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MULTILEVEL REGRESSION WITH MAXIMUM LIKELIHOOD AND RESTRICTED MAXIMUM LIKELIHOOD METHOD IN ANALYZING INDONESIAN READING LITERACY SCORES

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Abstract. *The multilevel regression model is a development of the linear regression model that can be used to analyze data that has a hierarchical structure. The problem with this data structure is that individuals in the same group tend to have the same characteristics, so the observations at lower levels are not independent. Education research often produces a hierarchical structure, one of which is PISA data, where students as level-1 nested within schools as level-2. In the PISA 2018 survey, reading literacy is the main focus. The data are sourced from the Organisation for Economic Co-operation and Development (OECD). The survey results show that the reading literacy scores of Indonesian students have decreased, thus placing Indonesia at 74th out of 79 countries. However, it is still very rare to research the reading literacy of Indonesian students' using a multilevel regression model. This study aims to apply a multilevel regression model to determine the factors influencing Indonesian reading literacy scores in PISA 2018 survey data. The results of this study indicate that the factors that influence response variables are gender, grade level, mother's education, facilities at home, age at school entry, student discipline behavior at school, and failing grade, while at the school level are the type of school and school location. The magnitude variance of student reading literacy scores can be explained by the explanatory variables the student level is 11,42%, and the school level is 60,66%, while the rest is explained by another factor outside the study.*

Keywords: *hierarchical data, multilevel regression model, reading literacy.*

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1. INTRODUCTION

Goldstein [1] introduced a multilevel regression model, which is a statistical analysis that has undergone an expansion of linear regression analysis. The expansion is based because the data used has a hierarchical structure, the structure arises due to the data obtained are the results of surveys conducted using the multistage sampling method, and there are variables that are defined at different levels [2]. If the analysis is carried out on data that has a hierarchical structure using linear regression analysis, it will result in a violation of assumptions, namely, there is the residual independence and heteroscedasticity of residual variance [3]. This happens because, in data with a hierarchical structure, individuals at the same level tend to have the same characteristics compared to individuals at different levels [3]. Thus, the observations at lower levels are not mutually independent. In addition to analyzing data with a hierarchical structure, the multilevel regression model can also be used for repeated measurement data and longitudinal data [3]–[5].

Another approach that can be used to analyze data with a hierarchical structure, repeated measurement data, and longitudinal data is by using the marginal model [6], repeated measures Analysis of Variance (ANOVA) [7], and Structural Equation Modeling (SEM) based on growth analysis known as latent curve models [3]. However, the multilevel regression model is the most popular model for analyzing data with a hierarchical structure and has several advantages, namely, multilevel regression models can be used to analyze information from several different levels that can be studied together in one statistical analysis and multilevel regression models can take into account the influence of variance at each level on the variance of responses so that the resulting model can provide more information than other analyzes [3], [8].

One example of data in the education field with a hierarchical structure is the Program for International Student Assessment (PISA) survey data, because the survey uses a two-stage stratified sample design as a sampling method, where students as level-1 are nested in schools as level-2 [9]. The PISA's survey is one of the international surveys in the field of education sponsored, regulated, and coordinated by the Organization for Economic Cooperation and Development (OECD), and the survey is routinely conducted every three years. The PISA survey aims to analyze whether students who are nearing the end of compulsory education have acquired the key knowledge and skills needed to participate in social and economic life fully [10]. There are three assessment subjects assessed in the PISA survey, namely mathematical literacy, reading literacy, and scientific literacy. In the 2018 PISA survey, a total of 79 countries participated the PISA survey, and reading literacy was the main focus of the assessment [11].

The research on PISA survey data that has been carried out by [12], [13], [14], and [15] can only find out the student factors that have a significant effect on PISA scores. So it is necessary to do further analysis to find out not only student factors but also school factors that have a significant effect on PISA scores. However, until now, research on the reading literacy of Indonesian students in the PISA survey using a multilevel regression model is still very rarely done. Therefore, this study aims to apply a multilevel regression model to determine the factors that affect Indonesian students' reading literacy scores in the 2018 PISA survey data.

