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1/3 SIMPSON'S RULE FOR ANALYSIS OF STRUCTURE DYNAMIC RESPONSE DUE TO EARTHQUAKE LOAD

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ABSTRACT

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In structural analysis, many calculations are encountered which are very complex, making it difficult to do with exact mathematical calculations. For easier analysis, numerical methods are needed to simplify the calculations. Complex building structures, such as towers, multi-story structures, and other buildings, are idealized for simplification into a single degree of freedom system (SDOF), assuming that the dynamic response of structures due to earthquake loads is horizontal. The analysis of this model is correlated with numerical analysis, so it can be completed quickly. The numerical method used in this study is the 1/3 Simpson Integral Method because this method is suitable for calculating dynamic structural responses such as structural displacement responses. The analysis procedure begins by entering the external forces on the structural system and calculating the resulting response value. The analysis can be repeated for a variety of different parameters, such as the mass of the structure, the damping ratio, and the stiffness of the structure. The structural response is calculated by sinusoidal dynamic load type for damped and undamped systems. The results of this study conclude that the relationship between the mass of the structure, the damping of the structure, and the stiffness of the structure with the displacement of the structure has an inverse relationship, where with high mass, high damping, and high stiffness, it can reduce the structure displacement.

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1. INTRODUCTION

Indonesia is an area that has a high vulnerability to earthquake hazards because it is located between three world plates, namely the Eurasian plate, Pacific plate, and Indo-Australian plate [1],[2],[3],[4],[5],[6],[7]. An earthquake is a sudden release of rock energy in the earth's crust due to the accumulation of old rock energy [8]. The accumulation of energy between these plates will release earthquake energy throughout the soil layer and affect the existing structures above the ground surface until the buildings are shaking [9],[10].

Vibration and shaking of the ground due to the earthquake suffered by the structure of the building will damage and collapse the building if it is not designed in accordance with earthquake-structure standards, even resulting in loss of life and property loss [11],[12]. The ground shaking due to the earthquake that affects the foundation of the structure is characterized by the magnitude of the ground acceleration. As a result, this ground acceleration will cause a deviation from a structure. The magnitude of this ground acceleration is influenced by three things, namely the source of the earthquake, the direction of propagation of earthquake waves, and the influence of local soil conditions (site). This means that a large earthquake source and close to where the building standars will cause large ground vibrations in the structure [13].

In planning the structure in an area with high seismic conditions, it is necessary to pay attention to the seismic aspects that affect the structure, such as earthquake loads due to ground shaking[14]. Earthquake load is a dynamic load that repeats and changes with time, so the analysis requires repetition and a long time [15],[16]. Every building structure has a dynamic response [17]. Acceleration and displacement of the structure is the response of the structure due to the dynamic load of the earthquake. The occurrence of this structural response can be repeated and periodic over time [18].

Dynamic loads due to earthquakes are not the same as other dynamic loads. Structural damage caused by ground shaking from random earthquake waves damages the structure from within and not as an external force. Earthquake loads occur because of the rock mass in the soil layer under the structure so that it can damage the building structure from the inside. In structural design, earthquake load is the design load for the calculation of structural strength [19].

In a certain existence, the structure may experience periodic dynamic loads, such as moving loads from trucks, cars, trains, and nuclear loads, as well as dynamic loads from earthquakes that occur repeatedly with a certain frequency. Based on this condition, it is necessary to review the response of the building structure with dynamic analysis, especially buildings that are close to earthquake sources. Dynamic analysis is the response of the structure to the dynamic load of the earthquake [20]. One of the analytical methods used to solve the above problems is Simpson's numerical integral analysis. This study aims to analyze the dynamic response of the structure due to the earthquake, such as displacement on portal structure with a structure system of single degree of freedom (SDOF).

Each structure has an infinite number of degrees of freedom, i.e., the number of coordinates needed to express the position of a system/mass at any given time, and has a natural frequency of as many degrees of freedom as it has. This means that if the type of structure is one floor, then the structure has one mass and a single degree of freedom system [21]. When the structure receives dynamic loads with a frequency close to the natural frequency, the structure will experience resonance which allows the structure to collapse.

Structures that experience horizontal displacement or displacement due to dynamic loads experienced by the structure are called structures with a single degree of freedom system. The structure with the SDOF system is modeled with a structure that has a single mass and displacement coordinates. For example, this type of structure with the SDOF system can be viewed as a simple single-level portal, as shown in Figure 1. Mathematically, the numerical model for the structure depicting the SDOF single degree of freedom system is described in Figure 2.

