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THE PROPERTIES OF ROUGH V-COEXACT SEQUENCE IN ROUGH GROUP

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Abstract

In ring and module theory, the concept of an exact sequence is commonly employed. The exact sequence is generalized into the U-exact sequence and the V-coexact sequence. Rough set theory has also been applied to a variety of algebraic structures, including groups, rings, modules, and others. In this study, we investigated characteristics of a rough V-coexact sequence in rough groups.

Keywords: *exact sequence, V-coexact sequence, rough group.*

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1. INTRODUCTION

One of the most fundamental concepts in algebraic structures is the exact sequence [1]. The concept of an exact sequence is developed in module theory into U-exact sequences, V -exact sequences, and X -sub-exact sequences. The generalization of the exact sequence is the U-exact sequence [2]. The V-coexact sequence is a dualization of the U-exact sequence. Anvanriyeh and Davvaz show how U-split sequences and projective modules are related in [3]. Generalization of Schanuel's Lemma and the relationship between quasi-exact sequences and their submodules can be obtained using the generalization of an exact sequences [4]. The generalization of Snake's Lemma and Five's Lemma was then studied in [5]. The X-sub-exact sequence is a generalization of the exact sequence [6]. The concept of an exact sequence is used to define an X-sublinearly independent set [7]. In 2018, the U-generator concept was introduced based on V-coexact sequences [8]. The concept of a U_v-generator and an X-sublinear independent module family were utilized to develop by (X, V)-basis and U-free modules in the same year [9].

Rough set theory is a mathematical concept initially introduced in 1982 [10]. Several concepts of algebraic structure in the rough set have been studied, including homomorphisms on rough sets [11], rough groups [12], rough subgroups [13], application of rough sets to computers [14], projective modules on rough sets [15], anti-homomorphism on rough prime ideals [16], and rough homomorphisms on rough set, rough group, and rough semigroups in approximation space [17]. Furthermore, Sinha gives a rough exact sequence of rough modules over rough rings [18].

Many researchers discuss the application of rough set theory in several aspects of science, including data mining and algebraic elements. In this research, we will give the properties of a rough V-coexact sequence in a rough group.

2. RESEARCH METHODS

The research methods rely on the upper and lower approximation spaces, the rough group, the exact sequence, the V-coexact sequence, and literature. We first define the rough set using its binary operation and define the rough V-coexact sequence of the rough groups. We also investigate the properties of the rough group and use the finite set to construct an example of the rough V-coexact sequence of the rough groups. Finally, we investigate the properties of the rough V-coexact sequence of rough groups.

The following are the stages of the research.

1. We define the rough V-coexact sequence of the rough groups.
2. We analyze the properties of the rough V-coexact sequence.
3. We construct the examples of the rough group, rough group homomorphisms, and rough V-coexact sequences by using the finite set.

3. RESULTS AND DISCUSSION

3.1. Rough V-Coexact Sequence in Rough Group

Motivated by the definition of the V-coexact sequence of the R-modules, we define the rough V-coexact sequence of the rough groups as follows.

Definition 1. Let (U, θ) be an approximation space, A, B, C the rough groups in (U, θ), and V the rough subgroups of A in (U, θ). If $f(\overline{V}) = \ker(g)$, this sequence

$$\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$$

is called rough V-coexact in A.

Next, we give the construction of a rough subgroup in an approximation space.

Example 1 Let $\mathbb{Z}_{16} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}, \overline{8}, \overline{9}, \overline{10}, \overline{11}, \overline{12}, \overline{13}, \overline{14}, \overline{15}\}$. We define $a\theta b$ if only if $a - b = 4k$ for some $k \in \mathbb{Z}$. From this equivalence relation, we have four equivalence classes in the following table.

Tabel 1. The Equivalence Classes of $\mathbb{Z}_{16}$

The Equivalence Class	The Element of the Class
E_1	$\{\overline{1},\overline{5},\overline{9},\overline{13}\}$
E_2	$\{\overline{2},\overline{6},\overline{10},\overline{14}\}$
E_3	$\{\overline{3},\overline{7},\overline{11},\overline{15}\}$
E_4	$\{\overline{0},\overline{4},\overline{8},\overline{12}\}$

Furthermore,we give three rough groups to form a rough V-coexact sequence of rough groups.
Let $X_1 = \{\overline{1},\overline{2},\overline{8},\overline{14},\overline{15}\}$. We have $\overline{X_1} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_{16}$.