2. RESEARCH METHODS

2.1 Data

The data used in this article is PISA's data performed by the OECD. The number of samples used in this study was as many as 7.961 students of junior high school grade 7 to high school grade 12 aged 15 years and came from 308 schools throughout Indonesia, where student data as level-1 nested within the school as level-2. The response variable used in this study is the reading literacy scores of Indonesian students. In addition, the explanatory variables used in this research have a hierarchical structure consisting of 15 variables at the student level and 2 variables at the school level. The explanatory variables at the student level consist of gender (X_1), grade level (X_2), mother education (X_3), father education (X_4), study desk at home (X_5), quiet room at home (X_6), many mobile phones with internet access at home (X_7), many computers at home (X_8), many books at home (X_9), age of entry to early childhood education (X_{10}), age of entry to elementary school (X_{11}), not listening to teachers (X_{12}), skipping school (X_{13}), being late for school (X_{14}), and failing grade (X_{15}), while the explanatory variables at the school level consist of type of school (Z_1) and location of school (Z_2).

2.2 Data Analysis Procedure

The data analysis procedures in this study are as follows:

1. Data input by combining student and school data and cleaning data.
2. Data exploration is done by presenting descriptive statistics of research variables, examination of the normality of response variables, and examination of multicollinearity between explanatory variables.
3. Building linear regression models without explanatory variables (Model 0), random intercept models without explanatory variables (Model 1), and random intercept models with explanatory variables (Model 2).
   a. Building a Model 0. The linear regression models without explanatory variables can be written as follows [3]:
   $$y_i = \beta_0 + e_i \tag{1}$$
   b. Building a Model 1. The random intercept models without explanatory variables (null model/intercept-only model) can be written as follows [3]:

   Level-1 model:
   $$y_{ij} = \beta_{0j} + e_{ij} \tag{2}$$
   Level-2 model:
   $$\beta_{0j} = \gamma_{00} + u_{oj} \tag{3}$$
   Combine model:
   $$y_{ij} = \gamma_{00} + u_{0j} + e_{ij} \tag{4}$$
   c. Building a Model 2. The multilevel regression model used in this observation is a 2-level regression model with random intercept because, in this study, it is assumed that the effect of each explanatory variable on the response variable for every group is the same. In addition, random coefficients models have a weakness, that is, the estimated results in these models are much less reliable than those in random intercepts models [3]. The random intercept models with explanatory variables can be written as follows:

   Level-1 model:
   $$y_{ij} = \beta_{0j} + \sum_{p=1}^{P} \beta_{pj} x_{pij} + e_{ij} \tag{5}$$
   Level-2 model:
   $$\beta_{0j} = \gamma_{00} + \sum_{q=1}^{Q} \gamma_{0q} z_{qj} + u_{0j} \tag{6}$$
   Combine model:
   $$y_{ij} = \gamma_{00} + \sum_{p=1}^{P} \gamma_{p0} x_{pij} + \sum_{q=1}^{Q} \gamma_{0q} z_{qj} + u_{oj} + e_{ij} \tag{7}$$
   where the subscript ij shows the individual i-th $(i=1,2,\ldots, n_j)$ in group j-th $(j=1,2,\ldots, J)$, y_{ij} is a response for individual i-th in group j-th, β_{0j} is the intercept for group j-th, β_{pj} is the regression coefficient for the explanatory variables level-1 ($p = 1,2,\ldots,P$), e_{ij} is residual at level-1 for individual i-th in group j-th and the assumed distribution $N(0,\sigma_e^2)$, γ_{00} is the intercept fixed or overall average in the variable y, γ_{0q} is the regression coefficient for the explanatory variables level-2 ($q = 1, 2,\ldots,Q$), and u_{0j} residual at the level-2 is for the group j-th and is assumed to have a distribution of $N(0,\sigma_{u0}^2)$.

   d. Estimating parameters using MLE for regression coefficients and REML for variance components with Newton-Raphson computational algorithm.
   e. Testing the significance of random effects. This test aims to determine whether the model with random effects is better than the model without random effects. Random effect significance testing is done with the following hypotheses:

   $H_0 : \sigma_{u0}^2 = 0$ (random effect is not significant)

   $H_1 : \sigma_{u0}^2 > 0$ (random effect is significant)

   The test statistics used are as follows:

$$LR = -2\ ln\left(\frac{L_0}{L_1}\right) \sim \chi^2_{(a,v)} \tag{8}$$

where
L_0 : the value of the likelihood function in a model without a random effect (Model 0)
L_1 : the value of the likelihood function in a model with a random effect (Model 1)

Reject H_0 if $p\ value < \alpha$ or $LR > \chi^2_{(\alpha,v)}$ with degrees of freedom (v), where v is the difference between the number of parameters of the two models [2].

