BAREKENG: Journal of Mathematics and Its Application
December 2022 Volume 16 Issue 4 Page 1325–1336
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol16iss4pp1325-1336

# OPTIMIZATION OF PORTFOLIO USING FUZZY SELECTION

**Rahmania Ayu Wardhani**[1], **Sugiyarto Surono**[2*], **Goh Khang Wen**[3]

*[1,2]Department of Mathematics, Faculty of Applied Science And Technology, Ahmad Dahlan University*
*Jl. Ringroad Selatan Tamanan Bantul, Yogyakarta, Indonesia*
*[3]Faculty of Data Science and Information Technology, INTI International University Malaysia*
*Bandar Nilai, Negeri Sembilan, Malaysia*

*Corresponding author's e-mail: [2*] sugiyarto@math.uad.ac.id*

***Abstract.*** *The problem of portfolio optimization concerns the allocation of the investor's wealth between several security alternatives so that the maximum profit can be obtained. One of the methods used is Fuzzy Portfolio Selection to understand it better. This method separates the objective function of return and the objective function of risk to determine the limit of the membership function that will be used. The goal of this study is to understand the application of the Fuzzy Portfolio Selection method over shares that have been chosen on a portfolio optimization problem, understand return and risk, and understand the budget proportion of each claim. The subject of this study is the shares of 20 companies included in Indonesia Stock Exchange from 1 January 2021 until 1 January 2022. The result of this study shows that from 20 shares, there are 10 shares that are suitable in the forming of an optimal portfolio, those are ADRO (0%), ANTM (43.3%), ASII (0%), BBCA (0%), BBRI (0%), BBTN (0%), BRPT (0%), BSDE (0%), ERAA (16%), and INCO (40.7%). The expected return from the portfolio is 0.0878895207 or 8.8% for the return and 0.0226022117 or 2.3% for the risk.*

***Keywords:*** *Fuzzy portfolio selection, Indonesia Stock Exchange, portfolio optimization, stocks.*

***Article info:***
*Submitted: 7th July 2022* *Accepted: 17th October 2022*

***How to cite this article:***
R. A. Wardhani, S. Surono and G. K. Wen, "OPTIMIZATION OF PORTFOLIO USING FUZZY SELECTION", *BAREKENG: J. Math. & App.*, vol. 16, iss. 4, pp. 1325-1336, Dec., 2022.

## 1. INTRODUCTION

*Multi-Criteria Decision-making* (MCDM) is a topic focused on determining the best alternative decision from other alternatives based on specific criteria as consideration. Based on its purpose, MCDM is divided into *Multi-Attribute Decision-making* (MADM) and *Multi-Objective Decision-making* (MODM), where MADM is used to solve problems in discreet areas. MADM often is used to do an evaluation or selection of several alternatives in a limited amount [1], % while MODM is the problem of decision-making in which the main characteristic of MODM problem is the said decision needs to achieve many purposes, but there are contradicting purposes inside it. *Multi-Objective Linear Programming* (MOLP) is one of the critical forms for explaining MODM problems, where the purpose of the linear function that will be optimized, whether maximized or minimized, is depended on the set of linear problem [2]. For several last years, there has been a lot of research on MODM, including [3], [4], [5], and [6]. Based on the results from said research, the MODM method is integrated with fuzzy to optimize the wanted goal.

The MODM is a problem of decision-making in which the main characteristic of the MODM problem is the existence of contradicting purposes. The MODM model considers the variable vector of decision, objective function, and constraint. The decision-making will be done by either maximizing or minimalizing the objective function. Since this problem is rarely seen and has a unique solution, the decision-making is hoped to be able to choose the solution over a group of efficient solutions (as an alternative).

*Fuzzy Multi-Objective Decision-making* (FMODM) can be applied in various fields, one of which is stock investment. The stock has become an investment instrument that is growing in popularity and is preferred by the public. Stock can be formed into an optimal portfolio. [7] has defined a portfolio as a group of assets in the shape of an investment owned by an investor or company. Those investments can be found in the form of deposits, gold, stock, property, obligation, etc. There are two functions in a stock portfolio that are maximizing return and minimalizing risk. [8] has shown that the investment of portfolio risk is dependent on the covariant between assets. The average return will determine the portfolio return, while the risk is dependent on covariant between assets in the portfolio.

Numerous researches have been done to solve and improve the Markowitz portfolio model. It's done to adapt the existing model to the finance market condition and demand from the capital market player. One of the focuses of the research in portfolio selection is the amount of return, risk, and budget proportion that is allocated in choosing the optimal portfolio [9],[10], and [11]. This can be understood because the more significant the involved security value in portfolio selection, the bigger chance of an optimal portfolio being formed. Many complicated securities in portfolio selection can be solved by classifying the data based on decided criteria. A security that is unable to fulfill the fixed criteria will not be used in the formation of an optimal portfolio [12],[13], and [14].

