BAREKENG: Jurnal Ilmu Matematika dan Terapan
March 2023 Volume 17 Issue 1 Page 0155–0164
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol17iss1pp0155-0164

MAX PLUS ALGEBRA OF TIMED PETRI NET FOR MODELLING SINGLE SERVER QUEUING SYSTEMS

Zumrotus Sya'diyah[1*]

STKIP Gotong Royong Masohi
Trans Seram Belakang Street, Negeri Haruru, Post Code 97514, Moluccas, Indonesia

*Corresponding author's e-mail: *zuma.yakuza@gmail.com*

ABSTRACT

This research modified a single server queuing system using timed Petri net. We add two places, a transition and its appropriate arcs. This research also considered all the holding times in the timed Petri net. We found that the Petri net is not stable but stabilizable according to Lyapunov stability criteria. The standard autonomous equation of the system is also determined. Furthermore, this system also has the eigenvalue which related to its periodical behavior, it is $\lambda = \max\left\{(C_2 + C_3 + C_4), (C_3 + C_4 + C_6), \left(\frac{C_2+2C_3+2C_4+C_6}{2}\right)\right\}$. *This means that the periodical behavior of the system only depends on the value of holding times of place W, R, B, and I.*

Article History:

Received: 9th September 2022
Revised: 6th December 2022
Accepted: 20th January 2023

Keywords:

Eigen Value;
Equation;
Lyapunov Stability;
Queuing System;
Single Server;
Standard Autonomous;
Timed Petri Net.

How to cite this article:

Z. Sya'diyah., "MAX PLUS ALGEBRA OF TIMED PETRI NET FOR MODELLING SINGLE SERVER QUEUING SYSTEMS," *BAREKENG: J. Math. & App.*, vol. 17, iss. 1, pp. 0155-0164, March 2023.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article • Open Access

1. INTRODUCTION

Queuing theory is a mathematical study in queuing or waiting lines, including its characteristic or any other properties of the system [1]. A queuing system can be defined as a dynamical system which has states that evolve in time by the occurrence of events at a possibly regular or irregular time intervals [2]. Modeling in queuing system is constructed and analyzed so that the lengths and waiting times of the queue can be predicted. Generally, queuing theory is mostly described in operation research, one of mathematic branches of field [3]. A queuing system is one of the crucial topics in humans' daily life, like buying a movie ticket, airport security, grocery check-out, mailing a package, getting a cup of coffee, etc. [4]

The queuing theory is mainly discussed using continuous mathematical systems, including various kinds of types according to its number of the server. But nowadays, there are many different points of view in describing any system, especially in mathematics. This happens to the queuing theory too. Not only described in the continuous equation system, there are many experts who describe the queuing theory in discrete event systems.

Petri net is one of the common tools in describing many discrete problems, as can be seen in [5], [6], [7], and [8]. Petri net is a tool of mathematical modeling in representing the state evolution of discrete event systems [9]. As the analyzing process, it is also common to use the max plus or min-max plus algebra on this matter.

Konigsberg is one of the experts who developed the model of queuing theory in a discrete event system point of view. In [2], he modeled various styles of queuing systems, including the break server of queuing. The model he built was using Petri net as the way he describes the condition and the event occurs in queuing theory. In that research, it is also using timed Petri net as the extension of the Petri net itself. The stability of the model also has been determined. But, in timed Petri net built before only has 2 holding times that were considered, and it also doesn't have an exact max plus standard autonomous equation of the timed Petri net yet.

In this paper, we modify the timed Petri net that had been built before by adding two places, a transition, and the appropriate arcs so that the model obtained represent the real condition more properly. This research also considers all the holding times in the place of the timed Petri net. We analyze the stability and the max plus standard autonomous equation is also determined. The result of this research purposively describes the queuing system more clearly.

2. RESEARCH METHODS

This research uses three major theories. They are timed Petri net, timed Petri net standard autonomous equation, and Lyapunov stability criteria of Petri net. So, in this section, we explain those two major theories and the assumption we used in building the timed Petri net.

2.1 Timed Petri Net

As we mentioned before, timed Petri net is the extension theory of Petri net. Petri net is 4-tuples of places, transitions, arcs, and weight of the arcs. Timed Petri net has a little difference from the usual Petri net. Timed Petri net has holding time in its place. So, if in the Petri net, a transition is called enable if there is enough token in the place so that the transition can be fired.