Next, we will prove that X_1 is a rough group.

Tabel 2. Cayley Table for X_1

$+_{16}$	$\overline{1}$	$\overline{2}$	$\overline{8}$	$\overline{14}$	$\overline{15}$
$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{9}$	$\overline{15}$	$\overline{0}$
$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{10}$	$\overline{0}$	$\overline{1}$
$\overline{8}$	$\overline{9}$	$\overline{10}$	$\overline{0}$	$\overline{6}$	$\overline{7}$
$\overline{14}$	$\overline{15}$	$\overline{0}$	$\overline{6}$	$\overline{10}$	$\overline{13}$

1. Table 2 shows that $x(+_{16})y \in \overline{X_1}$ for every $x, y \in X_1$,
2. the associative property is satisfied in $\overline{X_1}$,
3. there exist $\overline{0} \in \overline{X_1}$, such that $x(+_{16})\overline{0} = \overline{0}(+_{16})x = x$ for every $x \in \overline{X_1}$,
4. for every $x \in X_1$, there exist $y \in X_1$ such that $x(+_{16})y = \overline{0}$,

Tabel 3. Inverse Table for X_1

$x \in X_1$	Inverse of x
$\overline{1}$	$\overline{15}$
$\overline{2}$	$\overline{14}$
$\overline{8}$	$\overline{8}$

Based on Table 3, we can see that every element of X_1 has a rough inverse in X_1. So, it proves that X_1 is a rough group on $\mathbb{Z}_{16}$.

Let $X_2 = \{\overline{5},\overline{6},\overline{8},\overline{10},\overline{11}\}$, $\overline{X_2} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_{16}$. We will prove that X_2 is a rough group.

Tabel 4. Cayley Table for X_2

$+_{16}$	$\overline{5}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$\overline{11}$
$\overline{5}$	$\overline{10}$	$\overline{11}$	$\overline{13}$	$\overline{15}$	$\overline{0}$
$\overline{6}$	$\overline{11}$	$\overline{12}$	$\overline{14}$	$\overline{0}$	$\overline{1}$
$\overline{8}$	$\overline{13}$	$\overline{14}$	$\overline{0}$	$\overline{2}$	$\overline{3}$
$\overline{10}$	$\overline{15}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{5}$
$\overline{11}$	$\overline{0}$	$\overline{2}$	$\overline{3}$	$\overline{5}$	$\overline{6}$

1. Table 4, shows that $x(+_{16})y \in \overline{X_2}$ for every $x, y \in X_2$,
2. the associative property is satisfied in $\overline{X_2}$,
3. there exist $\overline{0} \in \overline{X_2}$, such that $x(+_{16})\overline{0} = \overline{0}(+_{16})x = x$, for every $x \in \overline{X_2}$,
4. for every $x \in X_2$, there exist $y \in X_2$ such that $x(+_{16})y = \overline{0}$,

Tabel 5. Inverse Table for X_2

$x \in \mathrm{X}_2$	Inverse of x
$\overline{5}$	$\overline{11}$
$\overline{6}$	$\overline{10}$
$\overline{8}$	$\overline{8}$

Based on Table 5, we can see that every element of X_2 has a rough inverse in X_2. Hence, it proves that X_2 is a rough group on $\mathbb{Z}_{16}$.

Let $\mathrm{X}_3 = \{\overline{0}, \overline{6}, \overline{8}, \overline{10}\}, \overline{\mathrm{X}_3} = \mathrm{E}_2 \cup \mathrm{E}_4 = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}, \overline{12}, \overline{14}\}$. Next, we will prove that X_3 is a rough group.