4. Calculating the value of the interclass correlation coefficient (ICC). The multilevel regression model in addition to being able to produce estimates for parameters, this model can also produce estimated values for intraclass correlation coefficients. By using the null model (Model 1), the intraclass correlation coefficient (ρ) can be estimated by Equation 9:

$$\rho = \frac{\sigma^2_{u0}}{\sigma^2_{u0} + \sigma^2_e} \tag{9}$$

If the ICC value is greater than 0,05 or 5% then it indicates that the variation between groups is greater than expected and the value implies that nesting in groups has an influence on the response of individuals so multilevel regression models are needed [16].

5. Significance test of the influence of explanatory variables level-1 and explanatory variables level-2 on the response variable simultaneously using the G test and partial using the t-test.

   a. Simultaneous testing of parameter significance can be performed using the G test, with the following hypothesis [3]:

   $H_0 : \gamma_{10} = \ldots = \gamma_{p0} = \gamma_{01} = \ldots = \gamma_{0q} = 0$

   H_1 : at least one $\gamma_{p0} \neq 0$ or $\gamma_{0q} \neq 0;\ p = 1, 2, \ldots, P;\ q = 1, 2, \ldots, Q$

   The test statistics used are as follows:

   $$G = -2\ ln\left(\frac{L_{(null\ model)}}{L_{(conditional\ model)}}\right) \sim \chi^2_{(\alpha,p)} \tag{10}$$

   where
   $L_{(null\ model)}$: the value of the likelihood function in the multilevel regression model without explanatory variables (model 1)
   $L_{(conditional\ model)}$: the value of the likelihood function in the multilevel regression model with explanatory variables (model 2)

   The decision rejects H_0, if $p\ value < \alpha$ or $G > \chi^2_{(\alpha,p)}$ with degrees of freedom (p), where p is the difference between the number of parameters between the conditional model and the null model.

   b. Partial parameter significance test is used to check the influence of each explanatory variable level-1 and level-2 individually on the variables and can be done using the t test, with the hypothesis at each level as follows [3]:

   Hypothesis for level-1 parameter:

   $$H_0 : \beta_{pj} = 0$$
   $$H_1 : \beta_{pj} \neq 0$$

   Hypothesis for level-2 parameter:

   $$H_0 : \gamma_{qj} = 0$$
   $$H_1 : \gamma_{qj} \neq 0$$

   The test statistic used is t-test, it can be formulated as follows:

   $$t(\beta_{pj}) = \frac{\hat{\beta}_{pj}}{SE(\hat{\beta}_{pj})} \text{ and } t(\gamma_{qj}) = \frac{\hat{\gamma}_{qj}}{SE(\hat{\gamma}_{qj})} \tag{11}$$

   In this case, the t-test follows the distribution of t-students, so H_0 is rejected if $p\ value < \alpha$ or $|t| > t_{(\alpha/2;\ v)}$ with degrees of freedom (v) using the Satterthwaite approximation [3].

6. Testing goodness of fit model using coefficient of determination. In multilevel regression modeling, the coefficient of determination can be used for goodness of fit test for models and the value of the

determination coefficient can be calculated at all existing levels [3]. The Formula for the coefficient of determination at level-1, is as follows [3]:

$$R_1^2 = 1 - \frac{\hat{\sigma}_{ep}^2}{\hat{\sigma}_{e0}^2} \tag{12}$$

where,

$\hat{\sigma}_{ep}^2$: the estimated value for the level-1 residual variance in the model with the explanatory variable p

$\hat{\sigma}_{e0}^2$: the estimated value for the level-1 residual variance in the model without the explanatory variable p

The coefficient of determination at level-2 [3]:

$$R_2^2 = 1 - \frac{\hat{\sigma}_{up}^2}{\hat{\sigma}_{u0}^2} \tag{13}$$

where,

$\hat{\sigma}_{up}^2$: the estimated value for the level-2 residual variance in the model with the explanatory variable p

$\hat{\sigma}_{u0}^2$: the estimated value for the level-2 residual variance in the model without the explanatory variable p

