BAREKENG: Journal of Mathematics and Its Application
December 2022 Volume 16 Issue 4 Page 1505–1514
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol16iss4pp1505-1514

# MODELING OF BOND YIELD CURVE USING CUBIC BEZIER CURVE

**Emy Siswanah**

*Department of Mathematics, Faculty of Science and Technology, Universitas Islam Negeri Walisongo Semarang*
*Prof. Hamka St., Ngaliyan, Semarang, 50185, Indonesia*

*Corresponding author's e-mail:* emysiswanah@walisongo.ac.id

***Abstract.*** *Investors attracted to Bonds have to analyze the Bond yield curve. In this study, the bond yield curve is modeled using a cubic bezier curve. The cubic bezier curve is flexible, precise, and simple to use and evaluate. The bonds used in this study are Surat Berharga Negara (Government Paper) Fix Rate type dated August 2nd–6th, 2021. Bond data are obtained from the Indonesia Stock Exchange https://www.idx.co.id. The results show that the bond yield curve that is formed varies because bond yields change every time following market developments. The cubic bezier curve is able to model the bond yield curve well. Cubic bezier curves have 4 control values that help guide the curve well. The MSE value obtained by the bezier curve is small in general. The MSE values of the cubic bezier curve for the Bond yield data, sequentially from the least to the greatest, are 0,098 on August 4th, 2021; 0,1719 on August 5th, 2021; 0,2161 on August 3rd, 2021; 0,2498 on August 6th, 2021; and 0,2906 on August 2nd, 2021.*

***Keywords:*** *Bond, cubic bezier curve, the MSE value, yield curve.*

***Article info:***

*Submitted: 2nd September 2022* *Accepted: 25th November 2022*

***How to cite this article:***
E. Siswanah, "MODELING OF BOND YIELD CURVE USING CUBIC BEZIER CURVE", *BAREKENG: J. Math. & App.*, vol. 16, iss. 4, pp. 1505-1514, Dec., 2022.

## 1. INTRODUCTION

Inarguably, investors put their funds into certain financial assets and desire profit. To gain profit, investors have to analyze the chosen instrument of investment. Analysis avails to the investors in finding the profit in prospect on the instrument of investment in the stock market. Moreover, it is also useful to make the investors confident in their decision.

Analysis of investment instruments on financial asset needs influencing factors of asset performance. It also applies to Bond investment. The willing investors in Bonds must analyze the company that publishes Bonds, coupons, their due times, yield, and Bond yield curve. The Bond yield curve is a curve that represents the correlation between yield to maturity and time to maturity. Through the Bond yield curve, investors can perceive the prospect of having the Bond until its due time. The Bond yield curve can be a reference for investors to understand the condition of the stock market. In fact, the slope of the bond yield curve is also important for investors to analyze [1]. Thus, the appropriate decision can be made.

There are three types of Bond yield curves: normal, inverted, and flat. The normal curve presents a long-term yield level above the short-term yield level. The inverted curve presents a short-term yield level above the long-term yield level. Flat curve presents an equal yield level between long-term and short-term.

The Nelson-Siegel method is a basic for modeling bond yield curves. The Nelson-Siegel method later developed into the Nelson-Siegel-Svensson and Extended Nelson-Siegel methods [2]. After that, other bond yield curve modeling methods were developed by the researchers. Several bond yield curve modeling methods are widely used by researchers, including Dynamic Nelson–Siegel Model [3]–[6], Factor augmented VAR and the Nelson and Siegel [7], the Nelson-Siegel model with GARCH and EGARCH volatility [8], Segmented Term Structure Models [9], dynamic natural cubic spline model [10], tractable dynamic factor models [11], and Machine Learning Techniques [12]. In modeling the bond yield curve, the parsimony method is needed [2]. Moreover, a curve pattern that minimizes error is required. The Bond yield always changes according to market conditions. Therefore, a proper curve that could model the Bond yield is needed, which is the parsimonious curve and the curve that attends to the flow of yield change so the error can be minimized.

In this research, the Bond yield curve is modeled using the bezier curve. Bezier curve is a smooth parameter curve [13], able to adjust the shape of the curve [14], and is often applied in computer graphics [15]–[17]. Bezier curves are used in constructing surfaces in engineering designs [18]. The Bezier curve has several control values to drive the curve. Bezier curve of $n$ order has $n + 1$ control value. In modeling the Bond yield curve, the bezier curve of order 3 or the cubic bezier curve is used. The cubic bezier curve is a flexible curve that follows the altering track [19]–[21]. Cubic bezier curve builds curves smoothly [22], more accurately than other approximation methods [15], [16]. The cubic bezier curve has four control values that could drive the curve. Thus, errors could be minimized. Cubic bezier curve is common for designing curves and surfaces, simple in the calculation, application, and evaluation [23].