In the several last years, lots of research about MODM on portfolios are being done, including [15], [16], and [17]. Based on the results of said research, MODM that is modeled using the mean-variant of Markowitz in portfolio selection with new modification will result in various function variants of risk and return, so maximizing return a minimalizing portfolio valued as fuzzy can be done as other alternatives.

In the new contribution to past studies, this paper will use FMODM that is integrated with *the fuzzy* approach in which the proposed model is a fuzzy bi-objective portfolio selection model that maximizes portfolio return and minimizes portfolio risk [18],[19],[20], and [18]. The writer uses *a fuzzy* interactive approach to solve the model, so the level of the desired goal in decision-making is in relation to the purpose of *return*, and risk is achieved as close as possible.

The problem of the bi-objective portfolio optimization model is a squared programming problem. Considering the fact that in the application of portfolio selection in real life in which the decisions are often arranged around the unclear aspiration of investor in the desired portfolio concerning return and risk, the framework of fuzzy is used to predict the need for linguistic type information in portfolio selection problems. It is assumed that the investor shows aspiration based on experience and prior knowledge, and the linear membership function is used to define the level of unclear aspiration of the investor.

## 2. RESEARCH METHODS

The research method used is as follows.

### 2.1 Data Source

The data that is used in this study is the data of monthly stock close price that is registered in Bursa Efek Indonesia (BEI)/ Indonesia Stock Exchange (IDX) in the span of 1 January 2020 until 1 January 2021, where the companies are included in the LQ45 market from the website https://finance.yahoo.com.

### 2.2 Multi-Objective Decision-making (MODM)

Multi-Objective Decision-making (MODM) considers the variable vector of decision, objective function, and constraint. Generally, the MODM problem can be written in the following formula:

$$\max f_k(\boldsymbol{x}) \tag{1}$$
$$s.t\ \boldsymbol{x} \in \boldsymbol{X} = \{\boldsymbol{x} \in R^n |\ g(\boldsymbol{x}) \leq b, \boldsymbol{x} \geq 0\}$$

where $f_k(\boldsymbol{x})$ symbolizes $k$-objective function that is contradictory to each other, $g(\boldsymbol{x}) \leq b$ symbolizes m-constraint and $x$ is a $n$-vector which is a return from the decision variable with $\boldsymbol{x} \in R^n$[2].

Stock is security ownership of the company's assets that publish shares. By owning a share of a certain company, the investor has the right over the company's income and wealth after being subtracted by payment of all company's duties. It is a securities type that is quite popular in the capital market. The reason is that compared to other types of investments, the stock can give a bigger return or profit in a relatively short period [14].

The return can be defined as the result obtained after an investment. To calculate return, the formula written below can be used [21]:

$$R_{it} = \frac{p_t - p_{(t-1)}}{p_{(t-1)}} \tag{3}$$

where

$R_{it}$ : Return of share $i$ in the time of $t$

$p_t$ : Close price of share in the time of $t$

$p_{(t-1)}$ : Close price of share in the time of $t-1$

and the expected return of share has a formula of [22]:

$$r_i = E[R_i] = \frac{1}{T}\sum_{t-1}^{T} R_{it} \tag{4}$$

where

$r_i$ : Expected Return of share i in the time of t

$R_{it}$ : Return of share i in the time of t

$T$ : Time period

The risk between shares can be defined using covariant matrix in which covariant $\sigma_{ij}$ is written in following formula:

$$\sigma_{ij} = \frac{1}{T}\sum_{t-1}^{T}(R_{it} - r_i)(R_{jt} - r_j) \tag{5}$$

where

$r_i$ : Expected Return of share $i$ in the time of $t$

$R_{it}$ : Return of share $i$ in the time of $t$

$r_j$ : Expected Return of share $j$ in the time of $t$

$R_{jt}$ : Return of share $j$ in the time of $t$

$T$ : Time period

The portfolio optimization model of bi-objective based on the work frame of mean-variance, which Markowitz previously proposed, models are able, at the same time, to maximize the portfolio return $(f_1(x))$ and minimalize portfolio risk $(f_2(x))$ ,which is written is written in the following formula:

$$\max f_1(x) = \sum_{i=1}^{n} r_i\, x_i$$
$$\max f_2(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j \qquad (6)$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1$$

$x_i \geq 0,\ \ i = 1,2,\cdots,\ n\ , j = 1,2,\cdots,\ n$

In which $r_i$ is expected return and $\sigma_{ij}$ is covariant between asset i and j.