7. Interpretation of level-1 and level-2 explanatory variables significantly affects student reading literacy scores.

3. RESULTS AND DISCUSSION

3.1 Exploratory Data

This study used data from the 2018 PISA survey for Indonesian students and the sample used was 7.961 junior high school students from grade 7 to senior high school grade 12 who was 15 years old (15 years 3 months to 16 years 2 months) and came from 308 schools throughout Indonesia. To see the characteristics of the data used data exploration by using descriptive statistics. The following Table 1 is a descriptive statistical table of the response variables used in this study:

Table 1. Descriptive statistics of response variables

Variabel	Min.	Max.	Mean	Std. Deviation
Y	176,31	613,46	396,8	78,71

Based on Table 1 shows that the reading literacy score of Indonesian students has the lowest score of 176,31 and the highest score of 613.46. While the average reading literacy score of Indonesian students in the 2018 PISA survey was 396.8, and the standard deviation was 78.67. The average shows that the average reading literacy score of Indonesian students is very low compared to the OECD average score, which is 487 [10].

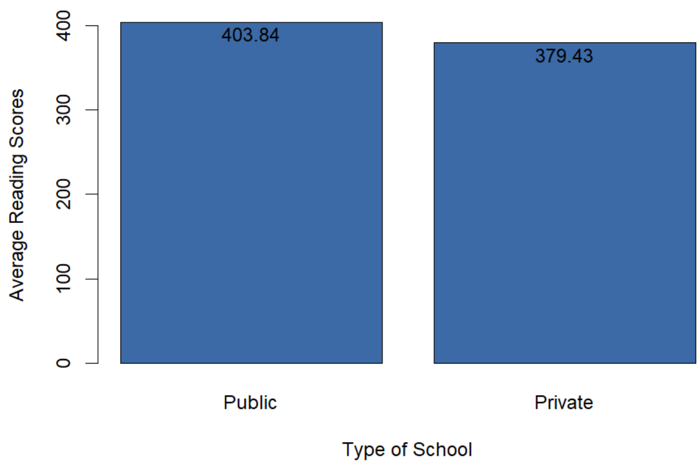

Figure 1. Average Reading Literacy Score of Students by Type of School

In the 2018 PISA survey, the number of samples of students attending public schools is greater than that of students attending private schools. Of the 7.961 Indonesian students, 71,1% or 5.662 students attended public schools, while the remaining 28,9% or 2.299 students attended private schools. Figure 1 shows students' average reading literacy scores by type of school. Based on the Figure 1, it shows that the average of reading literacy score of students who attend public schools is higher than the average reading literacy score of students who attend private schools.

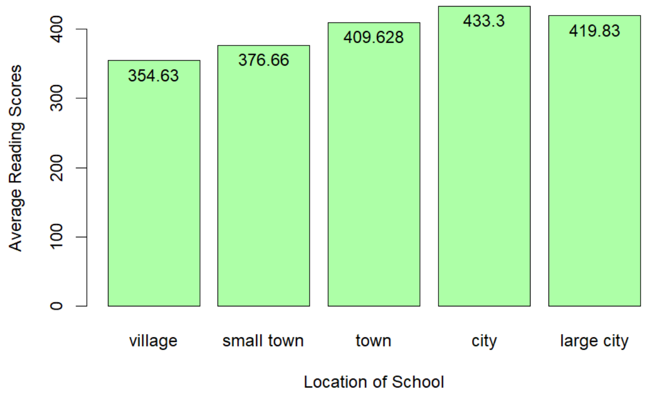

Figure 2. Reading Literacy Score of Students by Location of School

Meanwhile, the number of students based on school location was 1.116 (14%) students who attended schools in a village, 2.695 (33,9%) students who attended schools in a small town, 1.791 (22,5%) students who attended schools in a town, 1.782 (22,4%) students who attended schools in a city, and 577 (7,2%) students who attended schools in a large city. Based on Figure 2, it can be concluded that for students who attend school in towns, cities, and large cities, the reading literacy score of their students is above the reading literacy score of Indonesian students in general, which is more than 396,8. However, in contrast, for students who attend schools in villages and small towns, the average reading literacy score of their students is below the average reading literacy score of Indonesian students in general, which is less than 396,8.