## 2. RESEARCH METHODS

This research is a literature study on the bezier curve and is applied to Bond data. The used Bond in this research is the Bond of Surat Berharga Negara (Government Paper) Fix Rate type on August, 2–6 2021. Bond data are obtained from PT. Bursa Efek Indonesia (Indonesia Stock Exchange) via https://www.idx.co.id. Afterward, this data is processed to discover the equation model of the Bond yield curve.

## 3. RESULTS AND DISCUSSION

The general equation of the $n$-order bezier curve:

$$G(t_i) = \sum_{r=1}^{n} \binom{n}{r} t_i{}^r (1-t_i)^{n-r} X_r, \qquad t \in [0,1] \tag{1}$$

Control value of the curve is $X_i$. $n$-order bezier curve has $n + 1$ control values, which are $X_0$, $X_1$, $X_2$, ..., $X_n$. In the bezier curve equation, the t value represents the distance of the moving curve from starting point $X_0$ to the last point $X_n$. The t value is in 0 and 1 intervals, making the initial value $t_0 = 0$ and the final value $t_m = 1$. t value needs to be partitioned to $\Delta t$ which $\Delta t = \frac{1}{i-1}$. In the bezier curve, the searched value is $X_0$, $X_1$, $X_2$, ..., $X_n$ control value. To determine the control value, the least squares method is applied.

According to Equation (1), the cubic bezier curve (order 3) forms the Equation (2).

$$G(t_i) = (1-t_i)^3 X_0 + 3t_i(1-t_i)^2 X_1 + 3t_i{}^2(1-t_i)X_2 + t_i{}^3 X_3 \tag{2}$$

Cubic bezier curve has four control value, which are $X_0$, $X_1$, $X_2$, and $X_3$.

The modeled Bond yield curve uses the cubic bezier curve in the form of equation (2). In the equation of the cubic bezier curve, $t_i$ value is the estimation of time to maturity which the value is 0 – 1, and $G(t_i)$ is the estimation of Bond yield. In equation (2), there is a need for $t_i$ and control value of $X_0$, $X_1$, $X_2$, and $X_3$. The initial control value of $X_1$ and the final control value of $X_3$ were obtained from the Bond yield data. The temporary observation results of the control value between $X_1$ and $X_2$ is determined using the least squares method.

If there are $m$ data, $y_i$ is the Bond yield from observation results on the stock exchange and $G(t_i)$ is the Bond yield from estimation results. Therefore, the obtained least squares method equation:

$$GP = \sum_{i=1}^{m} [y_i - G(t_i)]^2 \tag{3}$$

$G(t_i)$ value in Equation (2) is substituted to Equation (3). Thus, Equation (4) is obtained.

$$P = \sum_{i=1}^{m} \{y_i - [(1-t_i)^3 X_0 + 3t_i(1-t_i)^2 X_1 + 3t_i{}^2(1-t_i)X_2 + t_i{}^3 X_3]\}^2 \tag{4}$$

Equation (4) is deducted partially on $X_1$ and $X_2$, then equaled to zero to minimize distance squares between the observation results yield data and the $G(t_i)$ estimation result yield data.

$$\frac{\partial P}{\partial X_1} = \sum_{i=1}^{m} \left(y_i - (1-t_i)^3 X_0 - t_i^3 X_3\right)(3t_i(1-t_i)^2) - 9\sum_{i=1}^{m} t_i^2 (1-t_i)^4 X_1 - 9\sum_{i=1}^{m} t_i^3 (1-t_i)^3 X_2 = 0 \tag{5}$$

$$\frac{\partial P}{\partial X_2} = \sum_{i=1}^{m} \left(y_i - (1-t_i)^3 X_0 - t_i^3 X_3\right)\left(3t_i^2(1-t_i)\right) - 9\sum_{i=1}^{m} t_i^3 (1-t_i)^3 X_1 - 9\sum_{i=1}^{m} t_i^4 (1-t_i)^2 X_2 = 0 \tag{6}$$

The answer to Equations (5) and (6):

$$X_1 = \frac{(A_2 C_1 - A_{12} C_2)}{(A_1 A_2 - A_{12} A_{12})} \tag{7}$$

$$X_2 = \frac{(A_1C_2 - A_{12}C_1)}{(A_1A_2 - A_{12}A_{12})} \tag{8}$$

with

$$A_1 = 9\sum_{i=1}^{m} t_i^2\,(1-t_i)^4$$

$$A_2 = 9\sum_{i=1}^{m} t_i^4\,(1-t_i)^2$$

$$A_{12} = 9\sum_{i=1}^{m} t_i^3\,(1-t_i)^3$$

$$C_1 = \sum_{i=1}^{m} 3t_i\,(1-t_i)^2\left[y_i - (1-t_i)^3X_0 - t_i^3X_3\right]$$

$$C_2 = \sum_{i=1}^{m} 3t_i^2\,(1-t_i)\left[y_i - (1-t_i)^3X_0 - t_i^3X_3\right]$$

Fix rate Bond data of August 2nd, 2021 is 35 data, of August 3rd, 2021 is 33 data, of August 4th, 2021 is 34 data, August 5th is 32 data, and August 6th, 2021 is 30 data. The Bond yield data presented is analyzed using the cubic bezier curve, so the Bond yield curve equation of the estimation result is obtained as Equation (2).