Tabel 6. Cayley Table for X_3

$+_{16}$	$\overline{0}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$+_{16}$
$\overline{0}$	$\overline{0}$	$\overline{6}$	$\overline{8}$	$\overline{10}$	$\overline{0}$
$\overline{6}$	$\overline{6}$	$\overline{12}$	$\overline{14}$	$\overline{0}$	$\overline{6}$
$\overline{8}$	$\overline{8}$	$\overline{14}$	$\overline{0}$	$\overline{2}$	$\overline{8}$
$\overline{10}$	$\overline{10}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{10}$

1. Table 6 shows that $\mathrm{x}(+_{16})\mathrm{y} \in \overline{\mathrm{X}_3}$ for every $\mathrm{x}, \mathrm{y} \in \mathrm{X}_3$,
2. The associative property is satisfied in $\overline{\mathrm{X}_3}$,
3. There exist $\overline{0} \in \overline{\mathrm{X}_3}$, such that $x(+_{16})\overline{0} = \overline{0}(+_{16})x = x$ for every $x \in \overline{\mathrm{X}_3}$,
4. For every $x \in \mathrm{X}_3$, there exist $y \in \mathrm{X}_3$ such that $x(+_{16})y = \overline{0}$.

Tabel 7. Inverse Table for X_3

$x \in \mathrm{X}_3$	Inverse of x
0	$\overline{0}$
$\overline{6}$	$\overline{10}$
$\overline{8}$	$\overline{8}$

Based on Table 7, we can see that every element of X_3 has a rough inverse in X_3. Hence, it proves that X_3 is a rough group on $\mathbb{Z}_{16}$.

Then $\mathrm{V} \subseteq \mathrm{X}_1$, let $\mathrm{V} = \{\overline{2}, \overline{8}, \overline{14}\}, \overline{\mathrm{V}} = \mathrm{E}_4 = \mathrm{E}_2 \cup \mathrm{E}_4 = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}, \overline{12}, \overline{14}\}$ is a rough subgroup of X_1. We can see that $\overline{2}(+_{16})\overline{14} = \overline{0} \in \overline{\mathrm{V}}$ and $(\overline{2})^{-1} = \overline{14}$. After that, we form a sequence $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ with f is identity function, and $g(a) = 2a$, for every $a \in \overline{\mathrm{X}_2}$. We have $\mathrm{V} \subseteq \mathrm{X}_1$. We will show the sequence $\overline{\mathrm{X}_1} \xrightarrow{\mathrm{f}} \overline{\mathrm{X}_2} \xrightarrow{\mathrm{g}} \overline{\mathrm{X}_3}$ is a rough V-coexact sequence. Since $f(\overline{V}) = \ker(\mathrm{g}) = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}, \overline{12}, \overline{14}\}$, we have $\overline{X_1} \xrightarrow{f} \overline{X_2} \xrightarrow{g} \overline{X_3}$ is rough V-exact sequence.

Next, we will give the properties of the rough V-coexact sequence.

Proposition 1 Let $\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$ is a rough exact sequence. If A' rough subgroup of A, B' rough subgroup of B, C' rough subgroup of c, and $\overline{A} = \overline{A'}, \overline{B} = \overline{B'}, \overline{C} = \overline{C'}$ then $\overline{A'} \xrightarrow{f} \overline{B'} \xrightarrow{g} \overline{C'}$ is is a rough exact sequence.

Proof. We know $\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$ is a rough exact sequence, then $\mathrm{im}(f) = \ker(g)$. Next, with homomorphism rough f and g in the same sequence, we have $\overline{A'} \xrightarrow{f} \overline{B'} \xrightarrow{g} \overline{C'}$ is is a rough exact sequence.

Moreover, we give a illustration of Proposition 1

Example 2 Let $\mathbb{Z}_{16}=\{\overline{0},\overline{1},\overline{2},\overline{3},\overline{4},\overline{5},\overline{6},\overline{7},\overline{8},\overline{9},\overline{10},\overline{11},\overline{12},\overline{13},\overline{14},\overline{15}\}$. We define $a\theta b$ if only if $a-b=4k$ for some $k\in\mathbb{Z}$. From this equivalence relation, we have four equivalence classes as follows:
$E_1=\{\overline{1},\overline{5},\overline{9},\overline{13}\}$,
$E_2=\{\overline{2},\overline{6},\overline{10},\overline{14}\}$,
$E_3=\{\overline{3},\overline{7},\overline{11},\overline{15}\}$,
$E_4=\{\overline{0},\overline{4},\overline{8},\overline{12}\}$.