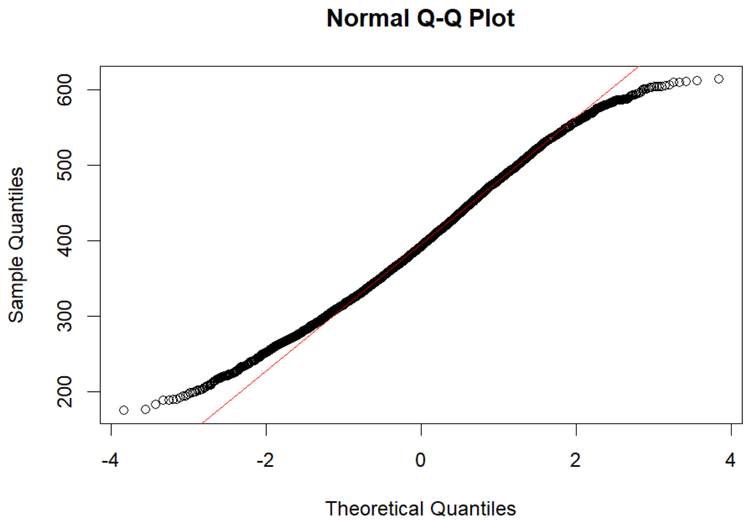

Figure 3. Normal Q-Q Plot of Response Variables

Before modeling students reading literacy scores that were assumed to be normally distributed, they were examined using the normal Q-Q plot. According to Figure 3, the points on the normal Q-Q plot for the response variable spread around the diagonal line and are linear, so the normal Q-Q plot can conclude that the response variables in the data used are normally distributed.

In multilevel regression models, freedom between explanatory variables either level-1 or level-2 is also a basic assumption that must be met so that before forming the model [3], Multicollinearity be examined between explanatory variables for level-1 and level-2 using polychoric correlation because all explanatory variables in level-1 and level-2 are of category type. Based on the examination of all explanatory variables, either level-1 or level-2, there are no explanatory variables that correlate very strongly or $\rho < 0{,}8$, which means that there is no multicollinearity, then all explanatory variables for level-1 and level-2 in the data can be used for modeling.

3.2 Multilevel Regression Model with Random Intercept

Before forming a multilevel regression model, the significance of the random effect was examined. Examination of random influence can be examined by comparing the value of the *deviance* between the regression model without explanatory variables (Model 0: $y_i = \beta_0 + e_i$) with the multilevel regression model without explanatory variables (Model 1: $y_{ij} = \gamma_{00} + u_{0j} + e_{ij}$) or by using Likelihood Ratio Tests (LRTs).

Table 2. Random Effect Significance Test Results between Model 0 and Model 1

	Npar	LogLik	*Deviance*	L. Ratio	Df	*p value*
Model 0	2	-46.051,01	92.102,03			
Model 1	3	-43.800,68	87.601,36	4.500,7	1	0,00*

*) significant at α = 5%

Table 2 shows that the value of the multilevel regression model *deviance* without explanatory variables (Model 1) is smaller than the value of the regression model *deviance* without explanatory variables (Model 0). This result shows that the multilevel regression model is more appropriate for explaining the data. In addition, based on the Likelihood Ratio Tests, obtained LR value of 4.500,7 with a *p-value* of 0,000. Because the value of $G > \chi^2_{(0,05;1)} = 3{,}84$ or *p-value* $< \alpha = 0{,}05$, then get the decision to reject H_0. In addition, it can be concluded that with a significance level of 5%, the random effect on the multilevel regression model is significant, meaning that there is a significant diversity of student reading literacy scores between schools in Indonesia. This also indicates that multilevel regression models (two levels) are more suitable in explaining the data used than linear regression models (one level).

Table 3. The Estimated Value of Residual Variance of Model 1

Residual Variance	Estimation
σ_e^2 (*Level*-1)	3.112,8
$\sigma_{u_o}^2$ (*Level*-2)	3.229,6

Based on Table 3 shows the estimated result for the residual variance of the multilevel regression model without explanatory variables at each level so that from these values can be calculated, the value of the interclass correlation coefficient (ICC): 3.229,26/(3.112,68 + 3.229,26) = 0,5091. These results indicate that the proportion of the variance in reading literacy scores of students that can be explained by the school structure without being influenced by any factor is 50,91%. Another interpretation is the expected correlation between two students randomly drawn at the same school on a reading literacy score of 0,5091. Because the value of the resulting interclass correlation coefficient is greater than 0,05, this reinforces the assumption that school structure provides great diversity and school structure also influences students reading literacy scores, so a multilevel regression model is needed to analyze the data.