The analysis of Bond yield data begins with determining the partition value of $\Delta t$. The value of $\Delta t$ for each data is shown in Table 1.

**Table 1. Δ*t* value**

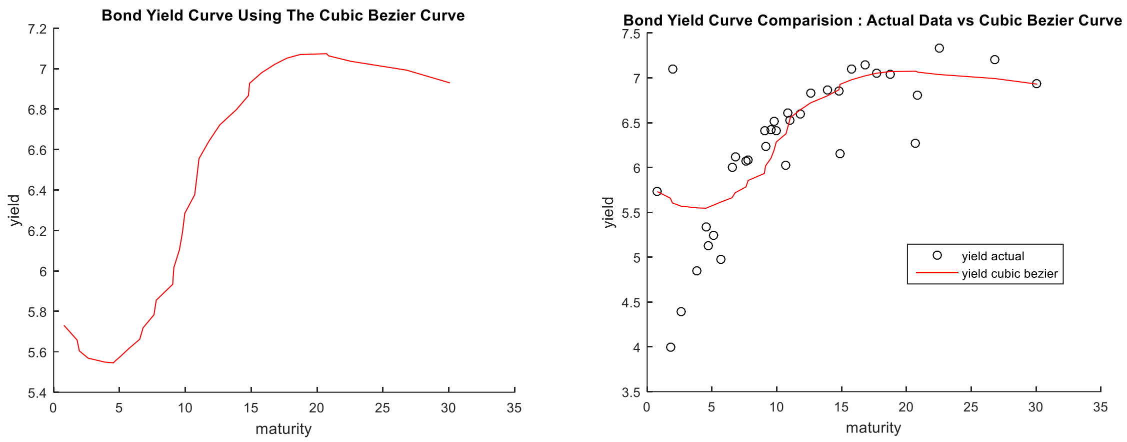
| No | Date | Δ*t* value |
|---|---|---|
| 1 | August 2nd, 2021 | 0.0294 |
| 2 | August 3rd, 2021 | 0.0313 |
| 3 | August 4th, 2021 | 0.0303 |
| 4 | August 5th, 2021 | 0.0323 |
| 5 | August 6th, 2021 | 0.0345 |

After the partition value is determined, yield data are sorted based on the least to greatest time to maturity. The next step, the most important, is to determine the control value $X_0$, $X_1$, $X_2$, and $X_3$. According to Bond yield data on August 2nd, 2021, the initial $X_0$ and the final $X_3$ on August 2nd, 2021 Bond data is $X_0 = 5.731$ and $X_3 = 6.93$. The control value of $X_0$ and $X_3$ obtained from the initial and the final data from observation results on Bond yield that has been sorted based on its time to maturity. The determined value between $X_1$ and $X_2$ based on Equations (7) and (8) is $X_1 = 4.7937$ and $X_2 = 7.7445$.

The equation of cubic bezier curve for August 2nd, 2021 Bond data:

$$G(t_i) = (1-t_i)^3\,5.731 + 3t_i(1-t_i)^2\,4.7937 + 3t_i{}^2(1-t_i)\,7.7445 + t_i{}^3 6.93 \tag{9}$$

The graphic of the Bond yield curve uses the cubic bezier curve for August 2nd, 2021 data that is presented in Figure 1(a). Figure 1(b) shows the comparison of the Bond yield curve between the actual observation result data and the cubic bezier curve estimation result yield data.




(a) (b)