The linear membership function is often used because of its simplicity and can be defined as repairing by using two points, and those are upper and lower acceptability. The continued explanation of the linear membership function is written below:

1. The Membership function of the objective function of the return portfolio can be defined as:

$$\mu_{f_1}(x) = \begin{cases} 1 & if \quad f_1(x) \geq f_1^R \\ \dfrac{f_1(x) - f_1^L}{f_1^R - f_1^L} & if\ f_1^L < f_1(x) < f_1^R \\ 0 & if \quad f_1(x) \leq f_1^L \end{cases}$$

In which $f_1^L$ is the worst lower limit (low aspiration level) and $f_1^R$ is the best upper limit (high aspiration level) from the return portfolio. The said explanation is illustrated in the following graph:

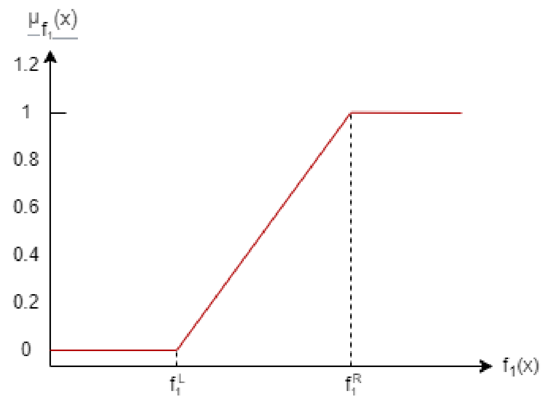


**Figure 1. Membership Function of Objective Function Return Portfolio**

2. Membership function of the objective function of the risk portfolio can be defined as:

$$\mu_{f_2}(x) = \begin{cases} 1 & if \quad f_2(x) \leq f_2^R \\ \dfrac{f_2^R - f_2(x)}{f_2^R - f_2^L} & if\ f_2^L < f_2(x) < f_2^R \\ 0 & if \quad f_2(x) \geq f_2^L \end{cases}$$

Where $f_2^L$ is the best lower limit (low aspiration level) and $f_2^R$ is the worst upper limit (high aspiration level) from the risk portfolio. The explanation is illustrated in the following graph:

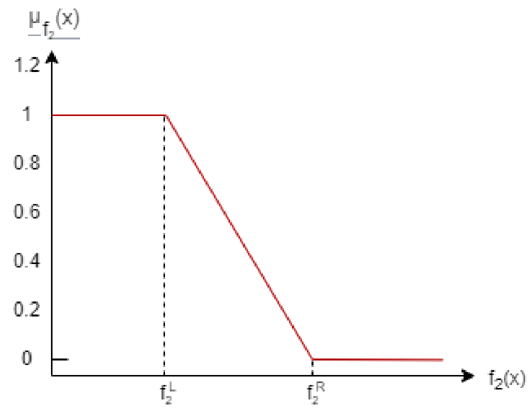


**Figure 2. Membership function of objective function risk portfolio**

The optimization model of fuzzy bi-objective in solving portfolio problems can be written in the following formula:

$$\max \lambda \tag{7}$$

that depends on

$$\lambda \leq \mu_{f_1}(x),$$

$$\lambda \leq \mu_{f_2}(x),$$

$$\sum_{i=1}^{n} x_i = 1,$$

$$x \geq 0, i = 1,2,3, \cdots, n,$$

$$0 \leq \lambda \leq 1$$

where λ is an additional variable that represents membership function.

Solving the lambda model by interactive fuzzy approach consists of several following steps:

1. Forming mathematic model in Equation (6)
2. Solving Equation (6) as the objective single problem in relation to objective function of return and risk. Mathematically,
   a. Objective function of return

   $$\max f_1(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \; and \; x_i \geq 0, \qquad i = 1,2,3, \cdots, n$$

   b. Objective function of return

   $$\max f_2(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \; and \; x_i \geq 0, \qquad i = 1,2,3, \cdots, n$$

   Assuming $x^1$ and $x^2$ are defined as the optimum solution by solving a single objective problem with an objective function of return and risk. If both solutions are defined as the same $x^1 = x^2 = (x_1, x_2, \cdots, x_n)$, an efficient solution can be obtained. If it doesn't stop, it's continued into the next step, Step 3.
3. Evaluation of obtained objective function

   Determining the worst lower limit ($f_1^L$) and the best upper limit ($f_1^R$) are done for the objective function of the return, while the best lower limit ($f_2^L$) and the worst upper limit ($f_2^R$) are done for the objective function of risk. The obtained limits are obtained and written in the following formulas:

$$f_1^R = f_1(x^1)$$

$$f_1^L = f_1(x^2)$$

$$f_2^L = f_2(x^2)$$

$$f_2^R = f_2(x^1)$$

4. Determining membership function for *return* and risk
5. Developing mathematic model in Equation 7 and solving it. Giving the solution to the investor [22].