In the timed Petri net, it is not just the amount of token and the weight of the arc which determined whether a transition is enable or not, but also the holding times in its previous places. This holding time is the time duration of a token in a place to make a transition become enable. So, we can say that timed Petri net is 6-tuples of places, transitions, arcs, weight of the arcs, the initial marking vector, and the time structures (holding times) in the places. This time duration in firing an enable transition can be applied in many daily problems, such as in queuing system.

We can see the example of Petri net and timed Petri net to see the different between them two below.

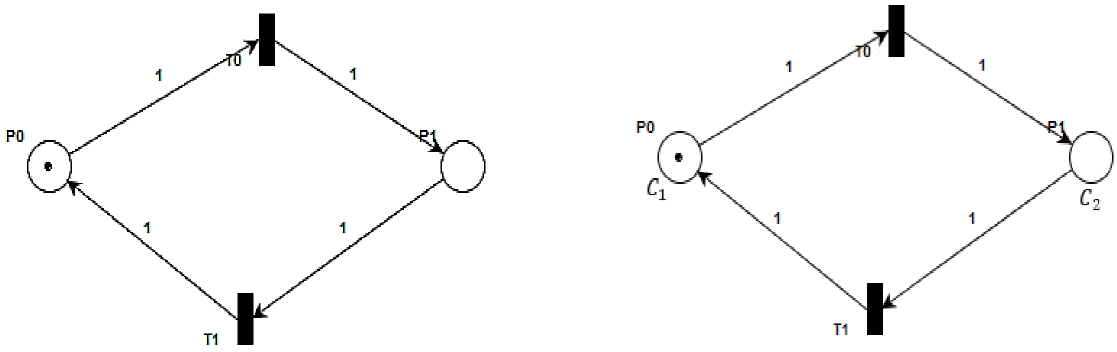

Figure 1. (a). The example of Petri net, (b). The example of timed Petri net

From **Figure 1**, we can see that in the timed Petri net, there are holding times in its places (the holding times can be included in all of the places or only several of them). We can see that in **Figure 1 (a)**, transition $\boldsymbol{T_0}$ is enable depends on the amount of token in the previous place. But, in the Figure 1 (b), transition $\boldsymbol{T_0}$ is enable depends not only on the amount of token but also the holding times in the previous place. Further information about Petri net and timed Petri net can be seen in **[5]**, **[6]**, **[7]**, **[8]**, **[10]**, **[11]**, and **[12]**.

2.2 Timed Petri Net Standard Autonomous Equation

Timed Petri net autonomous is one that each transition has at least an input place. The timed Petri net standard autonomous equation can be defined as follows:

$$\boldsymbol{x(k)} = \bigoplus_{m=0}^{M} \boldsymbol{A_m} \otimes \boldsymbol{x(k-m)}, \quad \boldsymbol{k \geq 0}$$

where $\boldsymbol{A_m} \in \mathbb{R}_{max}^{n\times n}$, $\boldsymbol{0 \leq m \leq M}$ and $\boldsymbol{x(m)} \in \mathbb{R}_{max}^{n\times n}$ with $\boldsymbol{-M \leq m \leq -1}$. This equation can be express as 1st order of differential equation with the state space is $\boldsymbol{\tilde{x}(k)}$ below:

$$\boldsymbol{\tilde{x}(k+1) = \tilde{A}(k) \otimes \tilde{x}(k)}$$

where $\boldsymbol{\tilde{x}(k)} = \left(\boldsymbol{x^T(k),\ x^T(k-1),\ \cdots,\ x^T(k+1-M)}\right)^{\boldsymbol{T}}, \boldsymbol{k \geq 0}$ and

$$\boldsymbol{\tilde{A}} = \begin{pmatrix} A_0^*\otimes A_1 & A_0^*\otimes A_2 & \cdots & \cdots & A_0^*\otimes A_M \\ E & \varepsilon & \cdots & \cdots & \varepsilon \\ \varepsilon & E & \ddots & \cdots & \varepsilon \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \varepsilon & \varepsilon & \cdots & E & \varepsilon \end{pmatrix} \text{ with } \boldsymbol{A_0^*} = \bigoplus_{i=0}^{n-1} \boldsymbol{A_0^{\otimes i}}$$

Further theory of timed Petri net can be found in **[13]**, **[2]**, **[11]**, **[14]**, and **[15]**.