Furthermore, we give three rough groups to form a rough exact sequence of rough groups.

Let $X_1=\{\overline{0},\overline{1},\overline{2},\overline{8},\overline{14},\overline{15}\}$. We have $\overline{X_1}$=$\mathbb{Z}_{16}$.
1. For every $x,y\in X_1$, $x(+_{16})y\in\overline{X_1}$;
2. The associative property is satisfied in $\overline{X_1}$;
3. There exists $\overline{0}\in\overline{X_1}$, such that for every $\bar{x}\in\overline{X_1}$, $\bar{x}(+_{16})\overline{0}=\overline{0}(+_{16})\bar{x}=\bar{x}$;
4. For every $\bar{x}\in X_1$, there exists $\bar{y}\in X_1$ such that $\bar{x}(+_{16})\bar{y}=\overline{0}$ or $\bar{y}=(\bar{x})^{-1}$, that is
$(\overline{0})^{-1}=\overline{0}\in X_1$, $(\overline{1})^{-1}=\overline{15}\in X_1$, $(\overline{2})^{-1}=\overline{14}\in X_1$, $(\overline{8})^{-1}=\overline{8}\in X_1$, $(1\overline{5})^{-1}=\overline{1}\in X_1$,.
$(\overline{14})^{-1}=\overline{2}\in X_1$.
So, X_1 is a rough group.

Let $X_2=\{\overline{0},\overline{5},\overline{6},\overline{8},\overline{10},\overline{11}\}$, then $\overline{X_2}$=$\mathbb{Z}_{16}$.
1. For every $x,y\in X_2$, $x(+_{16})y\in\overline{X_2}$;
2. The associative property is satisfied in in $\overline{X_2}$;
3. There exists $\overline{0}\in\overline{X_2}$, such that for every $\bar{x}\in\overline{X_2}$, $\bar{x}(+_{16})\overline{0}=\overline{0}(+_{16})\bar{x}=\bar{x}$;
4. For every $\bar{x}\in X_2$, there exist $\bar{y}\in X_2$ such that $\bar{x}(+_{16})\bar{y}=\overline{0}$ or $\bar{y}=(\bar{x})^{-1}$, that is
$(\overline{0})^{-1}=\overline{0}\in X_2$, $(\overline{5})^{-1}=\overline{11}\in X_2$, $(\overline{6})^{-1}=\overline{10}\in X_2$, $(\overline{8})^{-1}=\overline{8}\in X_2$, $(1\overline{0})^{-1}=\overline{6}\in X_2$,.
$(\overline{11})^{-1}=\overline{5}\in X_2$.
So, X_2 is a rough group.

Let $X_3=\{\overline{0},\overline{3},\overline{6},\overline{8},\overline{10},\overline{13}\}$, then $\overline{X_3}$=$\mathbb{Z}_{16}$.
1. For every $x,y\in X_3$, $x(+_{16})y\in\overline{X_3}$;
2. The associative property is satisfied in $\overline{X_3}$;
3. There exists $\overline{0}\in\overline{X_3}$, such that for every $\bar{x}\in\overline{X_3}$, $\bar{x}(+_{16})\overline{0}=\overline{0}(+_{16})\bar{x}=\bar{x}$;
4. For every $\bar{x}\in X_3$, there exist $\bar{y}\in X_3$ such that $\bar{x}(+_{16})\bar{y}=\overline{0}$ or $\bar{y}=(\bar{x})^{-1}$, that is
$(\overline{0})^{-1}=\overline{0}\in X_3$, $(\overline{3})^{-1}=\overline{13}\in X_3$, $(\overline{6})^{-1}=\overline{10}\in X_3$, $(\overline{8})^{-1}=\overline{8}\in X_3$, $(1\overline{0})^{-1}=\overline{6}\in X_3$,.
$(\overline{13})^{-1}=\overline{13}\in X_3$.
So, X_3 is a rough group.

Next, we form a sequence $\overline{X_1}\xrightarrow{f}\overline{X_2}\xrightarrow{g}\overline{X_3}$, where $f(a)=a$ mod 16, for every $a\in\overline{X_1}$ and g is an identity function. We have $\mathrm{im(f)}=\ker(\mathrm{g})=\mathbb{Z}_{16}$. Hence $\overline{X_1}\xrightarrow{f}\overline{X_2}\xrightarrow{g}\overline{X_3}$ is rough exact sequence.