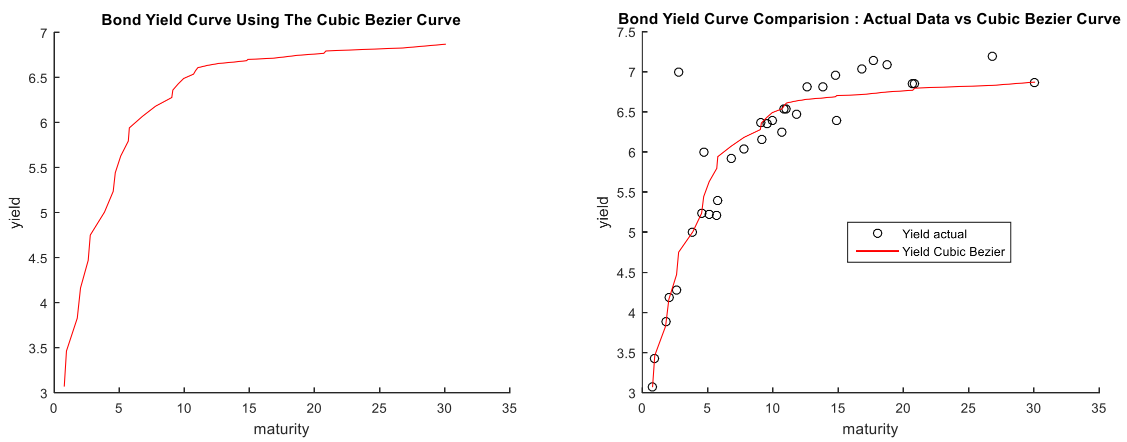
**Figure 1. (a) The Graphic of the Bond Yield Curve Using the Cubic Bezier Curve and (b) Comparing the Bond Yield Curve between Actual Data and Cubic Bezier Curve for August 2nd, 2021 Data.**

Figure 1(a) presents the cubic bezier curve encounters digression at the start of time to maturity T = 0.78 to $T = 4.54$. At the start $T =\ 0.78$, the yield value is 5.371 and at $T = 4.54$, the yield value decreases to 5.5454. The yield curve begins the increase after $T = 4.54$ to $T = 20.72$. The yield value decreases slowly after $T = 20.72$ to the last time to maturity. The greatest yield value is 7.0736 at $T = 20.72$ and the least yield value is 5.5454 at $T = 4.54$.

Figure 1(b) shows that from the beginning of time to maturity, there remains observation yield data unpassed by the cubic bezier curve. Its occurrence appears when the start of time to maturity is $T = 0.78$, the Bond yield of the observation results has a greater yield value than observation yield in several next time to maturity. When $T = 0.78$ to $T = 5.12$, the greatest observation yield value is at $T = 1.95$, which is 7.103. Several observation yield points have particular distances from the curve, so it is difficult for the curve to pass those points. The observation yield points that are not passed by the cubic bezier curve create a greater distance between observation yield data and estimation yield data. This occurrence influences the MSE value. The value of MSE for August 2nd, 2021 data is 0.2906. Each control value of $X_0$, $X_1$, $X_2$, and $X_3$ for August 3rd, 2021 data is $X_0 = 3.069$, $X_1 = 7.4525$, $X_2 = 6.3842$ and $X_3 = 6.87$. According to the obtained in-between value, the equation of the Bond yield curve that uses the bezier curve for August 3rd, 2021 data is presented as Equation (10).

$$G(t_i) = (1 - t_i)^3\, 3.069 + 3t_i(1 - t_i)^2\, 7.4525 + 3{t_i}^2(1 - t_i)\, 6.3842 + {t_i}^3 6.87 \qquad (10)$$

The formed Bond yield curve for August 3rd, 2021 Bond data is presented in Figure 2(a). The comparison of the Bond from observation and estimation results is presented in Figure 2(b).




(a) (b)

**Figure 2. (a) The Graphic of the Bond Yield Curve Using the Cubic Bezier Curve and (b) Comparing the Bond Yield Curve between Actual Data and the Cubic Bezier Curve for August 3rd, 2021 Data.**

In Figure 2(a), the modeled Bond yield curve with a cubic bezier curve shows that the longer the time to maturity is, the greater the Bond yield is obtained. From $T = 0.78$ to $T = 11.6097$, the cubic bezier curve is drastically increasing. After $T = 11.6097$, the cubic bezier curve is slowly increasing.

Figure 2(b) shows that the obtained cubic bezier curve passes most Bond yield points from observation results. There is one observation yield point that has a far enough distance from the cubic bezier curve, which is the point at $T = 2.78$, with 6.991. This observation yield is higher than other observation yields at the closest time to maturity with $T = 2.78$. The cubic bezier curve can efficiently model the cubic bezier curve. The obtained MSE from Bond yield data of the cubic bezier curve from estimation results on August 3rd, 2021 is 0.2161.

On Bond August 4th, 2021 data, the obtained control value is $X_0 = 3.043$, $X_1 = 7.575$, $X_2 = 6.3789$ and, $X_3 = 6.93$. According to these control values, the equation of the cubic bezier curve is presented in Equation (11).

$$G(t_i) = (1 - t_i)^3\, 3.043 + 3t_i(1 - t_i)^2\, 7.575 + 3{t_i}^2(1 - t_i)\, 6.3789 + {t_i}^3 6.93 \qquad (11)$$

The curved obtained from the cubic bezier curve estimation results on August 4th, 2021 data are presented in Figure 3(a) and 3(b). These Figures show the difference between Bond yield curve observation results and estimation results.

