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ON INCLUSIVE LOCAL IRREGULARITY VERTEX COLORING OF BOOK GRAPH FAMILY

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ABSTRACT

Let $G(V,E)$ is a simple and connected graph with $V(G)$ as the vertex set and $E(G)$ as the edge set. Vertex labeling on inclusive local irregularity vertex coloring is defined by mapping $l: V(G) \to \{1,2,\dots,k\}$ and the function of the inclusive local irregularity vertex coloring is $w^i: V(G) \to N$ with $w^i = l(v) + \sum_{u \in N(u)} l(u)$. In other words, an inclusive local irregularity vertex coloring is defined by coloring the graph so that its weight value is obtained by adding up the labels of the neighboring vertices and its label. The inclusive local irregularity chromatic number is defined as the minimum number of colors obtained from coloring the vertex of the inclusive local irregularity in graph G, denoted by $\chi_{lis}^i(G)$. In this paper, we learn about the inclusive local irregularity vertex coloring and determine the chromatic number on the book graphs family.

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1. INTRODUCTION

Graph theory is one of the discrete mathematics branches widely applied in various fields. A graph is defined as a pair of sets which are the infinite and non-empty sets of elements called vertices, whereas they are the edge set formed from the connecting line segments of two vertices [1]. There are several discussions in graph theory such as graph coloring. Graph coloring can be interpreted as giving a different color to each element of the graph, namely a vertex, edge, or neighboring region [2]. In this study, we used vertex coloring, thus the provision of color to all vertices on the graph so that every two neighboring vertices have a different color [2]. Vertex coloring on the graph is the giving of a different color to each vertex to a minimum so that the neighboring vertex has a different color [3]. The fundamental problem in the coloring of graph is to determine the minimum possible color needed to color any graph, which is then called the chromatic number [4]. The minimum positive integer *k* for graph *G* having *k*- node coloring is called the chromatic number *G*, and is annotated $\chi(G)$ [5]. A vertex coloring on a graph is said to be efficient if the number of colors used to color a graph is the chromatic number of the graph [6].

There are several special cases in the vertex coloring of a graph, one of them is the inclusive local irregularity vertex coloring. An inclusive local irregularity vertex coloring is a development of the local irregularity vertex coloring concept. Irregularity means that the labeling of vertices can be repeated and local means that the weight of neighboring vertices must have different values. While inclusive itself comes from the word include, and in this study, it is interpreted by adding the value of the vertex label itself. At the coloring of the vertex of local irregularity, the weight of the vertex is obtained by summing the value of the neighboring vertex [7]. Meanwhile, the weight of the vertex on the coloring of the inclusive local irregularity vertex is obtained by summing the value of the vertex adjacent to the value of the vertex itself.

The research related to the inclusive local irregularity vertex coloring started from the research on several simple graphs, which include path graphs, cycle graphs, and star graphs [8]. The research became the initial research on the inclusive local irregularity vertex coloring, which was then continued by other studies, such as inclusive local irregularity vertex coloring in tree graph families, which is studied by [9], inclusive local irregularity vertex coloring on fan graphs, firecracker graphs, and sun graphs by [10] and inclusive local irregularity vertex coloring on unicyclic graph families and star graph comb operations which is studied by [11][12].

Graph operation is a way to create a new graph which differs from the original graph [13]. One of the examples of graph operation is edge amalgamation. Edge amalgamation on graphs G_1 and G_2 is the fusion of the edge $e \in E(G_1)$ and edge$f \in E(G_2)$, denoted by $(G_1, G_2; e, f)$[14]. Book graphs can be interpreted as graphs resulting from edge amalgamation on cycle graphs C_n [7]. In this study, the book graph is denoted by B_n^m where m is the number of vertices on the cycle graph and *n* is the number of cycle graphs. The family of book graphs used in this study are book graphs (B_n^4), book graphs (B_n^5), and book graphs (B_n^6).