After that, we give $X_1'\subseteq X_1$.
Let $X_1'=\{\overline{1},\overline{2},\overline{8},\overline{14},\overline{15}\}$, then $\overline{X_1'}=\mathbb{Z}_{16}$. We can see X_1' is a subgroup rough of X, because every element in X_1' has element rough inverse in X_1', and every $x,y\in X_1'$, $x(+_{16})y\in\overline{X_1'}$.

Next, we give $X_2'\subseteq X_2$.
Let $X_2'=\{\overline{5},\overline{6},\overline{8},\overline{10},\overline{11}\}$, then $\overline{X_2'}=\mathbb{Z}_{16}$. We can see X_2' is a subgroup rough of X, because every element in X_2' has element rough inverse in X_2', and every $x,y\in X_2'$, $x(+_{16})y\in\overline{X_2'}$.

Furthermore, we give $X_3'\subseteq X_3$.
Let $X_3'=\{\overline{3},\overline{6},\overline{8},\overline{10},\overline{13}\}$, then $\overline{X_3'}=\mathbb{Z}_{16}$. We can see X_3' is a subgroup rough of X, because every element in X_3' has element rough inverse in X_3', and every $x,y\in X_3'$, $x(+_{16})y\in\overline{X_3'}$.
Next, we form a sequence $\overline{X_1'}\xrightarrow{f}\overline{X_2'}\xrightarrow{g}\overline{X_3'}$ with f, g homomorphism rough group is $f{:}\,a$ mod 16, and

g: identity function. Since $\overline{X_1}' \xrightarrow{f} \overline{X_2}'$ with $f: a \bmod 16$, and $\overline{X_2}' \xrightarrow{g} \overline{X_3}'$ with g: identity function, we can have $\mathrm{im}(f) = \ker(g) = \mathbb{Z}_{16}$. Hence $\overline{X_1}' \xrightarrow{f} \overline{X_2}' \xrightarrow{g} \overline{X_3}'$ is rough exact sequence.

After we construct rough V-coexact sequence, next we define the properties of rough group in approximation spaces with finite sets.

3.2. The Properties in a Rough Groups

Proposition 2 *Given an approximation space (U, θ), V the rough group in the approximation space (U, θ), and $X_1, X_2, X_3, \dots, X_n$ a subgroup of rough group V. If $\overline{X_1} \cap \overline{X_2} \cap \dots \cap \overline{X_n} = \overline{X_1 \cap X_2 \cap \dots \cap X_n}$, then $X_1 \cap X_2 \cap \dots \cap X_n$ is a rough subgroup of V of in approximation space (U, θ).*

Proof. Given a rough group V, $X_1, X_2, X_3, \dots, X_n$ a rough subgroup of V. We can show that $X_1 \cap X_2 \cap \dots \cap X_n$ is a rough subgroup V if $\overline{X_1} \cap \overline{X_2} \cap \dots \cap \overline{X_n} = \overline{X_1 \cap X_2 \cap \dots \cap X_n}$ as follows.
1. We have $X_1 \cap X_2 \cap \dots \cap X_n \neq \emptyset$.
2. For every $x, y \in X_1 \cap X_2 \cap \dots \cap X_n$, we have $x - y \in \overline{X_1}$, $x - y \in \overline{X_2}$, ...and $x - y \in \overline{X_n}$.

So, $X_1 \cap X_2 \cap \dots \cap X_n$ is a rough subgroup of V of in approximation space (U, θ). □

Furthermore, we give an example of rough subgroup V- of rough groups using the finite set as follows.

Example 3 Let $\mathbb{Z}_{50} = \{\overline{0}, \overline{1}, \overline{2}, \dots, \overline{49}\}$, we define $a\theta b$ if only if $a - b = 4k$ with $k \in \mathbb{Z}$, for every $a, b \in U$. We know that θ is an equivalence relation on U. From this equivalence relation, we have four equivalence classes in the following table.