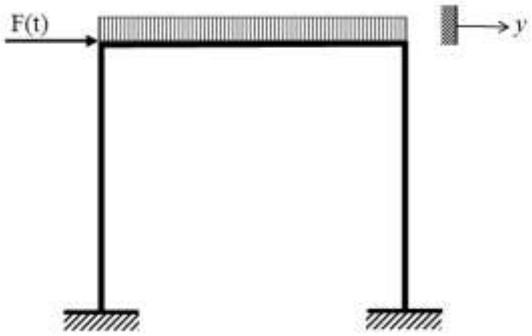

Figure 1. The portal structure of SDOF system

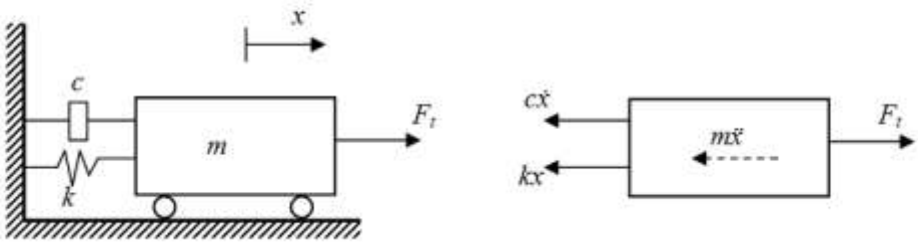

Figure 2. Idealization of the SDOF system and its free body diagram

The system model, as shown in **Figure 2**, is an idealization of the SDOF system, and the mass m is considered to be a rigid mass system. The roller supports stop the mass and can only move in the direction of simple horizontal translation. The single displacement coordinate x determines its position. The elastic resistance to displacement is provided by a weightless spring of stiffness k, and the energy loss (damping) mechanism is described by a damper c. The time-varying Ft load is an excitation loading mechanism that causes a dynamic response to the system. Based on the free body diagram in **Figure 2** above, the equation is obtained [22]:

$$m\ddot{x} + c\dot{x} + kx = F_t \tag{1}$$

where m is the mass of the system, c is the damping constant, and k is the spring constant, while $\ddot{x}$, $\dot{x}$, and x are the acceleration, velocity, and displacement/difference of the system, and F_t is the dynamic load originating from outside (excitation load), planned in the calculations. In the analysis, the earthquake load received by the structure is considered as an impulse load. Impulse load is a load that lasts for a very short time and suddenly. According to Newton's Laws of Motion, the impulse load is defined as,

$$m.dv/d\tau = F(\tau) \tag{2}$$

or,

$$dv = F(\tau)d\tau/m \tag{3}$$

where $F(\tau)d\tau$ is the impulse and dv is velocity, with the initial velocity we can assume the initial velocity of a mass m at time. The impulse load $F(\tau)d\tau$ of the structure acts on an undamped oscillator. When the oscillator changes in speed and acceleration, assuming the initial velocity and acceleration are zero, $\dot{x}(0) = 0$ and $\ddot{x}(0) = 0$ at time t and results in a change in displacement at τ time, so that:

$$dx(t) = \frac{F(\tau)d\tau}{m\omega}\sin(t-\tau) \tag{4}$$

The loading process can be said as a single short impulse in every increase in time $d\tau$, and each impulse forms a differential response at time t. Therefore, the total displacement at time t is an integral of the differential displacement $dx(t)$ at $t = 0$ to $t = t$, so that:

$$x(t) = \frac{1}{m\omega}\int_0^t F(\tau)\sin\omega(t-\tau)d\tau \tag{5}$$

Equation (5) above is known as the Duhamel integral, where the damped system response from **Equation (5)** is obtained by substituting the impulse load $F(\tau)d\tau$ and the initial velocity $dv = F(\tau)d\tau/m$, into the damped free vibration equation. For $x_0 = 0$ and $\dot{x}_0 = F(\tau)d\tau/m$ and substitute t-τ for t, we get the displacement at t_i:

$$dx(t) = e^{\xi\omega(t-\tau)} \frac{F(\tau)d\tau}{m\omega_D} \sin \omega_D (t-\tau) \tag{6}$$