### 2.3 Procedures

This research was conducted with the following procedures:

1. Studying literature that is related to portfolio, stock, and portfolio optimization with fuzzy.
2. Accessing research data through website https://finance.yahoo.com.
3. Choosing 20 company shares that are available in LQ45 market.
4. Calculating the return of each share and the expected return.
5. Determining which share that is going to be used in portfolio selection where the chosen share is the share that shows positive expected return.
6. Calculating the risk between the chosen shares.
7. Forming mathematic model, model of bi-objective optimization portfolio.
8. Solving the problem of model of bi-objective optimization portfolio as objective singular problem in relation of return objective function and risk. Mathematically, it can be written as,
   a. Objective function of return
   $$\max f_1(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \text{ } and \text{ } x_i \geq 0, \qquad i = 1,2,3,\cdots,n$$
   b. Objective function of return
   $$\max f_2(x) \text{ with constraint } \sum_{i=1}^{n} x_i = 1 \text{ } and \text{ } x_i \geq 0, \qquad i = 1,2,3,\cdots,n$$
9. Evaluating the objective function over the obtained result:

   Determining the worst lower limit ($f_1^L$) and the best upper limit ($f_1^R$) for objective function of return and the best lower limit ($f_2^L$) and the worst upper limit ($f_2^R$) for objective function of risk. The formula of obtained limits are:
   $$f_1^R = f_1(x^1)$$
   $$f_1^L = f_1(x^2)$$
   $$f_2^L = f_2(x^2)$$
   $$f_2^R = f_2(x^1)$$
10. Determining the membership function of return and risk.
11. Determining new model of the obtained membership function.
12. Doing an optimization on the new model and obtaining the result of portfolio optimization.

## 3. RESULTS AND DISCUSSION

The share that is used for optimization in this study is extracted from the calculation of the expected return from 20 shares in the span of 1 January 2020 until 1 January 2021, whether it has a positive value, negative value, or even zero value. A negative value on expected return means loss. Table 1 is the data of 10 shares that fulfill the criteria, it has a positive expected return.

**Table 1. Recapitulation of Positive Expected Return**

| Share Core | Minimum |
|---|---|
| ADRO | 0.005059047 |
| ANTM | 0.125256193 |
| ASII | 0.007389624 |
| BBCA | 0.006712295 |
| BBRI | 0.003012295 |
| BBTN | 0.023017824 |
| BRPT | 0.006903961 |
| BSDE | 0.013594574 |
| ERAA | 0.069381350 |
| INCO | 0.055436168 |

The results shown in Table 1 are obtained using Equation (4). The portfolio will be formed based on the result of positive *e*xpected returns, which are the companies with share codes of ADRO, ANTM, ASII, BBCA, BBRI, BBTN, BRPT, BSDE, ERAA, and INCO. The expected return of mentioned share codes will be used as an objective function of return in portfolio optimization.

Table 1 shows that the value of positive expected return from each company can be referred to as the amount of return value. The portfolio will be formed according to the result of the positive expected return value, which is shown in companies whose share codes are ADRO, ANTM, ASII, BBCA, BBRI, BBTN, BRPT, BSDE, ERAA, and INCO. The expected return value of share code ACES is calculated by adding ACES' share return value from February 2020 until January 2021 before being divided by the number of periods from February 2020 until January 2021. The expected return from the said share code will be used in portfolio optimization as the expected purpose function.

The risk of share portfolio optimization is calculated by the covariant matrix $\sigma_{ij}$. The risk calculation for each share code will be done for the total of 10 shares that have been selected using the mathematical model in Equation (5). Table 2 is a covariant matrix between the chosen ten shares.

**Table 2. Covariant Matrix**

| | ADRO | ANTM | ASII | $\cdots$ | INCO |
|---|---|---|---|---|---|
| ADRO | 0.014027 | 0.003895 | 0.007622 | $\cdots$ | 0.005608 |
| ANTM | 0.003895 | 0.063823 | 0.026913 | $\cdots$ | 0.020877 |
| ASII | 0.007622 | 0.026913 | 0.020617 | $\cdots$ | 0.011754 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| INCO | 0.005608 | 0.020877 | 0.11754 | $\cdots$ | 0.015859 |

The portfolio will be formed by using the result obtained from the covariant matrix and later will be used as an objective function of risk on portfolio optimization.