2.3 Lyapunov Stability of Petri Net

The system modeled with Petri net can be analyzed in its stability term state. A Petri net can be called a stable one if there is a strictly positive *m* vector, i.e., $\boldsymbol{\Phi}$, such that the following inequality is held:

$$\boldsymbol{\Delta v = e^T A^T \Phi \leq 0}$$

Because the firing vector, $\boldsymbol{e}$**,** is a non-negative one (at least one of its elements is positive). So, the inequality we express above can be simplified as follows:

$$\boldsymbol{A^T \Phi \leq 0}.$$

Furthermore, the system modeled by Petri net can be stabilizable if there is a firing vector, $\boldsymbol{e}$, such that the following expression is held:

$$\boldsymbol{Ae \leq 0}$$

The more massive explanation can be seen in [2], [16] and [17].

2.4 Assumptions in Modelling the Timed Petri Net

There are several assumptions that we used in developing and building the timed Petri net model of queuing system. They are:

1. We assume that the customer is arriving at the service area as soon as the previous customer is queuing in the line.
2. We assume that when the server is in the breaks condition, the customer who was being served queuing back in the line and waiting until the server is restored.
3. We assume that the server will be idle right away after the customer is finished being served.
4. All the places in the timed Petri net have holding time which is considered in determining the enabled transition.

3. RESULTS AND DISCUSSION

In this section we will discuss about the model we have been developed, its explanation and also the analysis result of the model will also be preserved. The model we developed is a representation of a queuing system which involves single server and the probability of down server is also considered.

3.1. Timed Petri Net Model of the Queuing System

The timed Petri net model of this type of queuing system can be seen in **Figure 2**. Where each place represents the following conditions:

A : Customer is arriving
W : Customer is waiting in the line/queuing
R : Customer is ready to be served
B : Customer is being served
L : Customer is leaving the service area
D : Server is out of order
I : Server is in idle condition

and the transition represents the following event:
q : Customer arrives to the queue
c : Customer is being called by the server
s : Server starts working
b : Server break or breakdown
r : Server is restored
f : Customer is finished to be served

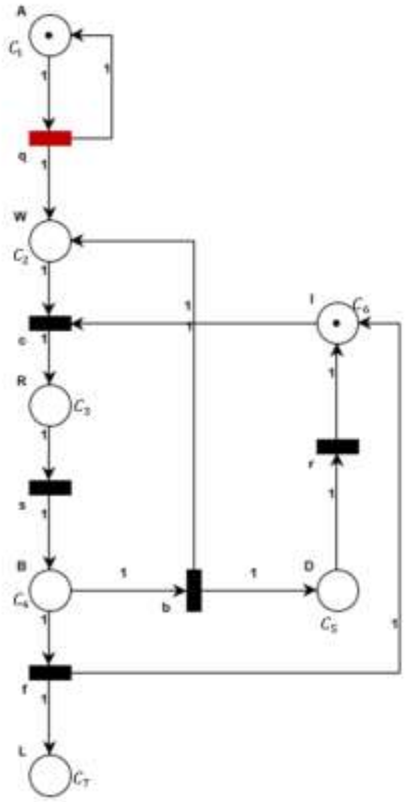

Figure 2. Timed Petri Net Model of Queuing System with Single Server and Down Server is Considered

The timed Petri net model above describes that the queuing system with single server and the server breakdown is considered. This timed Petri net is one that we developed from the model in [2]. We added two places, one transition with the appropriate arc, and an arc that connect the transition b to place W. This arc is added according to the assumption we made before that that when the server is in the breaks condition, the customer who was being served queuing back in the line and waiting until the server is restored.

3.2. Timed Petri Net Model of the Queuing System Stability

From the timed Petri net in Figure 2, we can get the following both forward and backward matrix incidence:

$$A_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } A_b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, we can get the matrix incidence as follows:

$$A = A_f - A_b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, we can explain the stability of the system according to the incidence matrix above. Using the Lyapunov stability criteria, we can let a vector $\Phi = [\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7]^T$ so that $A^T\Phi \leq \mathbf{0}$ which is sufficient to us just to find the vector $\Phi = [\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7]^T$ so that the equality $A^T\Phi = \mathbf{0}$ hold. With Gauss Jordan elimination to the matrix A^T, we can derive that:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \\ \Phi_5 \\ \Phi_6 \\ \Phi_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Then, we obtain that $\Phi_2 = \Phi_7 = 0$. So, we get that Φ is not a strictly positive vector.