2. RESEARCH METHODS

This research uses the axiomatic deductive method and pattern detection method. The axiomatic deductive method is defined as a research method that uses a theorem or axiom with the principles of deductive proof in mathematical logic to then be used in local irregularity vertex coloring on the book graph. While the pattern detection method in this study is used to formulate patterns and look for inclusive local irregularity chromatic numbers in a book graph. We used several definitions, lemmas, and observations in order to prove the theorem. The definitions, lemmas, and observations are as follows:

Definition 1. [15] *Given* $l: V(G) \to \{1,2,\dots,k\}$ *is a function of the label and* $w: V(G) \to N$ *is a weight function of e local irregularity vertex coloring, with* $w(u) \to \sum_{v \in N(u)} l(v)$. *Labeling is a local irregularity vertex coloring*

i. $opt(l) = \min\{\max(l_i)\,;\, label\ function\}$;
ii. $for\ every\ uv \in E(G), w(u) \neq w(v)$.

Definition 2. [15] *The minimum number of colors obtained by local irregularity vertex coloring of a graph G is called local irregularity vertex chromatic number, denoted by* $\chi_{lis}(G)$.

Inclusive local irregularity vsertex coloring [8] and has obtained a definition of inclusive local irregularity vertex coloring as follows:

Definition 3.[8] *Given* $l: V(G) \to \{1,2,\dots,k\}$ *is a function of the label and* $w^j: V(G) \to N$ *is a weight function of inclusive local irregularity vertex coloring, with* $w^i = l(v) + \sum_{u \in N(u)} l(u)$. *Labeling is an inclusive local irregularity vertex coloring which fulfill the following condition:*

i. $opt(l) = \min\{\max(l_i)\}; l_i$ *is an inclusive local irregularity vertex coloring*

ii. $for\ every\ uv \in E(G), w^i(u) \neq w^i(v)$

Definition 4. [8] *The minimum number of colors obtained by inclusive local irregularity vertex coloring of a graph G is called inclusive local irregularity vertex chromatic number, denoted by* $\chi_{lis}^i(G)$.

To make it easier to find an inclusive local irregularity vertex coloring in this study, the following lemma and observations will be used:

Lemma 1. [8] *Given that the graph G is a simple and connected graph, then* $\chi_{lis}^i(G) \geq \chi_{lis}(G)$.
Observation 1. *Local irregularity chromatic number of the book graph* B_n^4 with $n \geq 3$ $n \geq 3$ is $\chi_{lis}(B_n^4) = 4$.
Observation 2. *Local irregularity chromatic number of the book graph* B_n^5 with $n \geq 3$ is $\chi_{lis}(B_n^5) = 4$
Observation 3. *Local irregularity chromatic number of the book graph* B_n^6 with $n \geq 3$ is $\chi_{lis}(B_n^6) = 4$.

3. RESULTS AND DISCUSSION

In this article, we discuss the chromatic numbers of inclusive local irregularities in the book graph family, including book graphs (B_n^4), book graphs (B_n^5), and book graphs (B_n^6). The results about the inclusive local irregularity vertex chromatic number of the book graph family decompose in some theorems as follows:

Theorem 1. *Given a book graph* (B_n^4) *with* $n \geq 3$, *then the inclusive local irregularity chromatic number of the graph is* $\chi_{lis}^i(B_n^4) = 4$.

Proof. It is known (B_n^4) to be a book graph with a set of vertex $V(G) = \{x_i; 1 \leq i \leq 2n\} \cup \{y_1, y_2\}$ and a set of edges $E = \{x_i x_{i+1}; 1 \leq i \leq n\} \cup \{x_i y_1; 1 \leq i \leq n\} \cup \{x_j y_2; n+1 \leq j \leq 2n\} \cup \{y_1 y_2\}$. So that the cardinality of the vertex set and the edge set of book graph (B_n^4) are respectively $|V(B_n^4)| = 2n + 2$ and $|E(B_n^4)| = 3n + 1$. On book graph (B_n^4) with $n \geq 3$, we prove the upper bound and the lower bound of inclusive local irregularity vertex coloring in (B_n^4). If each vertex $v(B_n^4)$ is labeled with a value of 1, then the weight of the neighboring vertex is obtained namely $w^i(x_{1,1}) = l(x_{1,1}) + l(x_{1,2}) + l(y_1) = 1 + 1 + 1 = 3$ and $w^i(x_{1,2}) = l(x_{1,2}) + l(x_{1,1}) + l(y_2) = 1 + 1 + 1 = 3$. The two neighboring vertices are $v(x_{1,1})$ and $v(x_{1,2})$ have the same vertex weight. It is in contradiction with **Definition 1**, where each vertex $uv \in E(G)$ is such that $w(u) \neq w(v)$.