The data above is the calculation result of 10 companies in which the risks between companies are identified, and then the data is used to form an optimization model for the risk purpose function. The portfolio then will be formed by using the obtained result from the covariant matrix and later will be used as a risk purpose function in portfolio optimization. For example, identifying the asset risk between ANTM and ADRO can be found by subtracting ANTM's return in every period and multiplying it with ADRO's expected to return in every period before adding it with the result of subtraction between ADRO in each period with ADRO's expected return. The overall result then will be divided by the expected return.

According to the mathematical model in equation 6, a bi-objective portfolio optimization model will be formed by using the expected return value and the following covariant matrix. In which $x_i$ stood for the company's assets as written in Table 3.

**Table 3. Forming Optimization Model**

| Share Core | Symbol |
|---|---|
| ADRO | $x_1$ |
| ANTM | $x_2$ |
| ASII | $x_3$ |
| BBCA | $x_4$ |
| BBRI | $x_5$ |
| BBTN | $x_6$ |
| BRPT | $x_7$ |
| BSDE | $x_8$ |
| ERAA | $x_9$ |
| INCO | $x_{10}$ |

$$\max f_1(x) = 0.005059047x_1+0.125256193x_2+0.0073896241x_3+0.006712202x_4+0.003012295x_5 +0.023017824x_6+0.006903961x_7+0.013594574x_8+0.069381350x_9+0.055436168x_{10}$$

$$\begin{aligned}\min f_2(x) = {} & 0.014027x_1x_1+0.063823x_2x_2+0.020617x_3x_3+0.006214x_4x_4+0.015724x_5x_5 \\ & +0.072675x_6x_6+0.107596x_7x_7+0.0239071x_8x_8+0.039401x_9x_9+0.015859x_{10}x_{10} \\ & +0.003895x_1x_2+0.007622x_1x_3+0.003014x_1x_4+0.009273x_1x_5+0.002771x_1x_6 \\ & +0.002688\mathrm{x}x_1x_7+0.005300x_1x_8+0.000201x_1x_9+0.005608x_1x_{10}+0.026913x_2x_3 \\ & +0.014066x_2x_4+0.015750x_2x_5+0.02986x_2x_6+0.021001x_2x_7+0.026859x_2x_8 \\ & +0.032864x_2x_9+0.020877x_2x_{10}+0.007577x_3x_4+0.012476x_3x_5+0.016134x_3x_6 \\ & +0.008705x_3x_7+0.012245x_3x_8+0.016975x_3x_{10}+0.011754x_3x_{10}+0.007677x_4x_5 \\ & +0.015393x_4x_6-0.001939x_4x_7+0.008068x_4x_8+0.007249x_4x_9+0.005356x_4x_{10} \\ & +0.022054x_5x_6+0.001801x_5x_7+0.014135x_5x_8+0.011427x_5x_9+0.00913x_5x_{10} \\ & +0.014710x_6x_7+0.034540x_6x_8+0.023911x_6x_9+0.013107x_6x_{10}+0.01912x_7x_8 \\ & +0.031895x_7x_9+0.020655x_7x_{10}+0.017318x_8x_9+0.010113x_8x_{10}+0.016136x_9x_{10}\end{aligned}$$

with constraint function written as:

$$\sum_{i=1}^{n} x_i = 1,$$

In which variables $x_1, x_2, \dots \dots, x_{10}$ show the amount of the proportional investment value of happening $x_i$ is one of the happening is going to happen and no other possibilities left.

$$x_i \geq 0, \;\; i = 1,2,3,4,5,6,7,8,9,10.$$

The formula is obtained by changing the double-objective optimization model into a single-objective optimization model to determine the boundaries that are used to form the membership function. In determining the lower and upper limit, the bi-objective portfolio optimization model is formed into the following single-objective optimization model:

1. Objective Function of return

$$\max f_1(x) = 0.005059047x_1+0.125256193x_2+0.0073896241x_3+0.006712202x_4 +0.003012295x_5+0.023017824x_6+0.006903961x_7+0.013594574x_8 +0.069381350x_9+0.055436168x_{10}$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1,$$