For all parts of the differential displacement response of each load at time t, calculated by the damped system conditions,

$$x(t) = \frac{1}{m\omega_D} \int_0^t F(\tau) e^{-\xi\omega(t-\tau)} \sin \omega_D (t-\tau) d\tau \tag{7}$$

Equation (7) above, is known as the Duhamel Integral. The Duhamel integral solution of Equation (7) above is:

$$x(t) = [A_D(t) \sin \omega_D t - B_D(t) \cos \omega_D t] \frac{e^{-\xi\omega t}}{m\omega_D} \tag{8}$$

To get the value of A_D (t_i) and B_D (t_i), numerical evaluation is required. In this study, the numerical method used is the 1/3 Simpson's Rule. The Simpson's Rule is a fairly accurate numerical analysis and is a basic mathematical concept that has always been developed [23]. The general form of the integral of a function I(τ) is [24], [25]:

$$I(\tau) = \int_0^t F(\tau) d\tau \tag{9}$$

The A_D and B_D values of Equation (8) are calculated by numerical analysis, which is developed from the general form of numerical integration of Equation (9) above.

$$A_D(t_i) = A_D(t_{i-1}) + \int_{t_{i-1}}^{t_i} F(\tau) e^{\xi\omega t} \cos \omega_D \tau d\tau \tag{10}$$

$$B_D(t_i) = B_D(t_{i-1}) + \int_{t_{i-1}}^{t_i} F(\tau) e^{\xi\omega t} \sin \omega_D \tau d\tau \tag{11}$$

where m is mass (kg.sec^2/cm), $F(\tau)$ is a function of earthquake load (kg), is angular velocity (rad/sec), D is damped angular velocity (rad/sec), t_i is time to-i (dt), is the damping ratio, and $x(t_i)$ is the displacement (cm).

The basic formulation of the numerical integral derivative of Equation (9) for the 1/3 Simpson's Rule is as follows,

$$A(t) = \frac{d\tau}{3} \left(I_0 + 4I_1 + 2I_2 + \ldots + 4I_{n-1} + I_n \right) \tag{12}$$

where I_0, I_1, I_2, …, I_n is the integral value for each time.

2. RESEARCH METHODS

This research is a numerical approach. The required input data are system mass (m, kg.sec^2/cm), natural angular frequency (ω, rad/sec) and damped angular velocity, D, rad/sec), damping ratio (ξ, %), maximum time integral (t_{max}, dt), acceleration due to gravity (g, cm/sec^2), i time (t_i) and force at i time, (F_i).

Figure 3. Idealization of the SDOF system and its free body diagram

Based on Figure 3 above, the following formulation relationship applies,

$$m\omega^2 = \frac{F}{d} \tag{13}$$

where, m is the mass of the structure, is the angular velocity, F is the excitation load on the structure and d is the displacement.

2.1 Case Study

The case study for this research model is a simple portal structure with a single degree of freedom (SDOF) system, as shown in Figure 1, which is idealized as Figure 3. Based on the calculation results of the structural data, the mass, $m = 14.5$ kg.sec^2/cm, natural angular velocity $\omega = 17.755$ rad/sec, damped angular velocity $\omega_D = 17.03$ rad/sec, and stiffness $k = 4027.27$ kg/cm. Structural analysis was carried out with dynamic loading from earthquake loads of $F(t) = 1950\ cos\ t$, during $t = 30\ sec$.

2.2 Analysis Procedure

At first, data such as mass m, natural angular velocity and damped angular velocity ω_D, time t_i data, and loading F (t), are input. Furthermore, the displacement, $x(t)$, due to dynamic loads caused by the earthquake is calculated based on Equation (8). The analysis was carried out to see how the influence of mass m, damping ratio, and stiffness k varies on the displacement of the portal structure. The model variations of mass, damping ratio, and stiffness used are shown in Table 1.

Table 1. Variation of constant mass with different damping ratio and stiffness

Variation model	Mass (kg.sec^2/cm)	Stiffness (kg/cm)	Damping ratio (ξ)
1	14,5	4205,30	0
2	14,5	2012,14	0,5
3	14,5	8410,60	2

From Table 1 above, it is explained that there are three variations with the mass and stiffness used being $m = 14.5$ kg.sec^2/cm and $k = 4205.30$ kg/cm is variation I as a reference variation, with damping ratio $\xi = 0$. For variation II and III respectively with the reference mass being $m = 14.5$ kg.sec^2/cm, stiffness $k = 2012.14$ kg/cm and damping ratio $\xi = 0.5$, and mass $m = 14.5$ kg.sec^2/cm, stiffness $k = 8410.60$ kg/cm and damping ratio $= 2$.