According to Lyapunov stability criteria, the vector Φ must be a strictly positive one. So, we can conclude that the queuing system modeled in Figure 2, is not stable. It is also necessary to ensure whether the system is stabilizable or not using firing vector as explained.

Let a vector $e = [e_1, e_2, e_3, e_4, e_5, e_6]^T$ as firing vector of the system, so that the equation $Ae = \mathbf{0}$ hold. Through the Gauss-Jordan elimination of the matrix A we get the following equation:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

So that we can obtain the firing vector $e = [0, y, y, y, 0]^T$ which is a non-zero vector. So, we can deduce that the queuing system modeled in **Figure 2** is stabilizable.

3.3. Timed Petri Net Model of the Queuing System Standard Autonomous Equation and Its Eigenvalue

Our next step is to determine the timed Petri net standard autonomous equation, regarding to the max-plus algebra, of the system. The equation of the queuing system modeled in **Figure 2** can be derived as follows:

$$x_1(k) = C_1 \otimes x_1(k-1)$$
$$x_2(k) = C_2 \otimes x_1(k) \oplus C_2 \otimes x_4(k) \oplus C_6 \otimes x_6(k-1)$$
$$x_3(k) = C_3 \otimes x_2(k)$$
$$x_4(k) = C_4 \otimes x_3(k)$$
$$x_5(k) = C_5 \otimes x_4(k)$$
$$x_3(k) = C_4 \otimes x_3(k)$$

where:

$x_1(k)$ denotes the time of customer arrives to the queue at k-period
$x_2(k)$ denotes the time of customer is being called by the server at k-period
$x_3(k)$ denotes the time of service starts at k-period
$x_4(k)$ denotes the time of server breaks or in breakdown condition at k-period
$x_5(k)$ denotes the time of service is restored at k-period
$x_6(k)$ denotes the time of customer is finished to be served at k-period

So, we can describe the max-plus equation above in matrix expression below:

$$\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \end{bmatrix} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & C_3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & C_4 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & C_5 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & C_4 & \varepsilon & \varepsilon & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \end{bmatrix} \oplus \begin{bmatrix} C_1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & C_2 & \varepsilon & C_6 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix} \otimes \begin{bmatrix} x_1(k-1) \\ x_2(k-1) \\ x_3(k-1) \\ x_4(k-1) \\ x_5(k-1) \\ x_6(k-1) \end{bmatrix}$$

From that equation, we can determine the following corresponding matrices:

$$A_0 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & C_3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & C_4 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & C_5 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & C_4 & \varepsilon & \varepsilon & \varepsilon \end{bmatrix} \text{ and } A_1 = \begin{bmatrix} C_1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & C_2 & \varepsilon & C_6 \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

Then, we also can get the following matrices:

$$A_0^{\otimes 2} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & C_3 + C_4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & C_4 + C_5 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & C_3 + C_4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}, A_0^{\otimes 3} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 + C_4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & C_3 + C_4 + C_5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 + C_4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix},$$

$$A_0^{\otimes 4} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 + C_4 + C_5 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}, \text{ and } A_0^{\otimes 5} = \boldsymbol{\mathcal{E}}(6 \times 6)$$

And we can compute the matrices of A_0^* and $\tilde{A}$ as follows:

$$A_0^* = \bigoplus_{\boldsymbol{i=0}}^{\boldsymbol{5}} \boldsymbol{A_0^{\otimes i}} = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 & C_3 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 + C_4 & C_3 + C_4 & C_4 & \varepsilon & \varepsilon & \varepsilon \\ C_2 + C_3 + C_4 + C_5 & C_3 + C_4 + C_5 & C_4 + C_5 & C_5 & \varepsilon & \varepsilon \\ C_2 + C_3 + C_4 & C_3 + C_4 & C_4 & \varepsilon & \varepsilon & \varepsilon \end{bmatrix}$$