$x_i \geq 0, \;\; i = 1,2,3,4,5,6,7,8,9,10.$

2. Objective Function of Risk

$$\begin{aligned}\min f_2(x) = {} & 0.014027x_1x_1+0.063823x_2x_2+0.020617x_3x_3+0.006214x_4x_4+0.015724x_5x_5 \\ & +0.072675x_6x_6+0.107596x_7x_7+0.0239071x_8x_8+0.039401x_9x_9+0.015859x_{10}x_{10} \\ & +0.003895x_1x_2+0.007622x_1x_3+0.003014x_1x_4+0.009273x_1x_5+0.002771x_1x_6 \\ & +0.002688\text{x}x_1x_7+0.005300x_1x_8+0.000201x_1x_9+0.005608x_1x_{10}+0.026913x_2x_3 \\ & +0.014066x_2x_4+0.015750x_2x_5+0.02986x_2x_6+0.021001x_2x_7+0.026859x_2x_8 \\ & +0.032864x_2x_9+0.020877x_2x_{10}+0.007577x_3x_4+0.012476x_3x_5+0.016134x_3x_6 \\ & +0.008705x_3x_7+0.012245x_3x_8+0.016975x_3x_{10}+0.011754x_3x_{10}+0.007677x_4x_5 \\ & +0.015393x_4x_6-0.001939x_4x_7+0.008068x_4x_8+0.007249x_4x_9+0.005356x_4x_{10} \\ & +0.022054x_5x_6+0.001801x_5x_7+0.014135x_5x_8+0.011427x_5x_9+0.00913x_5x_{10} \\ & +0.014710x_6x_7+0.034540x_6x_8+0.023911x_6x_9+0.013107x_6x_{10}+0.01912x_7x_8 \\ & +0.031895x_7x_9+0.020655x_7x_{10}+0.017318x_8x_9+0.010113x_8x_{10}+0.016136x_9x_{10}\end{aligned}$$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0,\ \ i = 1,2,3,4,5,6,7,8,9,10$$

After doing optimization for each objective function, whether it's maximizing or minimalizing the objective function of risk and return, the following Table 4 obtained result:

**Table 4. The Optimization Result of Objective Function of Return and Risk**

| | $\boldsymbol{x^1}$ | $\boldsymbol{x^2}$ |
|---|---|---|
| Return $\left(f_1(x)\right)$ | 0.125256 | 0.00301229 |
| Risk $\left(f_2(x)\right)$ | 0.063823 | 0.00445511 |

with

$$f_1^R = f_1(x^1)$$
$$f_1^L = f_1(x^2)$$
$$f_2^L = f_2(x^2)$$
$$f_2^R = f_2(x^1)$$

Therefore, the membership function for objective function of expected return and portfolio risk are:

$$\mu_{f_1}(x) = \begin{cases} 1 & if\ f_1(x) \geq 0.125256 \\ \frac{f_1(x)-0.00301229}{0.125256-00301229} & if\ 0.00301229 < f_1(x) < 0.125256 \\ 0 & if\ f_1(x) \leq 0.00301229 \end{cases}$$

and

$$\mu_{f_2}(x) = \begin{cases} 1 & if\ f_2(x) \leq 0.00445511 \\ \dfrac{0.063823 - f_2(x)}{0.063823 - 0.00445511} & if\ 0.00445511 < f_2(x) < 0.063823 \\ 0 & if\ f_2(x) \geq 0.063823 \end{cases}$$

After the membership function is obtained for the objective function of expected return and risk, according to the mathematical model in Equation (7), a new model of bi-objective fuzzy optimization can be formed to solve the portfolio selection problem and can be written as the following formula:

$$Max \quad \lambda$$

in which $\lambda$ is an additional variable that represents membership function

$0.005059047x_1$+$0.125256193x_2$+$0.0073896241x_3$+$0.006712202x_4$+$0.003012295x_5$
+$0.023017824x_6$+$0.006903961x_7$+$0.013594574x_8$+$0.069381350x_9$+$0.055436168x_{10}$

$-0.12224371\lambda \geq 0.00201299,$
$0.014027x_1x_1$+$0.063823x_2x_2$+$0.020617x_3x_3$+$0.006214x_4x_4$+$0.015724x_5x_5$
+$0.072675x_6x_6$+$0.107596x_7x_7$+$0.0239071x_8x_8$+$0.039401x_9x_9$+$0.015859x_{10}x_{10}$
+$0.003895x_1x_2$+$0.007622x_1x_3$+$0.003014x_1x_4$+$0.009273x_1x_5$+$0.002771x_1x_6$+$0.002688\mathrm{x}x_1x_7$
+$x_7 0.005300x_1x_8$+$0.000201x_1x_9$+$0.005608x_1x_{10}$+$0.026913x_2x_3 + 0.014066x_2x_4$
+$0.015750x_2x_5$+$0.02986x_2x_6$+$0.021001x_2x_7$+$0.026859x_2x_8$+$0.032864x_2x_9$+$0.020877x_2x_{10}$
+$0.007577x_3x_4$+$0.012476x_3x_5$+$0.016134x_3x_6 + 0.008705x_3x_7$+$0.012245x_3x_8$
+$0.016975x_3x_9$+$0.011754x_3x_{10}$+$0.007677x_4x_5$+$0.015393x_4x_6$-$0.001939x_4x_7$
+$0.008068x_4x_8$+$0.007249x_4x_9$+$0.005356x_4x_{10} + 0.022054x_5x_6$+$0.001801x_5x_7$
+$0.014135x_5x_8$+$0.011427x_5x_9$+$0.00913x_5x_{10}$+$0.014710x_6x_7$+$0.034540x_6x_8$
+$0.023911x_6x_9$+$0.013107x_6x_{10}$+$0.01912x_7x_8 + 0.031895x_7x_9$+$0.020655x_7x_{10}$
+$0.017318x_8x_9$+$0.010113x_8x_{10}$+$0.016136x_9x_{10} + 0.5936788\lambda \leq 0.063823,$