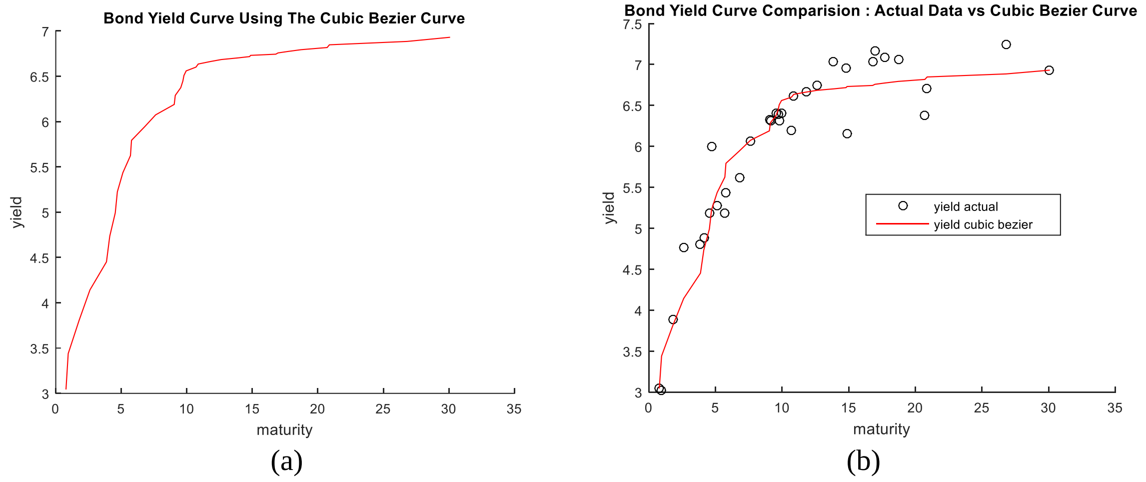



**Figure 3. (a) The Graphic of the Bond Yield Curve Using the Cubic Bezier Curve and (b) Comparing Bond Yield Curve between Actual Data and Cubic Bezier Curve for August 4th, 2021 Data.**

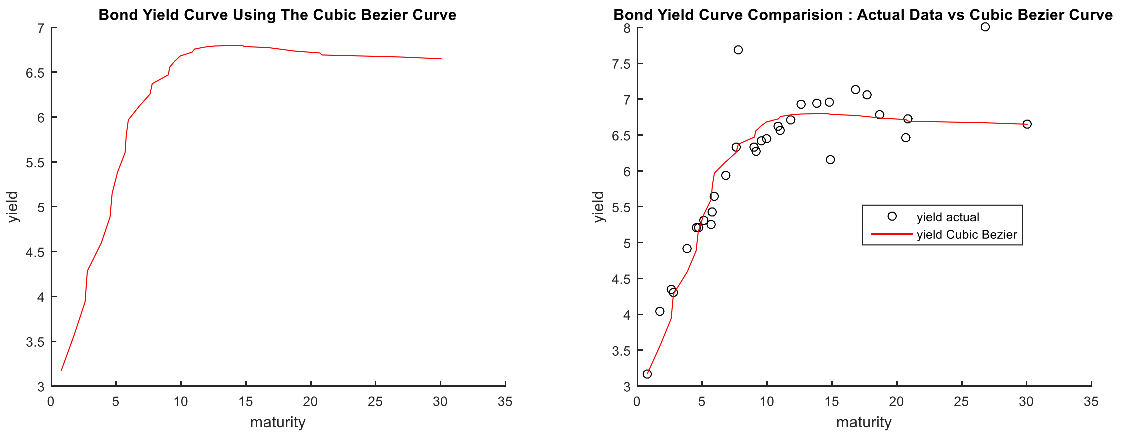
Figure 3(a) shows that the longer the time to maturity is from the Bond, the greater the Bond yield obtained. The Bond yield has a significant increase from the start of time to maturity to $T = 9.95$ time to maturity. At $T = 9.95$, the value of Bond yield is 6.5594. After $T = 9.95$, the value of Bond yield is slowly increasing.

According to Figure 3(b), the cubic bezier curve almost passes all points. The cubic bezier curve has a near enough distance to every Bond yield point from observation results. The obtained cubic bezier curve is highly effective in reaching Bond yield from observation results. It could be seen from the result of MSE from Bond data on August 4th, 2021, which is 0.098.