$$\tilde{A} = A_0^* \otimes A_1 = \begin{bmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_1 + C_2 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ C_1 + C_2 + C_3 & \varepsilon & \varepsilon & C_2 + C_3 & \varepsilon & C_3 + C_6 \\ C_1 + C_2 + C_3 + C_4 & \varepsilon & \varepsilon & C_2 + C_3 + C_4 & \varepsilon & C_3 + C_4 + C_6 \\ C_1 + C_2 + C_3 + C_4 + C_5 & \varepsilon & \varepsilon & C_2 + C_3 + C_4 + C_5 & \varepsilon & C_3 + C_4 + C_5 + C_6 \\ C_1 + C_2 + C_3 + C_4 & \varepsilon & \varepsilon & C_2 + C_3 + C_4 & \varepsilon & C_3 + C_4 + C_6 \end{bmatrix}$$

So, using matrix $\tilde{A}$ above, we can get the timed Petri net standard autonomous equation $\boldsymbol{\tilde{x}(k+1)} = \boldsymbol{\tilde{A} \otimes \tilde{x}(k)},$ with $\boldsymbol{\tilde{x}(k)} = [x_1(k), x_2(k), x_3(k), x_4(k), x_5(k), x_6(k)]^{\boldsymbol{T}}$. The matrix $\tilde{A}$ can be interpreted into a graph as shown in Figure 3 below. It is necessary to be drawn so that we can find the exact eigenvalue, as one of the characteristics of the system related to its periodic behavior.

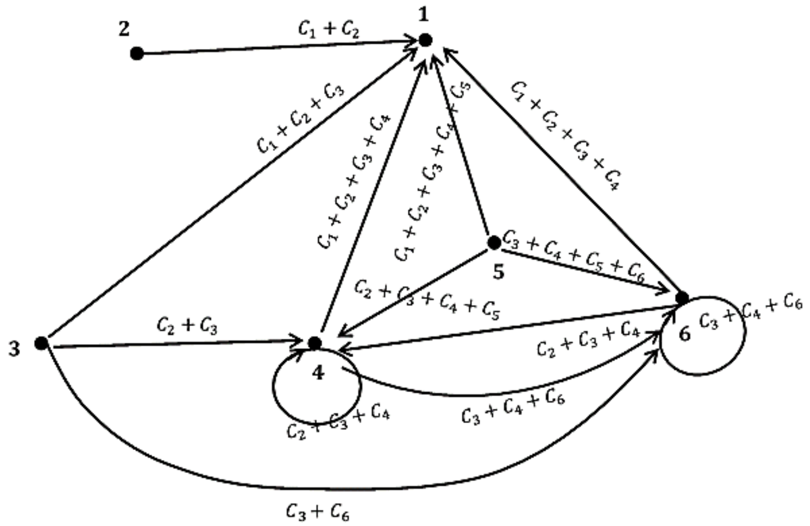

Figure 3. The Graph of Timed Petri Net Standard Autonomous

So, we can determine the circuits and their average weight of graph in **Figure 3** can be seen in **Table 1**.

Table 1. Circuits and the Average Weights of Graph Autonomous

Circuit	Average Weight
4 – 4	$C_2 + C_3 + C_4$
6 – 6	$C_3 + C_4 + C_6$
6 – 4 – 6	$\frac{C_2 + 2C_3 + 2C_4 + C_6}{2}$

Finally, we can obtain the eigenvalue of the system modeled with timed Petri net in **Figure 2** of queuing system is:

$$\lambda = \max\left\{(C_2 + C_3 + C_4), (C_3 + C_4 + C_6), \left(\frac{C_2 + 2C_3 + 2C_4 + C_6}{2}\right)\right\}$$

So, this maximum value will be determined the periodical behavior of the system. It means that the queuing system will be oscillated in period of λ.

4. CONCLUSIONS

From the explanation and the research result we mentioned before, the queuing system with a single server and the break or breakdown server is considered can be modeled in timed Petri net. The system maybe not stable but stabilizable. This queuing system will be oscillated in period of $\lambda = \max\left\{(C_2 + C_3 + C_4), (C_3 + C_4 + C_6), \left(\frac{C_2+2C_3+2C_4+C_6}{2}\right)\right\}$. So, it is depended on the value of holding times of place W, R, B, and I. As for further research, this system can be simulated into several examples of holding times to show the periodical behavior of the system. It also can be considered that the holding times is the interval one for the development of the timed Petri net in queuing system theory.