Table 2. Variation of damping ratio and constant stiffness with different structural mass

Variation model	Damping ratio (ξ)	Stiffness (kg/cm)	Mass (kg.sec^2/cm)
1	2	4205,30	7,25
2	2	4205,30	14,5
3	2	4205,30	29

Based on Table 2 above, the next analysis was carried out with different variations of the fixed stiffness and mass of the structure, namely, $k = 4205.30$ kg/cm and the mass was respectively 7.25 kg.sec^2/cm, 14.5 kg.sec^2/cm, and 29 kg.sec^2/cm.

3. RESULTS AND DISCUSSION

Based on the results of the dynamic response analysis of the planned earthquake load using the 1/3 Simpson's Rule, the influence of mass variation, damping ratio, and stiffness on the displacement of this simple portal structure can be seen in the following description.

3.1. Displacement with Constant Mass-Stiffness Structure and Variation of Damping Ratio

This reference variation is provided with data from the mass and stiffness of the reference where, $m = 14.5$ kg.sec^2/cm and $k = 4205.30$ kg/cm. The dynamic response plot of the displacement due to dynamic loads and variations in mass and stiffness above, can be seen in **Figure 4** below.

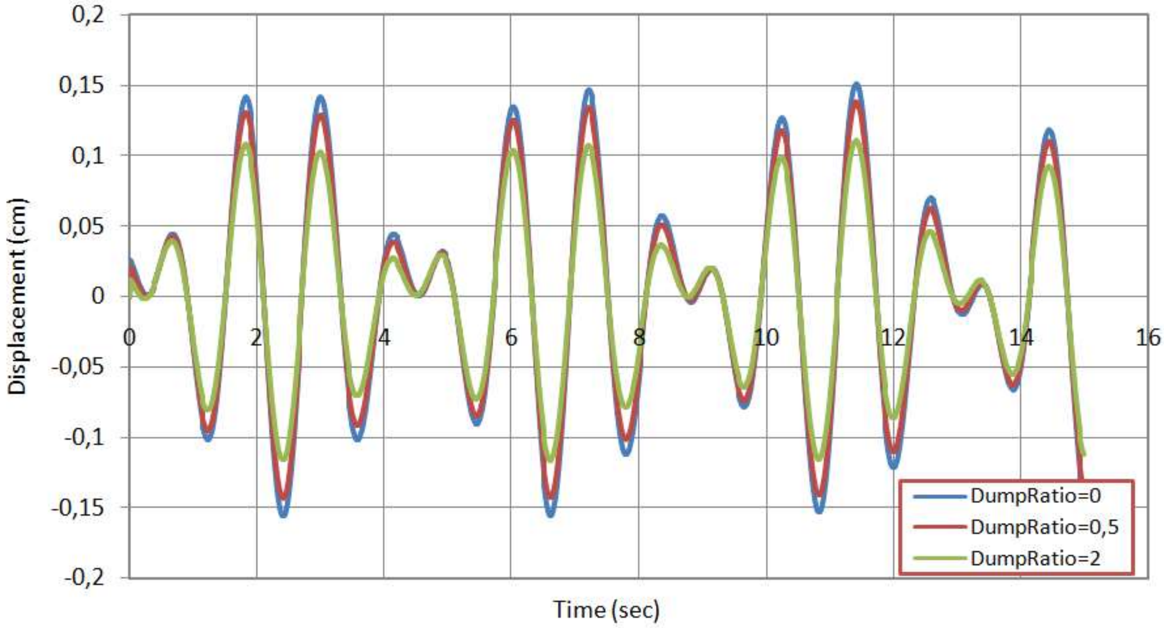

Figure 4. Displacement-time plot with constant mass-stiffness and variation of damping ratio

From **Figure 4** above, it shows that with a high damping ratio, the deviation is small. The maximum deviation, $y(t)_{max}$, for each damping ratio successively is, for $\xi = 0$ is $y(t)_{max} = 0.15$ cm, $\xi = 0.5$ obtained $y(t)_{max} = 0.14$ cm and $\xi = 2$ obtained $y(t)_{max} = 0.11$ cm. This explains that the damping ratio is inversely proportional to the displacement value.

