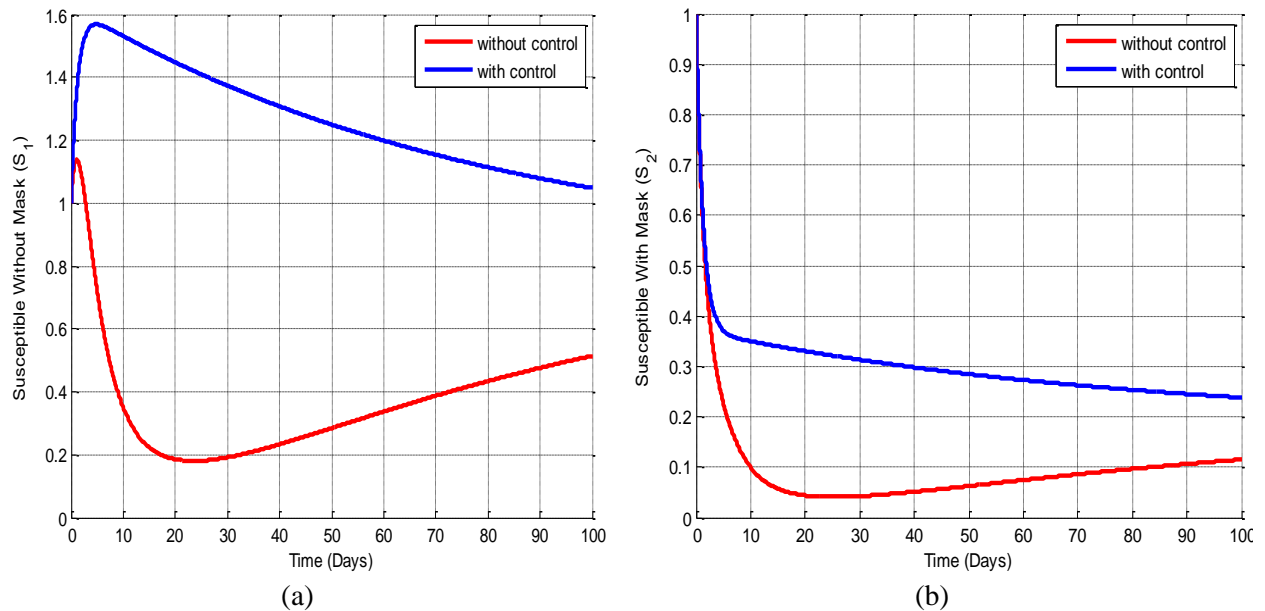
Figure 1. The behavior solutions of sub-population without and with control: (a) Susceptible without masks, (b) Susceptible with masks

Based on the simulation in figure 1, part (a) demonstrates that susceptible populations without masks have decreased starting at the beginning of the simulation time. This indicates that the susceptible population has entered into infected Covid-19 or into susceptible population with masks. Interestingly, if the control of $z_1(t), z_2(t)$ and $z_3(t)$ is decided to apply into the Covid-19 model, then the computation results show a rise at the beginning of time simulation. Additionally, from $t = 8$ through the end of the simulations, the susceptible population will decrease, although not as crisp as before the procedure is performed. In the meanwhile, part (b) provides the simulation results of susceptible populations with masks have decreased from the beginning of time to the ending of the 100th day. However, by giving controls resulted in a significant decrease in susceptible with masks, than the controls were not implemented. In other words, the probability of a susceptible population spreading to an infected population persists.

Figure 2 below illustrates the behavior of an infected subpopulation's solution with and without a mask. Figure 1, part (a) clearly demonstrates, that the infected Covid-19 population without a mask before controlling grew at the start of the simulation period, and the quantity of infected reduced in time around the 85th day. In contrast, after applying the controls to the model, it appears that the number of infected populations decreased at the beginning of the simulation. Meanwhile, the numerical simulation of part (b) in figure 2 shows that the number of populations infected with masks has increased significantly when the control process is not implemented. Similarly, when $z_1(t), z_2(t)$ and $z_3(t)$ are governed, the population rise is as strong as previously, as depicted by the solution curve in blue. In this situation, it means that the optimal control utilized was already in line with the functional objective of the previously defined, which is to minimize the infected subpopulations without and using a masks.