Next, it will be investigated that the chromatic numbers of inclusive local irregularities on the book graph (B_n^4) with $n \geq 3$ is $\chi_{lis}^i(B_n^4) = 4$. Based on Lemma 1 which says that "For graphs G which are simple and connected graphs, then $\chi_{lis}^i(G) \geq \chi_{lis}(G)$" is obtained the lower bound of chromatic numbers of inclusive local irregularity from the graph book is $\chi_{lis}^i(G) \geq \chi_{lis}(G) = 4$ with $n \geq 3$. Then to prove the value $\chi_{lis}^i(B_n^4)$, will be found the upper bound of the chromatic number of inclusive local irregularities through the function of labeling the vertex defined with $l: V \to B_n^4 = \{1,2\}$ and the weight function of the vertex inclusive local irregularity. The labeling of inclusive local irregularity vertex on the book graph (B_n^4) is as follows.

$$l(y_i) \to 1, for\ i = 1$$
$$l(y_i) \to 2, for\ i = 2$$

$$l(x_{i,j}) \to 1, for\ 1 \leq i \leq n, j = 1$$

$$l(x_{i,j}) \to 2, for\ 1 \le i \le n, j = 2$$

So that the weight of the vertex obtained based on the definition of inclusive local irregularity vertex coloring is as follows.

$$w^i(y_i) \to n+3, for\ i = 1 \text{ and } w^i(y_i) \to 2n+3, for\ i = 2.$$

$$w^i(x_{i,j}) \to 4, for\ 1 \le i \le n, j = 1 \text{ and } w^i(x_{i,j}) \to 5, for\ 1 \le i \le n, j = 2$$

Based on the results of the calculation of the weight of the vertex above $|w^i(B_n^4)| = 4$ with $n \ge 3$ as the upper bound for the graph book B_n^4 . Thus, it is obtained the lower bound through Observation 1 and the upper bound through the vertex weight function, we got the value the inclusive local irregularity chromatic number of the graph (B_n^4) is $4 = \chi_{lis}^i(G) \ge \chi_{lis}(G) = 4$. Then it can be said based on Definition 2 that the chromatic number of inclusive local irregularities on the book graph (B_n^4) with $n \ge 3$ is 4 or $\chi_{lis}^i(B_n^4) = 4$.■

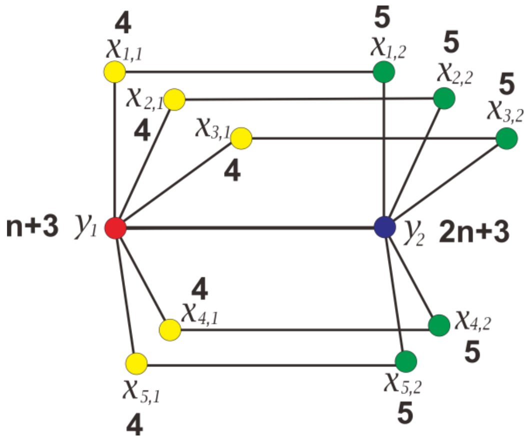

Figure 1. An inclusive local irregularity vertex coloring of book graph (B_n^4)

Figure 1 is an illustration of the inclusive local irregularity vertex coloring of the book graph (B_n^4) with $n = 5$ and obtained the results of chromatic numbers of inclusive local irregularity $\chi_{lis}^i(B_5^4) = 4$.

Theorem 2. *Given a book graph* (B_n^5) *with* $n \ge 3$, *then the inclusive local irregularity chromatic number of the graph is* $\chi_{lis}^i(B_n^5) = 5$.