3.2. Displacement with Fixed Damping Ratio and Variation of Changed Structure Mass

The analysis with this model uses a variation of data with a fixed damping ratio and a variable mass variation of the structure, as shown in Table 1, where the damping ratio used is $\xi = 2$, and the structural mass m, respectively, 7.25 kg.sec^2/cm, 14.5 kg.s^2/cm and 29 kg.sec^2/cm. The displacement caused by this condition is shown in **Figure 5** below.

From **Figure 5**, it shows that with a high structural mass, it results in a small displacement. The maximum displacement, $y(t)_{max}$, for each structural mass in a row is, for $m = 7.25$ kg.sec^2/cm is $y(t)_{max} = 0.222$ cm, $m = 14.5$ kg.sec^2/cm is obtained $y(t)_{max} = 0.055$ cm and $m = 29$ kg.sec^2/cm obtained $y(t)_{max} = 0.111$ cm.

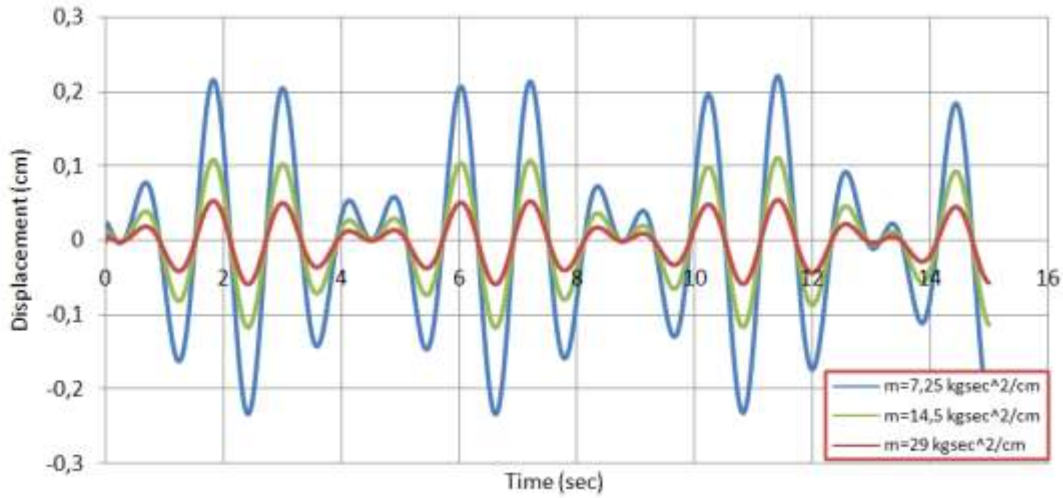

Figure 5. Displacement-time plot with a constant mass-stiffness and variation of damping ratio

3.3. Displacement with Constat Structure Mass-Damping Ratio and Variation of Stiffness Change

The analysis with this model uses data variations with a fixed mass-damping ratio and variable stiffness, k variations, such as **Table 1**, where the damping ratio used is ξ=2, the structural mass m = 14.5 kg.sec^2/cm, and the stiffness k is respectively 4205.30 kg/cm, 2012,14 kg/cm, and 8410.60 kg/cm. The magnitude of the displacement caused by this condition is as shown in **Figure 6** below.