with constraint function:

$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0,\ \ i = 1,2,3,4,5,6,7,8,9,10.$$

The above model is optimized, thus, there will be an obtained result of expected return, risk, and budget proportion that will be allocated from the portfolio, which is written as follows:

**Table 5. The Result of Return and Risk**

| $\lambda$ | ***Return*** | **Risk** |
|---|---|---|
| 0.694328 | 0.694328 | 0.0226022117 |

**Table 6. Budget Proportion of Optimization Result**

| **Share Code** | **Minimum** |
|---|---|
| ADRO | $0.1149577 \times 10^{-7}$ |
| ANTM | 0.4328280 |
| ASII | $0.2856066 \times 10^{-8}$ |
| BBCA | $0.7728622 \times 10^{-8}$ |
| BBRI | $0.4587149 \times 10^{-8}$ |
| BBTN | $0.6810672 \times 10^{-8}$ |
| BRPT | $0.3030953 \times 10^{-8}$ |
| BSDE | $0.4877755 \times 10^{-8}$ |
| ERAA | 0.1601465 |
| INCO | 0.4070255 |

Table 5 is the result of the new model optimization with variable $\lambda$, where the obtained value of optimized return is 0.0878895207 while the risk is 0.0226022117. Table 6 is the result of new model optimization with variable λ, where the budget proportion that is allocated for share code ANTM is 43.3%, and for the other share codes like ADRO, ASII, BBCA, BBRI, BBTN, and BRPT is close to zero, so the budget proportion that is allocated is 0%, on the other side the share code of ERAA has a value of 16%, and INCO is 40.7%.

## 4. CONCLUSIONS

The study explains the method of choosing a share that will be optimized according to criteria that have been decided by the writer. The chosen share then will be formed into a bi-objective optimization model using ten shares that have fulfilled the criteria. *The bi-objective* optimization model then will be changed into a single-objective with the purpose of finding the limit that will be used in the making of the fuzzy optimization model. The optimized fuzzy optimization model results in an optimal result where the return of the optimized portfolio is 8.8%, and the risk of the optimized portfolio is 2.3%.

## AKNOWLEDGEMENT

Thanks to the mathematics laboratory Department of Mathematics FAST Universitas Ahmad Dahlan Yogyakarta for the facilities and infrastructure so that this research can be completed.

## REFERENCES

[1] S. Kusumadewi, S. Hartati, A. Harjoko, and R. Wardoyo, "Fuzzy Multi-Attribute Decision-making (Fuzzy MADM)," *Yogyakarta Graha Ilmu*, pp. 78–79, 2006.

[2] J. Lu, G. Zhang, D. Ruan, and F. Wu, "Multi-objective group decision-making: methods, software and applications.," *J. Lu*, vol. 6, 2007.

[3] E. Raei, M. Reza Alizadeh, M. Reza Nikoo, and J. Adamowski, "Multi-objective decision-making for green infrastructure planning (LID-BMPs) in urban storm water management under uncertainty," *J. Hydrol.*, vol. 579, no. January, p. 124091, 2019, doi: 10.1016/j.jhydrol.2019.124091.

[4] E. SUSANTI, I. INDRAWATI, R. SITEPU, A. NABILA, and R. WULANDARI, "Optimasi Pendistribusian Produk Menggunakan Model Fuzzy Multiobjektif Cyclical Inventory Routing Problem," *E-Jurnal Mat.*, vol. 9, no. 1, p. 96, 2020, doi: 10.24843/mtk.2020.v09.i01.p285.

[5] M. Bagheri, A. Ebrahimnejad, S. Razavyan, F. Hosseinzadeh Lotfi, and N. Malekmohammadi, *Fuzzy arithmetic DEA approach for fuzzy multi-objective transportation problem*, no. July 2021. Springer Berlin Heidelberg, 2020.