Table 4. G Test Results Between Model 1 and Model 2

	Npar	LogLik	*Deviance*	G	Df	*p value*
Model 1	3	-43.800,68	87.601,36			
Model 2	60	-43.063,92	86.127,84	1.473,5	57	0,00

*) significant at α = 5%

In the next stage, explanatory variables level-1 (students) and level-2 (schools) are included in the model and parameter estimation methods used in this model are MLE for regression coefficients and REML for variance components. To determine the explanatory variables both at level-1 (students) and at level-2 (schools) that significantly affect students ' reading literacy scores in Indonesia, it is necessary to test the parameters' significance simultaneously or partially.

The results of testing the significance of parameters simultaneously using the G test in Table 4, obtained G value of 1.473,5 and a *p value* of 0,000. Because the value of G > $\chi^2_{(0,05;57)}$ = 75,62 or *p value* < α = 0,05 then obtained the decision to reject H_0. It can be concluded that at a significance level of 5%, at least one explanatory variable affects the reading literacy score of students in Indonesia in the 2018 PISA survey. In addition, it can also be interpreted that random intercept models with explanatory variables (model 2) are more suitable for use in data than random intercept models without explanatory variables (model 1).

The results of testing the significance of parameters partially using the t-test showed that there are 12 explanatory variables level-1 that significantly affect the reading literacy score of students, namely, gender (X_1), grade level (X_2), mother education (X_3), study desk at home (X_5), many mobile phones with internet access at home (X_7), many computers at home (X_8), many books at home (X_9), age of entry to early childhood education (X_{10}), age of entry to elementary school (X_{11}), not listening to teachers (X_{12}), skipping school (X_{13}), and failing grade (X_{15}) and there are 2 explanatory variables level-2 that significantly affect the reading literacy score of students, namely, the type of school (Z_1) and the location of school (Z_2). Based on the results of this study are in line with research conducted by [12], [13], [14], and [15].

Table 5. The Estimated Value of Residual Variance of Model 2

Residual Variance	Estimation
σ_e^2 (*Level*-1)	2.757,299
$\sigma_{u_o}^2$ (*Level*-2)	1.270,383

Based on Table 5, estimation variance components for every level in a random intercept model with explanatory variables (model 2), where the estimate of level-1 (students) residual variance shows the diversity of reading literacy scores between students in schools $\left(\hat{\sigma}_e^2 = 2.757{,}299\right)$, while the estimation level-2 (school) residual variance shows the diversity of the average reading literacy scores between schools ($\hat{\sigma}_{u_o}^2$ = 1.270,383). The estimated value of the variance of level-1 (students) shows a value that is significant, indicating that the diversity of reading literacy scores among students in the school value is significant. These results are along with the preliminary assumption that reading literacy scores between students in the same school are homogeneous, so the random effect of schools needs to be included in the model to overcome the violation of the heteroscedasticity assumption and residual independence in the model. The level-2 variance component, namely the level-2 (school) residual variance, has a reasonably large estimated value in model 2, this indicates that the school has a significant influence on students or can be said to be quite heterogeneous between schools and can also be interpreted that there is considerable diversity for schools caused by two explanatory variables of level-2 in the model, namely, the type of school (Z_1) and the location of school (Z_2).

The proportion of variance at each level that can be described by the model can be obtained from the comparison of models involving explanatory variables and models without explanatory variables [17]. Based on the calculation, the coefficient of determination for level-1 $\left(R_1^2\right)$ of 0,1142 means that the variance of students reading literacy scores that can be explained by explanatory variables at level-1 is equal to 11,42%, while the rest is explained by other variables that are not examined in this study. Meanwhile, the calculation of the coefficient of determination for level-2 $\left(R_2^2\right)$ obtained a value of 0,6066 means that the variance of students reading literacy scores that can be explained by explanatory variables at level-2 is equal to 60,66%, while the rest is explained by other variables that are not examined in this study.

4. CONCLUSIONS

The multilevel regression model can be applied in analyzing the reading literacy score of Indonesian students in the 2018 PISA data. This is because the results of random effect testing show that the random influence on the multilevel regression model gives a significant effect. Factors that effect the reading literacy score of Indonesian students at the student level are gender, grade level, mother education, study desk at home, many mobile phones with internet access at home, many computers at home, many books at home, age of entry to early childhood education, age of entry to elementary school, not listening to teachers, skipping school, and failing grade.

Meanwhile, the factors that affect the reading literacy score of Indonesian students at the school level are the type of school and the location of school.

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