Tabel 8. The Equivalence Classes of $\mathbb{Z}_{50}$

The Equivalence Class	The Element of the Class
E_1	$\{\overline{0}, \overline{4}, \overline{8}, \overline{12}, \overline{16}, \overline{20}, \overline{24}, \overline{28}, \overline{32}, \overline{36}, \overline{40}, \overline{44}, \overline{48}\}$
E_2	$\{\overline{1}, \overline{5}, \overline{9}, \overline{13}, \overline{17}, \overline{21}, \overline{23}, \overline{25}, \overline{29}, \overline{33}, \overline{37}, \overline{41}, \overline{45}, \overline{49}\}$
E_3	$\{\overline{2}, \overline{6}, \overline{10}, \overline{14}, \overline{18}, \overline{22}, \overline{26}, \overline{30}, \overline{34}, \overline{38}, \overline{42}, \overline{46}\}$
E_4	$\{\overline{3}, \overline{7}, \overline{11}, \overline{15}, \overline{19}, \overline{23}, \overline{27}, \overline{31}, \overline{35}, \overline{39}, \overline{43}, \overline{47}\}$

Give $X = \{\overline{4}, \overline{5}, \overline{6}, \overline{8}, \overline{10}, \overline{13}, \overline{15}, \overline{21}, \overline{29}, \overline{35}, \overline{37}, \overline{40}, \overline{42}, \overline{44}, \overline{45}, \overline{46}\} \subseteq \mathbb{Z}_{50}$. Then $\underline{X} = \emptyset$, $\overline{X} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_{50}$. Hence the rough set is $\mathrm{Apr}(X) = (\underline{X}, \overline{X}) = (\{\}, \{\overline{0}, \overline{1}, \overline{2}, \dots, \overline{49}\})$. Next, we define the binary operation $(+_{50})$ on $\mathbb{Z}_{50}$. We will show that X is a rough group.
1. For every $a, b \in X$, $a(+_{50})b \in \overline{X}$,
2. Association property holds in $\overline{X}$,
3. There exist $0 \in \overline{X}$ such that for every $x \in X$, $x(+_{50})0 = 0(+_{50})x = x$,
4. In the following table, we can show that every element of X has a rough inverse in X.

Tabel 9. Inverse Table for X

$x \in X$	Inverse of x
$\overline{4}$	$\overline{46}$
$\overline{5}$	$\overline{45}$
$\overline{6}$	$\overline{44}$
$\overline{8}$	$\overline{42}$
$\overline{10}$	$\overline{40}$
$\overline{13}$	$\overline{37}$
$\overline{15}$	$\overline{35}$
$\overline{21}$	$\overline{29}$

Basic in Table 9, every element of X has an inverse in X. So, it proves that X is a rough group on $\mathbb{Z}_{50}$.
If we choose a subset of X that is $X_1 = \{\overline{4}, \overline{8}, \overline{10}, \overline{40}, \overline{42}, \overline{46}\}$, we have $\underline{X_1} = \emptyset$ and $\overline{X_1} = E_1 \cup E_3$, so X_1 is a rough set. Then, $X_2 = \{\overline{4}, \overline{8}, \overline{12}, \overline{38}, \overline{42}, \overline{46}\}$, we have $\underline{X_2} = \emptyset$ and $\overline{X_2} = E_1 \cup E_3$, so X_2 is a rough set. We can see that every element of X_1 has a rough inverse in X_1, and for every $x, y \in X_1, x, y \in \overline{X_1}$ so that X_2. Hence X_1 and X_2 are rough groups in $(\mathbb{Z}_{50}, \theta)$, so that it can be said to be a rough group. Furthermore, we will show that $X_1 \cap X_2$ is a rough group in approximation space $\mathbb{Z}_{50}$. Based on the two examples above, we can see $\overline{X_1} \cap \overline{X_2} = \overline{X_1 \cap X_2} = E_1 \cup E_3$. So, $X_1 \cap X_2$is rough subgroup X.