Proof. It is known (B_n^5) to be a book graph with a set of vertex $V(G) = \{x_i; 1 \le i \le 3n\} \cup \{y_1, y_2\}$ and a set of edges $= \{x_i y_1; 1 \le i \le n\} \cup \{x_i x_{i+n}; 1 \le i \le 3n-n\} \cup \{x_j y_2; 2n+1 \le j \le 3n\} \cup \{y_1 y_2\}$. So that the cardinality of the vertex set and the edge of book graph (B_n^5) are respectively $|V(B_n^5)| = 3n+2$ and $|E(B_n^5)| = 4n+1$. On the book graph (B_n^5) with $n \ge 3$ will be proved the upper bound and the lower bound of inclusive local irregularity vertex coloring. If each vertex $v\ (B_n^5)$ is labeled with a value of 1, then the weight of the neighboring vertex is obtained namely $w^i(x_{1,1}) = l(x_{1,1}) + l(x_{1,2}) + l(y_1) = 1+1+1 = 3$ and $w^i(x_{1,2}) = l(x_{1,2}) + l(x_{1,1}) + l(x_{1,3}) = 1+1+1 = 3$. The two neighboring vertices are $v(x_{1,1})$ and $v(x_{1,2})$ have the same vertex weight. It is in contradiction with Definition 1, where each vertex $uv \in E(G)$ is such that $w^i(u) \ne w^i(v)$.

Next it will be investigated that the chromatic numbers of inclusive local irregularities on the graph book (B_n^5) with $n \ge 3$ is $\chi_{lis}^i(B_n^5) = 5$. Based on Lemma 1 which says that "For graphs G which are simple and connected graphs, then $\chi_{lis}^i(G) \ge \chi_{lis}(G)$" is obtained the lower bound of chromatic numbers of inclusive local irregularity from the graph book is $\chi_{lis}^i(G) \ge \chi_{lis}(G) = 4$ with $n \ge 3$. Then to prove the value $\chi_{lis}^i(B_n^5)$, will be found the upper bound of the chromatic number of inclusive local irregularities through the function of labeling the vertex defined with $l: V \to B_n^5 = \{1,2\}$ and the weight function of the vertex inclusive local irregularity. The labeling of inclusive local irregularity vertex on the book graph (B_n^5) is as follows.

$$l(y_i) \to 1, for\ i = 1$$

$$l(y_i) \to 2, for\ i = 2$$

$$l(x_{i,j}) \to 1, for\ 1 \le i \le n, j = 1,2$$

$$l(x_{i,j}) \to 2, for\ 1 \le i \le n, j = 3$$

So that the weight of the vertex obtained based on the definition of inclusive local irregularity vertex coloring is as follows.

$$w^i(y_i) \to n + 3, for\ i = 1 \text{ and } w^i(y_i) \to 2n + 3, for\ i = 2.$$

$$w^i(x_{i,j}) \to 3, for\ 1 \le i \le n, j = 1 \text{ and } w^i(x_{i,j}) \to 4, for\ 1 \le i \le n, j = 2$$
$$\text{and } w^i(x_{i,j}) \to 5, for\ 1 \le i \le n, j = 3$$

Based on the results of the calculation of the weight of the vertex above $|w^i(B_n^5)| = 5$ with $n \ge 3$ as the upper bound for the graph book (B_n^5). Thus obtained the lower bound through observation 2 and the upper bound through the vertex weight function, it is obtained the value $5 = \chi_{lis}^i(G) \ge \chi_{lis}(G) = 4$. Then it can be said based on **Definition 2** that the chromatic number of inclusive local irregularities on the book graph (B_n^5) with $n \ge 3$ is 5 or $\chi_{lis}^i(B_n^5) = 5$.■

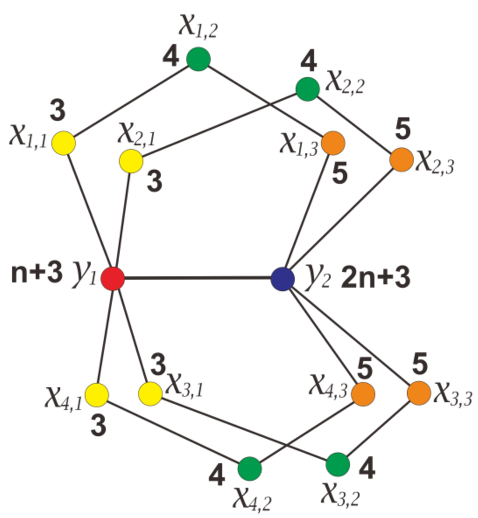

Figure 2. An inclusive local irregularity vertex coloring of book graph (B_n^5)

Figure 2 is an illustration of the inclusive local irregularity vertex coloring of the book graph (B_n^5) with $n = 4$ and obtained the results of chromatic numbers of inclusive local irregularity $\chi_{lis}^i(B_4^5) = 5$.