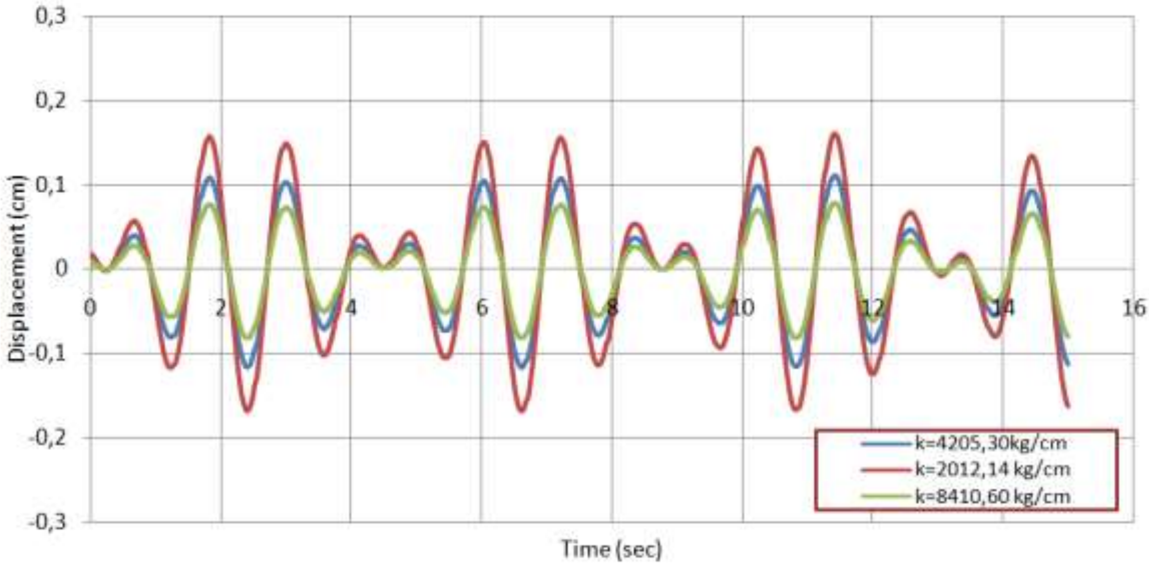

Figure 6. Displacement-time plot with a constant mass-damping ratio and structural stiffness variations

From **Figure 6** above, it shows that by increasing the stiffness of the structure, it can reduce the displacement. The maximum displacement, $y(t)_{max}$, for each structural stiffness in a row is, for k = 2012,14 kg/cm, then $y(t)_{max}$ = 0.16 cm, k = 4205.30 kg /cm obtained the maximum displacement, $y(t)_{max}$ = 0.11 cm and k = 8410.60 kg/cm obtained the maximum displacement $y(t)_{max}$ = 0.08 cm.

4. CONCLUSIONS

Based on the results of the analysis of the response of the simple portal structure of the SDOF system with the 1/3 Simpson's Rule with the dynamic earthquake loads that have been carried out above, it can be concluded that the relationship between the mass of the structure, damping and stiffness of the structure with the displacement of the structure, has an inverse relationship. This means that if the mass is increased, the structural displacement will decrease, and opposite. If the damping is increased, the displacement will decrease, and vice versa. If the stiffness of the structure is increased, the displacement will decrease, and vice versa. This can be seen in **Figures 4**, **Figures 5** and **Figures 6**.

REFERENCES

[1] R. Imani, U. D. Arman, and W. Boy, "Correlation between type of ground-based on b-value and the impact on buildings due to Sumatra earthquakes," *MATEC Web Conf.*, vol. 02013, no. 229, 2018.

[2] R. Imani, "Seismic Pattern Based on Analysis of Fractal Variations Before the 2009 Sumatra Earthquake with Zmap Software," *J. Islam. Sci. Technol.*, vol. 4, no. 1, 2019.

[3] Refrizon, I. H. Arif, L. Kurnia, and O. Tria, "Analisis Percepatan Getaran Tanah Maksimum dan Tingkat Kerentanan Seismik Daerah Ratu Agung Kota Bengkulu," *Pros. Semirata FMIPA Univ. Lampung*, pp. 323–328, 2013.

[4] A. P. R, M. S. Purwanto, and A. Widodo, "Identifikasi Percepatan Tanah Maksimum (PGA) dan Kerentanan Tanah Menggunakan Metode Mikrotremor Jalur Sesar Kendeng," *J. Geosaintek*, vol. 3, no. 2, p. 107, 2017.

[5] E. Purnomo, E. Purwanto, and A. Supriyadi, "Analisis Dinamik Respon Spektrum Menggunakan Software Etabs (Studi Kasus: Bangunan Hotel di Semarang)," *Matriks Tek. Sipil*, vol. 2, no. 4, pp. 569–576, 2014.

[6] B. Baehaki, S. Soelarso, and N. Fitria, "Analisis Perilaku Struktur pada Sistem Ganda Apartemen 9 Lantai Menggunakan Metode Time History Analysis Sesuai Peraturan Sni 1726:2012," *J. Fondasi*, vol. 7, no. 1, 2018.

[7] S. J. Akbar and Y. Candra, "Analisa Nilai Simpangan Horizontal (Drift) pada Struktur Tahan Gempa Menggunakan Sistem Rangka Bresing Eksentrik Type Braced V," *Teras J.*, vol. 7, no. 2, p. 301, 2018.