[6] N. Chiadamrong and N. Sutthibutr, "Integrating a weighted additive multiple objective linear model with possibilistic linear programming for fuzzy aggregate production planning problems," *Int. J. Fuzzy Syst. Appl.*, vol. 9, no. 2, pp. 1–30, 2020, doi: 10.4018/IJFSA.2020040101.

[7] G. Primajati and A. Ahmad, "A Analisis Portfolio Investasi dengan Metode Mean Varian Dua Konstrain," *J. VARIAN*, vol. 2, no. 1, pp. 24–30, 2018, doi: 10.30812/varian.v2i1.319.

[8] H. Markowitz, "Portfolio Selection Harry Markowitz," *J. Finance*, vol. 7, no. 1, pp. 77–91, 1952.

[9] W. Chen and W. Xu, "A Hybrid Multiobjective Bat Algorithm for Fuzzy Portfolio Optimization with Real-World Constraints," *Int. J. Fuzzy Syst.*, vol. 21, no. 1, pp. 291–307, 2019, doi: 10.1007/s40815-018-0533-0.

[10] W. Zou, C. Li, and N. Zhang, "A T-S Fuzzy Model Identification Approach Based on a Modified Inter Type-2 FRCM Algorithm," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1104–1113, 2018, doi: 10.1109/TFUZZ.2017.2704542.

[11] R. C. Tsaur, C. L. Chiu, and Y. Y. Huang, "Fuzzy portfolio selection in covid-19 spreading period using fuzzy goal programming model," *Mathematics*, vol. 9, no. 8, pp. 1–15, 2021, doi: 10.3390/math9080835.

[12] Y. Li, B. Wang, A. Fu, and J. Watada, "Fuzzy portfolio optimization for time-inconsistent investors: a multi-objective dynamic approach," *Soft Comput.*, vol. 24, no. 13, pp. 9927–9941, 2020, doi: 10.1007/s00500-019-04504-3.

[13] M. Tavana, G. Khosrojerdi, H. Mina, and A. Rahman, "A hybrid mathematical programming model for optimal project portfolio selection using fuzzy inference system and analytic hierarchy process," *Eval. Program Plann.*, vol. 77, no. July, p. 101703, 2019, doi: 10.1016/j.evalprogplan.2019.101703.

[14] G. Primajati, A. Z. Amrullah, and A. Ahmad, "Analisis Portfolio Investasi dengan Metode Multi Objektif," *J. Varian*, vol. 3, no. 1, pp. 6–12, 2019, doi: 10.30812/varian.v3i1.476.

[15] P. Yuli Utami, Y. Arkeman, A. Buono, and I. Hermadi, "Peningkatan Performansi Multi Objektif Nsga-Ii Dengan Operator Mutasi Adaptif Pada Kasus Portfolio Reksadana Saham," *Cybernetics*, vol. 3, no. 02, p. 72, 2020, doi: 10.29406/cbn.v3i02.2194.

[16] E. B. Tirkolaee, A. Goli, M. Hematian, A. K. Sangaiah, and T. Han, "Multi-objective multi-mode resource constrained project scheduling problem using Pareto-based algorithms," *Computing*, vol. 101, no. 6, pp. 547–570, 2019, doi: 10.1007/s00607-018-00693-1.

[17] H. Li, Z. Xu, and W. Wei, "Bi-objective scheduling optimization for discrete time/cost trade-offin projects," *Sustain.*, vol. 10, no. 8, pp. 1–15, 2018, doi: 10.3390/su10082802.

[18] A. Kameli, N. Javadian, and A. Daghbandan, "Multi-period and Multi-objective Stock Selection Optimization Model Based on Fuzzy Interval Approach," *Int. J. Eng.*, vol. 32, no. 9, pp. 1306–1311, 2019, doi: 10.5829/ije.2019.32.09c.11.

[19] D. Wu, C. T. Lin, J. Huang, and Z. Zeng, "On the Functional Equivalence of TSK Fuzzy Systems to Neural Networks, Mixture of Experts, CART, and Stacking Ensemble Regression," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 10, pp. 2570–2580, 2020, doi: 10.1109/TFUZZ.2019.2941697.

[20] N. S. Al-Mumtazah and S. Surono, "Quadratic Form Optimization with Fuzzy Number Parameters: Multiobjective Approaches," *Int. J. Fuzzy Syst.*, vol. 22, no. 4, pp. 1191–1197, 2020, doi: 10.1007/s40815-020-00808-x.

[21] P. Protter, M. Capinski, and T. Zastawniak, *Mathematics for Finance: An Introduction to Financial Engineering*, vol. 111, no. 10. 2004.

[22] P. Gupta, *Portfolio Optimization*, vol. 306. Springer, 2021.