(a)   (b)
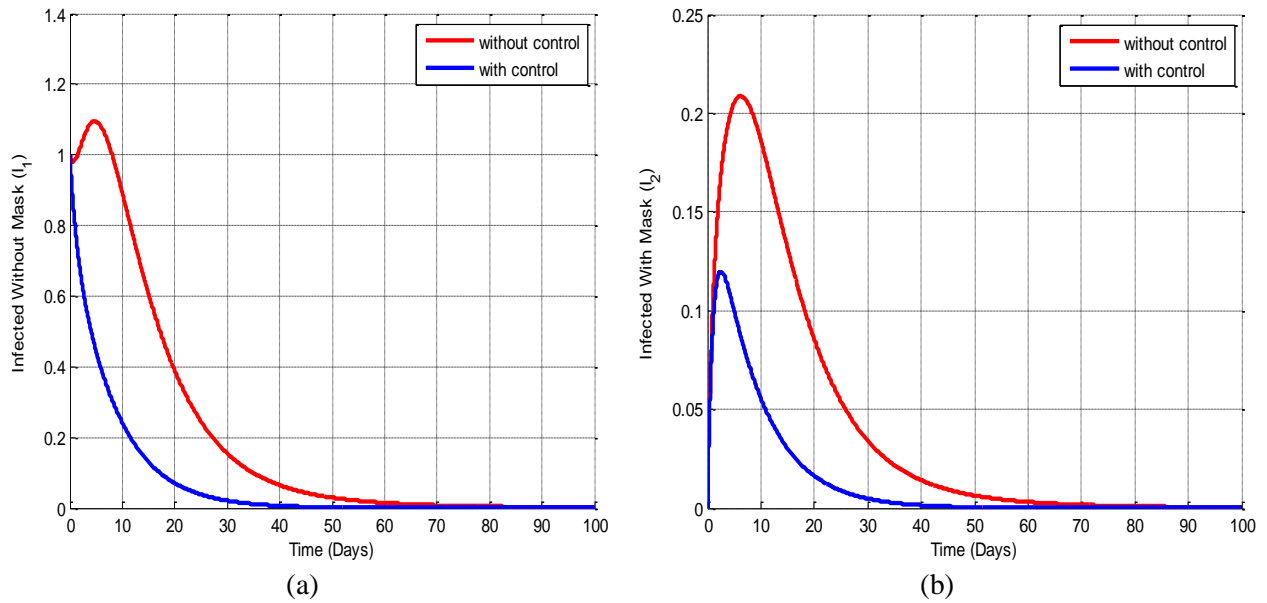
Figure 2. The behavior solutions of sub-population without and with control: (a) Infected without masks, (b) Infected with masks

(e)   (f)
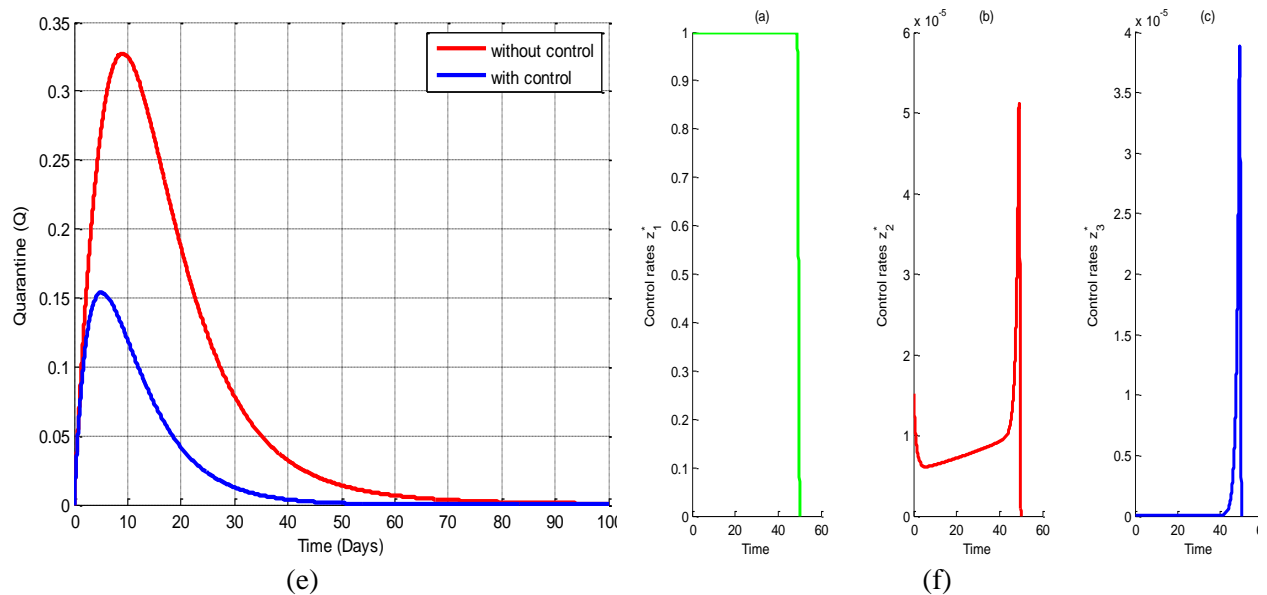
Figure 3. (a) The behavior solutions of Quarantine, (b) The profile of control rate z_1^*, z_2^*, and z_3^*

Figure 3, part (a), depicts the isocline solution of the quarantine population, before going to the control process shows immoderate. It provides a condition that the infected population is very high, then quarantine strategy is needed to prevent the COVID-19 infectious. Meanwhile, after the control was carried out, it was seen that the quarantine population continued to increase at the beginning of time, but not as high as before the control was carried out. Part (b) of Figure 3, shows the profile of the control variables $z_1(t)$, $z_2(t)$ and $z_3(t)$. Where the control variable $z_1(t)$ must be applied maximally from the beginning of the time after time to the end of time, $t = 100$. Meanwhile, the behavior of the control variables $z_2(t)$ and $z_3(t)$ are seem growth at the end of the simulation. Therefore, based on the simulation appear that controlling direct contact between susceptible populations and infected populations ($z_1(t)$) is more effective than the control variables $z_2(t)$ and $z_3(t)$ in the form of efforts to wear masks by sub-populations.

4. CONCLUSIONS

In this study, an optimal control of controlling the spread COVID-19 has been presented by analytically theorem and numerically simulation. In order to manage COVID-19, three control variables are used. The first control variable is an effort to reduce direct contact between susceptible and infected populations without a mask. Conversely, the second and third control variables lead to the formation of a policy initiative to wear masks on susceptible and infected populations, respectively. Whereas, an optimal control works to minimize infected populations without and with a mask, as well as the weight of the control process. Finally, the numerical simulation suggests that an optimal control already be carried out to hold down of spreading COVID-19, and it is in line with the governed objective function.

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