REFERENCES

[1] Martha, G.A. , Siswanto, and Tri Atmojo Kusmayadi, "Petri Net and Max-Plus Algebra Model in Bank Queue with Two Servers in Each Service," *Journal of Physics: Conference Series*, vol. 06, no. 1, pp. 1-8, August 2019.

[2] Z. R. Konigsberg, "Timed Petri Nets Modeling And Lyapunov/Max-Plus Algebra Stability Analysis For A Type Of Queuing Systems," *International Journal of Pure and Applied Mathematics*, vol. 86, no. 2, pp. 301-323, January 2013.

[3] N. K. Krivulin, "The Max-Plus Algebra Approach in Modelling of Queueing Networks," in *Summer Computer Simulation Conference, pp. 485-490*, Portland , 1996.

[4] F. D. Utami, Analysis Of Queue Systems In Optimization Of Services At Bank BJB Rawamangun Branch Office, Jakarta: Bachelor Degree [Thesis], STIE Indonesia, 2020.

[5] Freddo, Francine, Sandro Sawicki, Rafael Z. Frantz, et al, "Using timed and coloured Petri nets for modelling, simulation, and analysis of integration solutions," *Proc. Of International Journal of Web Engineering and Technology,* vol. 14, no. 1, pp. 231-238, 2019.

[6] Kindler, E., Bergenthum, R, "Algorithms and Tools for Petri Nets," in *Proceedings of the Workshop AWPN, DTU Compute*, Lyngby, Denmark, 2017.

[7] Patela, Ajay M, Anand Y. Joshib, "Modeling and Analysis of a Manufacturing System with Deadlocks to Generate the Reachability Tree using Petri Net System," in *International Conference On Design And Manufacturing (IConDM)*, Chennai, India, 2013.

[8] Qin, Jianhlong, Na Zhao, Zhongwen Xie, et al, "Business Process Modelling based on Petri nets," *MATEC Web of Conference* , vol. 139, no. 1, pp. 105-113, 2017.

[9] Azizah,Farah , Subiono, "Max Plus Algebra and Petri Net Application onv Scheduling of Ship Engine Component’s Spare Part Ordering," *International Journal Of Computing Science And Applied Mathematics,* vol. 4, no. 1, pp. 1-9, February 2018.

[10] Bera, D., Hee, van, K. M., Sidorova, N, "Discrete timed Petri nets," *Computer science reports,* vol. 1303, no. 1, pp. 198-213, 2013.

[11] Popova-Zeugmann, Louchka, "Timed Petri Nets," in *Time and Petri Nets, eBook Edition,* New York, Springer Heidelberg New York Dordrecht London, 2013, pp. 139-168.

[12] Tristono, Tomi, Setiyo Daru Cahyono ,Sutomo, et al, "Model petri net of adaptive traffic lights and its collaboration with a special event," in *MATEC Web of Conferences 147*, Bandung, Indonesia, 2018.

[13] A. Afif, Aplikasi Petri Net Dan Aljabar Max-Plus Pada Sistem Jaringan Kereta Api Di Jawa Timur, Surabaya: Magister [Thesis], Program Magister Jurusan Matematika, FMIPA: INSTITUT TEKNOLOGI SEPULUH NOPEMBER (ITS), Available: http://repository.it, 2015.

[14] S. R. P. W. Pramesthi, "Model Petri Net Sistem Jaringan Antrean Multichannel Tak-Siklik 5 Server," *TRANSFORMASI: Jurnal Pendidikan Matematika & Matematika,* vol. 2, no. 2, pp. 40-50, December 2018.

[15] Subiono, "Aljabar Max Plus," in *Aljabar Min-Max Plus dan Terapannya*, Surabaya, Jurusan Matematika-ITS, 2015, pp. 111-150.

[16] Subiono, Zumrotus Sya’diyah, "Lyapunov-Max-Plus-Algebra Stability in Predator-prey Systems Modeled with Timed Petri Net," *IPTEK: The Journal for Technology and Science,* vol. 22, no. 3, pp. 117-121, 2011.

[17] Z. Sya’diyah, "Kestabilan Model Petri Net Dari Sistem Pembayaran Tagihan Listrik PT. PLN (Persero) Rayon Ambon Timur," *BAREKENG: Jurnal Ilmu Matematika dan Terapan,* vol. 15, no. 4, p. 601–606, 2021.