Theorem 3. *Given a book graph (B_n^6) with $n \ge 3$, then the inclusive local irregularity chromatic number of the graph $\chi_{lis}^i(B_n^6) = 5, for\ n = 3$ and $\chi_{lis}^i(B_n^6) = 6, for\ n \ge 4$.*

Proof. It is known (B_n^6) to be a book graph with a set of vertex $V(G) = \{x_i; 1 \le i \le 4n\} \cup \{y_1, y_2\}$ and a set of edges $E = \{x_i x_{i+n}; 1 \le i \le 4n - n\} \cup \{x_i y_1; 1 \le i \le n\} \cup \{x_j y_2; 3n + 1 \le j \le 4n\} \cup \{y_1 y_2\}$. So that the cardinality of the vertex and the cardinality of the edge of book graph (B_n^6) are respectively $|V(B_n^6)| = 4n + 2$ and $|E(B_n^6)| = 5n + 1$. On the book graph (B_n^6) with $n \ge 3$, will be proved the upper bound and the lower bound of the inclusive local irregularity vertex coloring. If each vertex $v(B_n^6)$ is labeled with a value of 1, then the weight of the neighboring vertex is obtained namely $w^i(x_{1,1}) = l(x_{1,1})l(x_{1,2}) + l(y_1) = 1 + 1 + 1 = 3$ and $w^i(x_{1,2}) = l(x_{1,2}) + l(x_{1,1}) + l(x_{1,3}) = 1 + 1 + 1 = 3$. The two neighboring vertex are $v(x_{1,1})$ and $v(x_{1,2})$ have the same vertex weight. It is in contradiction with definition 1, where the inclusive local irregularity should fulfill $for\ every\ uv \in E(G), w(u) \ne w(v)$.

Based on Lemma 1 which says that "For graphs G which are simple and connected graphs, then $\chi_{lis}^{i}(G) \geq \chi_{lis}(G)$" is obtained the lower bound of chromatic numbers of inclusive local irregularity from the graph book is$\chi_{lis}^{i}(G) \geq \chi_{lis}(G) = 4$ with $n \geq 3$. Then to prove the value $\chi_{lis}^{i}(B_n^6)$, will be found the upper bound of the chromatic number of inclusive local irregularities through the function of labeling the vertex defined with $l: V \to B_n^6 = \{1,2\}$ and the weight function of the vertex inclusive local irregularity.

Case 1. For $n = 3$

The labeling of inclusive local irregularity vertex on the book graph (B_n^6) is as follows.

$$l(y_i) \to 1, for\ i = 1$$

$$l(y_i) \to 2, for\ i = 2$$

$$l(x_{i,j}) \to 1, for\ 1 \leq i \leq n, j = 1,2$$

$$l(x_{i,j}) \to 2, for\ 1 \leq i \leq n, j = 3,4$$

So that the weight of the vertex obtained based on the definition of inclusive local irregularity vertex coloring is as follows.

$$w^i(y_i) \to 6, for\ i = 1 \text{ and } w^i(y_i) \to 9, for\ i = 2.$$

$$w^i(x_{i,j}) \to 3, for\ 1 \leq i \leq n, j = 1 \text{ and } w^i(x_{i,j}) \to 4, for\ 1 \leq i \leq n, j = 2$$
$$\text{and } w^i(x_{i,j}) \to 5, for\ 1 \leq i \leq n, j = 3 \text{ and } w^i(x_{i,j}) \to 6, for\ 1 \leq i \leq n, j = 4$$

Based on the results of the calculation of the weight of the vertex above $|w^i(B_n^6)| = 5$ with $n = 3$ as the upper bound for the graph book (B_n^6). Thus obtained the lower bound through observation 3 and the upper bound through the vertex weight function obtained the value $5 = \chi_{lis}^{i}(G) \geq \chi_{lis}(G) = 4$. Then it can be said based on **Definition 2** that the chromatic number of inclusive local irregularities on the book graph (B_n^6) with $n = 3$ is 5 or $\chi_{lis}^{i}(B_n^6) = 5$.