Moreover, we give proposition about cross product in rough group.
Proposition 3 *If M_1, M_2 are a rough group in the approximation space (U, θ), then $M_1 \times M_2$ ar a rough group in approximation space $(U \times U, \theta^2)$ with $(a_1, a_2)\theta^2(b_1, b_2)$ if only if $(a_1\theta b_1, a_2\theta b_2)$ for every $(a_1, a_2), (b_1, b_2) \in M_1 \times M_2$.*

Proof First, we show that θ^2 is an equivalence relation in $U \times U$.
1. Given $(a, b) \in U \times U$, then $(a, b)\theta^2(a, b)$. So, θ^2 is reflective,
2. given $(a, b), (c, d) \in U \times U$ with $(a, b)\theta^2(c, d)$, then $(c, d)\theta^2(a, b)$. So, θ^2 is symmetrical,
3. given $(a, b), (c, d), (e, f) \in U \times U$ with $(a, b)\theta^2(c, d)$ and $(c, d)\theta^2(e, f)$, then $(a, b)\theta^2(e, f)$. So, θ^2 is transitive.

So, it proves θ^2 is an equivalence relation in $U \times U$.

Next, we will show $M_1 \times M_2$ is a rough group in approximation space $(U \times U, \theta^2)$.
We know $\langle M_1, *_1\rangle$ and $\langle M_2, *_2\rangle$ are rough group defined by binary operation $M_1 \times M_2$ is $(a_1, b_1) * (a_2, b_2) = (a_1 *_1 a_2, b_1 *_2 b_2)$.
1. For every $(a_1, b_1), (a_2, b_2) \in M_1 \times M_2, (a_1 *_1 a_2, b_1 *_2 b_2) \in M_1 \times M_2$.
2. Association property holds in $M_1 \times M_2$.
3. There exist $(e_1, e_2) \in \overline{M_1 \times M_2}$ such that for every $a_1 \in M_1$ and $a_2 \in M_2$, $e_1 *_1 a_1 = a_1 *_1 e_1 = a_1$ and $e_2 *_2 a_2 = a_2 *_2 e_2 = a_2$.
4. For every $(a_1, a_2) \in M_1 \times M_2$ has rough invers $({a_1}^{-1}, {a_2}^{-1}) \in M_1 \times M_2$, such that $(a_1, a_2) * ({a_1}^{-1}, {a_2}^{-1}) = (a_1 *_1 {a_1}^{-1}, a_2 *_2 {a_2}^{-1}) = (e_1, e_2)$.

So, it proves $M_1 \times M_2$ is rough group rough in approximation space $(U \times U, \theta^2)$. □

Next, we give the illustration of Proposition 3.

Example 4 Given a non-empty set $\mathbb{Z}_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$, by definition $\mathbb{Z}_6$ is a relation on expressed as $a\theta b$ with $a, b \in \mathbb{Z}_6$ if and only if $a - b = 2k$ with $k \in \mathbb{Z}$. We have equivalence classes of approximation space $(\mathbb{Z}_6, \theta)$ as follows:
$E_1 = \{\overline{0}, \overline{2}, \overline{4}\}$
$E_2 = \{\overline{1}, \overline{3}, \overline{5}\}$.

Moreover, given rough group $G = \{\overline{1}, \overline{2}, \overline{4}, \overline{5}\}$, and $\overline{G} = \mathbb{Z}_6$. G is called rough group satisfies the following conditions:
1. $a(+_6)b \in \bar{G}$, for every $a, b \in G$,
2. association property hold in G, i.e. $(a + b) + c = a + (b + c)$, for every $a, b, c \in G$,
3. there is a rough identity element $0 \in \overline{G}$ such that $0(+_6)x = x(+_6)0 = x$, for every $x \in G$,
4. every element x of G has rough invers y in G such that $x(+_6)y = y(+_6)x = 0$.