Defining control values $X_0$ and $X_3$ use the sorted Bond yield data based on the least to the greatest time to maturity, which are $X_0 = 3.172$ and, $X_3 = 6.65$. The control value between, $X_1$ and $X_2$, is based on the calculation result using equations (7) and (8), which are $X_1 = 7.4546$ and $X_2 = 6.8509$. Equation (12) is obtained from the equation of the cubic bezier curve on August 5th, 2021 data.

$$B(t_i) = (1 - t_i)^3\, 3.172 + 3t_i(1 - t_i)^2\, 7.4546 + 3{t_i}^2(1 - t_i)\, 6.8509 + {t_i}^3 6.65 \qquad (12)$$

The obtained cubic bezier curve as Equation (12) is presented in Figure 4(a). The comparison of the Bond yield curve between yield data from observation results and yield data from estimation results can be seen in Figure 4(b).




(a) (b)

**Figure 4. (a) The Graphic of the Bond Yield Curve Using the Cubic Bezier Curve and (b) Comparing the Bond Yield Curve between Actual Data and the Cubic Bezier Curve for August 5th, 2021 Data.**

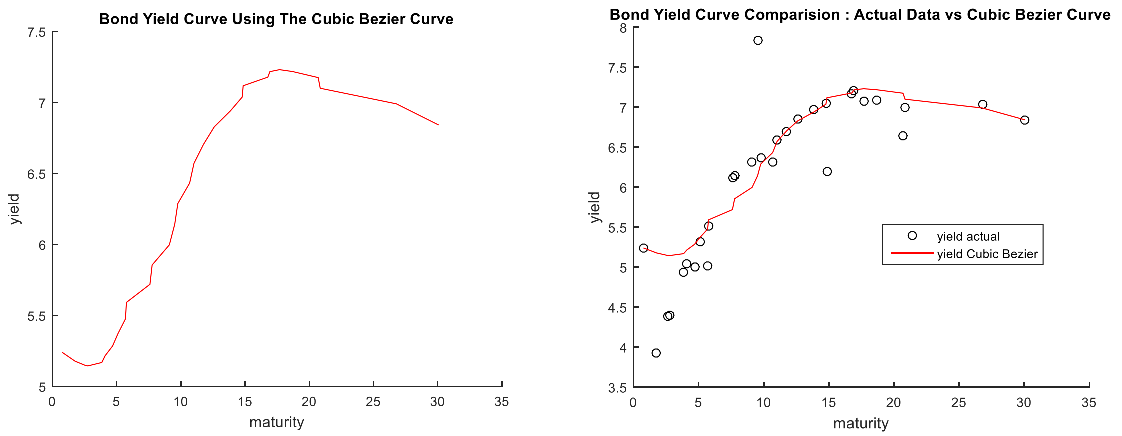
According to Figure 4(a), from the initial time to maturity $T = 0.77$ to $T = 12.61$, the Bond yield is constantly increasing along with the longer time to have the Bond (longer time to maturity). Afterward, the Bond yield seems to be constant then experiences digression that starts at time to maturity $T = 14.87$ to the latest data with $T = 30.04$.

According to Figure 4(b), the cubic bezier curve tends to have a close enough distance to the Bond yield from observation results (actual yield). There are two actual yield points that have a far enough distance to the cubic bezier curve, which is when the time to maturity $T = 7.78$ with 7.683 yield and $T = 14.87$ with 6.15 yield. The cubic bezier curve can model the Bond yield curve efficiently for Bond data on August 5th, 2021. The obtained MSE by the cubic bezier curve is 0.1719.

Based on the Bond data on August 6th, 2021, the obtained control values of the cubic bezier curve are $X_0 = 5.24$, $X_1 = 4.4801$, $X_2 = 8.4533$, and $X_3 = 6.843$. The equation of the obtained cubic bezier curve that uses these control values is presented in Equation (13).

$$B(t_i) = (1 - t_i)^3\, 5.24 + 3t_i(1 - t_i)^2\, 4.4801 + 3{t_i}^2(1 - t_i)\, 8.4533 + {t_i}^3 6.843 \tag{13}$$

Equation (13) generates the cubic bezier curve as shown in Figure 5(a). Figure 5(b) presents the difference between the Bond yield curve from observation data and estimation data.




(a) (b)

**Figure 5. (a) The Graphic of the Bond Yield Curve Using the Cubic Bezier Curve and (b) Comparing the Bond Yield Curve Actual Data and the Cubic Bezier for August 6th, 2021 Data.**

According to Figure 5(a), the cubic bezier curve experiences digression from the beginning to time to maturity $T = 4.68$. Afterward, the cubic bezier curve is increasing and reaches its peak on time to maturity $T = 17.68$ with 7.2308 yield. Then, the cubic bezier curve is decreasing.

According to Figure 5(b), several points have a far enough distance from the cubic bezier curve. These points cannot be passed by the cubic bezier curve. The point with the farthest distance is point (1.76, 3.92), with 1.76 times to maturity and 3.92 yields. The other points are (2.59, 4.386), (2.76, 4.402), (9.52, 783), and (14.85, 6.197). Because several points have far enough distances, the obtained MSE form Equation (13) is 0.2498.