Case 2. To $n \geq 4$

The labeling of inclusive local irregularity vertex on the book graph (B_n^6) is as follows.

$$l(y_i) \to 1, for\ i = 1$$

$$l(y_i) \to 2, for\ i = 2$$

$$l(x_{i,j}) \to 1, for\ 1 \leq i \leq n, j = 1,2$$

$$l(x_{i,j}) \to 2, for\ 1 \leq i \leq n, j = 3,4$$

So that the weight of the vertex obtained based on the definition of inclusive local irregularity vertex coloring is as follows.

$$w^i(y_i) \to n + 3, for\ i = 1 \text{ and } w^i(y_i) \to 2n + 3, for\ i = 2.$$

$$w^i(x_{i,j}) \to 3, for\ 1 \leq i \leq n, j = 1 \text{ and } w^i(x_{i,j}) \to 4, for\ 1 \leq i \leq n, j = 2$$
$$\text{and } w^i(x_{i,j}) \to 5, for\ 1 \leq i \leq n, j = 3 \text{ and } w^i(x_{i,j}) \to 6, for\ 1 \leq i \leq n, j = 4$$

Based on the results of the calculation of the weight of the vertex above $|w^i(B_n^6)| = 6$ with $n \geq 4$ as the upper bound for the book graph (B_n^6). Thus obtained the lower bound through observation 3 and the upper bound through the vertex weight function obtained the value $6 = \chi_{lis}^{i}(G) \geq \chi_{lis}(G) = 4$. Then it can be said based on **Definition 2** that the chromatic number of inclusive local irregularities on the book graph (B_n^6) with $n \geq 4$ is 6 or $\chi_{lis}^{i}(B_n^6) = 6$.■

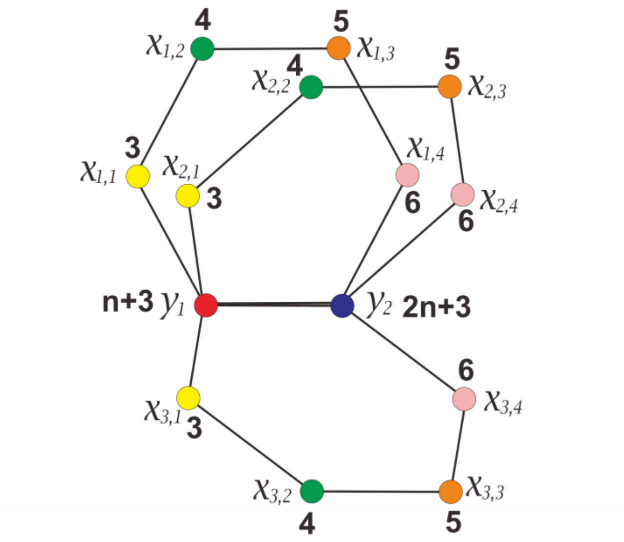

Figure 3. An inclusive local irregularity vertex coloring of book graph (B_n^6)

Figure 3 is an illustration of the inclusive local irregularity vertex coloring of the book graph (B_n^6) with $n = 3$ and obtained the results of chromatic numbers of inclusive local irregularity $\chi_{lis}^{i}(B_3^6) = 5$.

4. CONCLUSIONS

Based on the results and the discussion, three new theorems were obtained on the topic of inclusive local irregularity vertex coloring in the book graph family. The chromatic number of inclusive local irregularity obtained from the book graph (B_n^4) is 4 and chromatic of from the book graph (B_n^5) is 5. While the chromatic number of inclusive local irregularity in the book graph (B_n^6), there are 2 cases where the chromatic number processed for $n = 3$ is 5 and $n \geq 4$ is 6.

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