[8] R. Faizah and D. Widodo, "Analisis Gaya Gempa Rencana pada Struktur Bertingkat Banyak dengan Metode Dinamik Respon Spektra (189S)," *Strukt. Konf. Nas. Tek. Sipil Univ. Sebel. Maret -Surakarta*, vol. 7, no. 1, pp. 24–26, 2013.

[9] J. Mahto, S. Roy, J. Kuswaha, and R. Prasad, "Displacement Analysis of Cantilever Beam Using FEM Package," *Int. J. Mech. Eng. Technol.*, vol. 4, no. 3, pp. 75–78, 2013.

[10] G. A. Pratiwi and Widodo, "Analisis dan Desain Struktur Beton Bertingkat Banyak Berdasarkan Perbandingan Analisis Respons Spektrum dan Dinamik Riwayat Waktu," *J. Tek.*, vol. Volume XXI, no. 1, pp. 281–293, 2017.

[11] Republik Indonesia, *SNI-1726-2002 Standar Perencanaan Ketahanan Gempa untuk Struktur Bangunan Gedung*, vol. 7798393, no. April. 2002.

[12] Soelarso, Z. Darwis, and R. Sugara, "Analisa Simpangan pada Struktur Gedung 10 Lantai Menggunakan SNI 03-1726-2002 dan SNI 03-1726-2012," *J. Fondasi*, vol. 4, no. 1, pp. 120–126, 2015.

[13] BMKG, *Ulasan Guncangan Tanah Akibat Gempabumi Donggala 28 September 2018*, no.1, September 2018.

[14] C. D. Matani, H. Manalip, R. S. Windah, and S. O. Dapas, "Analisa Menara Air Akibat Gempa Menggunakan Solusi Numerik Integral Duhamel," *J. Sipil Statik*, vol. 1, no. 4, pp. 298–304, 2013.

[15] O. Dewayanti, E. J. Kumaat, S. O. Dapas, and R. S. Windah, "Perhitungan Simpangan Struktur Bangunan Bertingkat (Studi Komparasi Model Pembalokan Arah Radial Dan Grid)," *J. Sipil Statik*, vol. 1, no. 11, pp. 689–695, 2013.

[16] Mukmin, "Riwayat Waktu pada Gedung di MataramStudi Perbandingan Pembebanan Gempa Dinamik Respon Spektrum dan Dinamik," 2015.

[17] A. Fitriansyah and W. Pawirodikromo, "Pengaruh Soft-Story terhadap Simpangan Struktur akibat Variasi Kandungan Frekuensi Gempa," *J. Tek.*, vol. 4, no. 1, pp. 1–9, 2018.

[18] R. Suryanita and H. Sarfika, "Respons Struktur SDOF Akibat Beban Sinusoidal dengan Metode Integral Duhamel," *J. Tek. Sipil*, vol. 7, no. 3, pp. 266–278, 2007.

[19] R. Suryanita, Mudjiatko, and H. Sarfika, "Respon Struktur Sistem Derajat Kebebasan Tunggal Akibat Beban Dinamis dengan Pola Pembebanan Segitiga ," *J. Sains dan Teknol.*, vol. 5, no. 2, 2006.

[20] S. Apriwelni, A. Yudi, and T. Tilameo, "Displacement Analysis Due to Time History Load Case Study Building C and D Itera," in *IOP Conference Series: Earth and Environmental Science*, vol. 830, no. 1, 2021.

[21] Anonymous, "Sistem dengan Derajat Kebebasan Tunggal." 2019.

[22] M. Islam, "Perilaku Dinamik Plat Lantai Berbentang Panjang dengan dan Tanpa Pengaku Rangka Batang," *J. Tek. Inersia*, vol. 1, no. 2, pp. 10–19, 2010.

[23] N. M. R. R. C. Perbani and Rinaldy, "Penerapan Hitungan Volume Metode Simpson untuk Menghitung Volume Kapal dan Topografi Darat," *J. Rekayasa Hijau*, vol. 2, no. 1, pp. 90–100, 2018.

[24] M. J. Uddin, M. M. Moheuddin, and M. Kowsher, "A New Study of Trapezoidal, Simpson's 1/3 and Simpson's 3/8 Rule of Numerical Integral Problem," *Appl. Math. Sci. An Int. J.*, vol. 6, no. 4, pp. 1–13, 2019.

[25] F. Rumlawang and H. Batkunde, "Sifat-sifat Integral Riemann-Stieltjes," *Barekeng*, vol. 1, no. 2, pp. 25–30, 2007.