Based on the Bond yield data from August 2nd – August 6th, 2021, the Bond yield curve obtained varies from the cubic bezier curve from estimation results. These occur because the Bond yield always changes following the market development. However, the Bond yield curve of the cubic bezier curve from estimation results in general on August 2nd – August 6th, 2021 data is quite similar. The curve of August 2nd is similar to the curve of August 6th. The curves for August 3rd, 4th, and 5th are also quite similar, but there are some differences for August 5th, 2021. On this date, after the significant progression of the curve, constant, and slow digression, there is no slow progression as the Bond yield curve on August 3rd and 4th, 2021. All the obtained Bond yield curves have significant progressions of Bond yield value in the particular period of time to maturity. After the significant progression, the Bond yield increases slowly, constantly, or decreases slowly.

The MSE of the cubic bezier curve for Bond yield data from the least to the greatest sequentially is the cubic bezier curve for Bond data on August 4th, 5th, 3rd, 6th, and 2th, 2021. The obtained MSE from the least to the greatest sequentially is 0.098, 0.1719, 0.2161, 0.2498, and 0.2906. The most approaching cubic bezier curve to Bond yield is the cubic bezier curve on August 4th, 2021. In Bond data on this date, the cubic bezier curve has the least MSE, which is 0.098. The greatest MSE is obtained from the cubic bezier curve on August 2nd, 2021, which is 0.2906.

The curve bezier cubic in general can efficiently model the Bond yield curve. The obtained MSE value by the bezier curve, in general, is classified as the least. This is suited to Rababah and Jaradat stating that the cubic bezier curve can minimize the error and be more accurate than the approximation method [16]. Okumura states the cubic bezier curve has good performance in representing geometric shapes [15]. Several advantages of the bezier curve make it suitable to model the curve or any geometric shape, including in modeling the Bond yield curve. Alvarez-Trejo et al. state the cubic bezier curve is often used in designing curves and surfaces, simple in the calculation, and simple to use and evaluate [23]. From the result of the research, it can be seen the cubic bezier curve can approach and follow the direction of Bond yield points from observation results. The cubic bezier curve has four control values that function to direct the curve efficiently. The control value has the role of controlling the direction of the curve, so the curve has the minimum distance to the observation yield point. This is suitable to the result of the study of Long et al., which states that the cubic bezier curve is a flexible curve that follows the alteration of direction [20].

The disadvantages of the cubic bezier curve are that the curve has to pass the first and last data. If the first data or last data is the outlier, then it makes the curve leave other points. The leaving curve from several points makes the greater MSE value. This type of curve occurs on the cubic bezier curve for Bond data on August 2nd and 6th, 2021.

## 4. CONCLUSIONS

The cubic bezier curve can efficiently model the Bond yield curve. The obtained MSE values by the bezier curve are little in general. The cubic bezier curve MSE for Bond yield data for the least to the greatest sequentially is the cubic bezier curve for Bond data on August 4th, 5th, 3rd, 6th, and 2nd, 2021. The obtained MSE from the least to the greatest sequentially is 0.098, 0.1719, 0.2161, 0.2498, and 0.2906.

## REFERENCES

[1] J. Hännikäinen, "When does the yield curve contain predictive power? Evidence from a data-rich environment," *Int. J. Forecast.*, vol. 33, no. 4, pp. 1044–1064, Oct. 2017, doi: 10.1016/J.IJFORECAST.2017.05.006.