Next, we have a non-empty set $G \times G$ is a rough group in approximation space $(\mathbb{Z}_6 \times \mathbb{Z}_6, \theta^2)$.
$\mathbb{Z}_6 \times \mathbb{Z}_6 = \{(\overline{0}, \overline{0}), (\overline{0}, \overline{1}), \ldots, (\overline{0}, \overline{5}), (\overline{1}, \overline{0}), (\overline{1}, \overline{1}), \ldots, (\overline{1}, \overline{5}), (\overline{2}, \overline{0}), (\overline{2}, \overline{1}), \ldots, (\overline{2}, \overline{5}),$
$= (\overline{3}, \overline{0}), (\overline{3}, \overline{1}), \ldots, (\overline{3}, \overline{5}), (\overline{4}, \overline{0}), (\overline{4}, \overline{1}), \ldots, (\overline{4}, \overline{5}), (\overline{5}, \overline{0}), (\overline{5}, \overline{1}), \ldots, (\overline{5}, \overline{5})\}$
Relation of θ^2 has equivalence classes as follows:
$E_1 = \{(\overline{0}, \overline{0}), (\overline{0}, \overline{2}), (\overline{0}, \overline{4}), (\overline{2}, \overline{0}), (\overline{2}, \overline{2}), (\overline{2}, \overline{4}), (\overline{4}, \overline{0}), (\overline{4}, \overline{2}), (\overline{4}, \overline{4})\}$,
$E_2 = \{(\overline{0}, \overline{1}), (\overline{0}, \overline{3}), (\overline{0}, \overline{5}), (\overline{2}, \overline{1}), (\overline{2}, \overline{3}), (\overline{2}, \overline{5}), (\overline{4}, \overline{1}), (\overline{4}, \overline{3}), (\overline{4}, \overline{5})\}$,
$E_3 = \{(\overline{1}, \overline{0}), (\overline{1}, \overline{2}), (\overline{1}, \overline{4}), (\overline{3}, \overline{0}), (\overline{3}, \overline{2}), (\overline{3}, \overline{4}), (\overline{5}, \overline{0}), (\overline{5}, \overline{2}), (\overline{5}, \overline{4})\}$,

$E_4 = \{(\bar{1},\bar{1}),(\bar{1},\bar{3}),(\bar{1},\bar{5}),(\bar{3},\bar{1}),(\bar{3},\bar{3}),(\bar{3},\bar{5}),(\bar{5},\bar{1}),(\bar{5},\bar{3}),(\bar{5},\bar{5})\}$.
Furthermore, given $G = \{\bar{1},\bar{2},\bar{4},\bar{5}\}$, we have
$G \times G = \{(\bar{1},\bar{1}),(\bar{1},\bar{2}),(\bar{1},\bar{4}),(\bar{1},\bar{5}),(\bar{2},\bar{1}),(\bar{2},\bar{2}),(\bar{2},\bar{4}),(\bar{2},\bar{5}),$
$= (\bar{4},\bar{1}),(\bar{4},\bar{2}),(\bar{4},\bar{4}),(\bar{4},\bar{5}),(\bar{5},\bar{1}),(\bar{5},\bar{2}),(\bar{5},\bar{4}),(\bar{5},\bar{5})\}$,
$\overline{G \times G} = E_1 \cup E_2 \cup E_3 \cup E_4 = \mathbb{Z}_6 \times \mathbb{Z}_6$.

We can say $G \times G$ is a rough group, because it satisfies all the properties of rough groups i.e. for every $(a,b) \in G \times G$, then $(a+b) \in \overline{G \times G}$. The association property holds in $\overline{G \times G}$. There exist $(0,0) \in \overline{G \times G}$ such that for every $(x,y) \in G \times G, (x,y)(+_{50})(0,0) = (0,0)(+_{50})(x,y) = (x,y)$. Every element in $G \times G$ has a rough inverse in $G \times G$. Hence, $G \times G$ is a rough group.

4. CONCLUSIONS

The rough V-coexact sequence of the rough groups is the generalization of the rough exact sequence of rough groups. If (U,θ) is an approximation space, A,B,C are the rough groups in (U,θ), and V is the rough subgroups of A in (U,θ), then the sequence $\overline{A} \xrightarrow{f} \overline{B} \xrightarrow{g} \overline{C}$ is a rough V-coexact if $f(\overline{V}) = \ker(g)$. If M_1, M_2 are a rough group in the approximation space (U,θ), then $M_1 \times M_2$ ar a rough group in approximation space $(U \times U, \theta^2)$ with $(a_1,a_2)\theta^2(b_1,b_2)$ if only if $(a_1\theta b_1, a_2\theta b_2)$ for every $(a_1,a_2),(b_1,b_2) \in M_1 \times M_2$.

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