[2] M. N. Ali *et al.*, “Estimating the Yield Curve for the Malaysian Bond Market Using Parsimony Method,” *Procedia Econ. Financ.*, vol. 31, no. 15, pp. 194–198, 2015, doi: 10.1016/s2212-5671(15)01220-4.
[3] H. Kaya, “Forecasting the yield curve and the role of macroeconomic information in Turkey,” *Econ. Model.*, vol. 33, pp. 1–7, Jul. 2013, doi: 10.1016/J.ECONMOD.2013.03.013.
[4] A. Abdymomunov, “Predicting output using the entire yield curve,” *J. Macroecon.*, vol. 37, pp. 333–344, Sep. 2013, doi: 10.1016/J.JMACRO.2013.05.002.
[5] J. Levant and J. Ma, “A dynamic Nelson-Siegel yield curve model with Markov switching,” *Econ. Model.*, vol. 67, pp. 73–87, Dec. 2017, doi: 10.1016/J.ECONMOD.2016.10.003.
[6] M. Guidolin and M. Pedio, “Forecasting and trading monetary policy effects on the riskless yield curve with regime switching Nelson–Siegel models,” *J. Econ. Dyn. Control*, vol. 107, p. 103723, Oct. 2019, doi: 10.1016/J.JEDC.2019.103723.
[7] F. Vieira, M. Fernandes, and F. Chague, “Forecasting the Brazilian yield curve using forward-looking variables,” *Int. J. Forecast.*, vol. 33, no. 1, pp. 121–131, Jan. 2017, doi: 10.1016/J.IJFORECAST.2016.08.001.
[8] W. Ullah and K. M. Bari, “The Term Structure of Government Bond Yields in an Emerging Market,” *J. Econ. Forecast. Inst. Econ. Forecast.*, vol. 3, pp. 5–28, 2018.
[9] C. Almeida, A. Simonsen, and J. Vicente, “Forecasting Bond Yields with Segmented Term Structure Models,” *Work. Pap. Ser. 288*, pp. 1–44, 2012.
[10] P. Feng and J. Qian, “Forecasting the yield curve using a dynamic natural cubic spline model,” *Econ. Lett.*, vol. 168, pp. 73–76, Jul. 2018, doi: 10.1016/J.ECONLET.2018.04.009.
[11] C. Gerhart and E. Lütkebohmert, “Empirical analysis and forecasting of multiple yield curves,” *Insur. Math. Econ.*, vol. 95, pp. 59–78, Nov. 2020, doi: 10.1016/J.INSMATHECO.2020.08.004.
[12] O. Castello and M. Resta, “Modeling the Yield Curve of BRICS Countries: Parametric vs. Machine Learning Techniques,” *Risks*, vol. 10, no. 2, 2022, doi: https://doi.org/10.3390/risks10020036.
[13] Z. Duraklı and V. Nabiyev, “A new approach based on Bezier curves to solve path planning problems for mobile robots,” *J. Comput. Sci.*, vol. 58, p. 101540, Feb. 2022, doi: 10.1016/J.JOCS.2021.101540.
[14] M. A. Fortes and E. Medina, “Fitting missing data by means of adaptive meshes of Bézier curves,” *Math. Comput. Simul.*, vol. 191, pp. 33–48, Jan. 2022, doi: 10.1016/J.MATCOM.2021.07.025.
[15] A. Okumura, “Optimization of the collection efficiency of a hexagonal light collector using quadratic and cubic Bézier curves,” *Astropart. Phys.*, vol. 38, no. 1, pp. 18–24, 2012, doi: 10.1016/j.astropartphys.2012.08.008.
[16] A. Rababah and M. Jaradat, “Approximating offset curves using Bezier curves ´ with high accuracy,” *Int. J. Electr. Comput. Eng.*, vol. 10, no. 2, pp. 1648–1654, 2020.
[17] M. Louzazni and S. Al-Dahidi, “Approximation of photovoltaic characteristics curves using Bézier Curve,” *Renew. Energy*, vol. 174, pp. 715–732, Aug. 2021, doi: 10.1016/J.RENENE.2021.04.103.
[18] G. Hu, C. Bo, G. Wei, and X. Qin, “Shape-adjustable generalized Bézier surfaces: Construction and it is geometric continuity conditions,” *Appl. Math. Comput.*, vol. 378, p. 125215, Aug. 2020, doi: 10.1016/J.AMC.2020.125215.
[19] D. Brander, J. A. Bærentzen, A. S. Fisker, and J. Gravesen, “Bézier curves that are close to elastica,” *Comput. Des.*, vol. 104, pp. 36–44, Nov. 2018, doi: 10.1016/J.CAD.2018.05.003.
[20] L. Chen, D. Qin, X. Xu, Y. Cai, and J. Xie, “A path and velocity planning method for lane changing collision avoidance of intelligent vehicle based on cubic 3-D Bezier curve,” *Adv. Eng. Softw.*, vol. 132, no. November 2018, pp. 65–73, 2019, doi: 10.1016/j.advengsoft.2019.03.007.
[21] O. Coskun and H. S. Turkmen, “Multi-objective optimization of variable stiffness laminated plates modeled using Bézier curves,” *Compos. Struct.*, vol. 279, p. 114814, Jan. 2022, doi: 10.1016/J.COMPSTRUCT.2021.114814.
[22] G. Bai and W. Fang, “Application of Quasi-Cubic Bézier Curves in the Blending of Tubes with Different Radiuses,” *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 612, no. 3, 2019, doi: 10.1088/1757-899X/612/3/032171.
[23] A. Álvarez-Trejo, E. Cuan-Urquizo, A. Roman-Flores, L. G. Trapaga-Martinez, and J. M. Alvarado-Orozco, “Bézier-based metamaterials: Synthesis, mechanics and additive manufacturing,” *Mater. Des.*, vol. 199, p. 109412, Feb. 2021, doi: 10.1016/J.MATDES.2020.109412.