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THE APPLICATION OF GUMBEL COPULA TO ESTIMATE VALUE AT RISK WITH BACKTESTING IN TELECOMMUNICATION STOCK

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ABSTRACT

The Value at Risk (VaR) method refers to a statistical risk measurement tool used to determine the maximum loss of an investment, while the distribution that must be met is the normal distribution. This is not in line with the actual situation because the distribution of the return value is found to be not normally distributed but depends on market conditions that occurred at that time, thus invalidating the VaR estimate and resulting in greater portfolio risk. Therefore, in this study, the estimation of risk value will be carried out using the Gumbel Copula method, which can model the dependency structure between stocks and is flexible enough to model financial return data from https://finance.yahoo.com/. The parameter estimates produced by the Gumbel Copula method are then used to calculate the VaR at 90% and 99% confidence levels. The resulting VaR values are 0,076 and 0.231. To test the feasibility of the VaR model, backtesting was carried out, and it concluded that the VaR value obtained was valid and suitable for use in the risk assessment of PT. XL Axiata Tbk and PT. Telkomunikasi Indonesia Tbk.

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1. INTRODUCTION

Risk measurement is very important in investment and is often the center of attention for investors. The measurement results can be interpreted as a possibility of loss or destruction. Therefore, risk measurement needs to be done so that the risk is at a controlled level so as to reduce the occurrence of losses in investing [1]. One measuring tool that is growing rapidly and is very popular in use today is Value at Risk (VaR) which was popularized by J. P. Morgan in 1994. VaR is an estimate of the maximum loss that will be obtained over a certain period of time at a certain level of confidence [2]. VaR has become the standard model for measuring internal risk (Internal Model Approach) as set by the Bank for International Settlement since 1996 through the Basel Committee on Banking Supervision (BCBS) in BASEL II, which is used by financial analysts to calculate the market risk of an asset or portfolio [3].

Even though it has become a standard reference in risk management, so far, the VaR calculation is assumed to be normally distributed, whereas in reality, many financial data are not found to be normally distributed [4]. This may lead to inaccurate VaR estimations, increasing the actual portfolio risk over the calculated risk. Therefore, in this study, the calculation is not assumed to be normally distributed but adjusted to the distribution of the stock return value itself.

Parameter estimation for VaR calculation using the Gumbel Copula method. Gumbel Copula is one of the sub-Copulas of the Archimedean Family, where this family has a unique generator for each sub-Copula that is part of it. It is an asymmetric copula with a higher probability concentrated in the right tail [5]. It is especially well suited to random variables with positive correlation, as well as those whose high values exhibit stronger correlation than their low ones. The Gumbel Copula model reflects the upper tail dependence with high sensitivity [6]. Therefore, in this study, Copula from the Archimedean Family will be used, namely Gumbel Copula.

VaR as an estimate and every estimation calculation certainly have problems with changes in the value of an asset that does not have a fixed pattern, the VaR calculation accuracy test results are required. To check whether the results obtained from VaR calculations are consistent and reliable, it must be verified [7]. The method used in validating risk models is known as the backtesting method [8]. Backtesting is the application of quantitative methods to determine whether the estimated risk of a model is consistent with the assumptions underlying the model being tested [7].

Based on this description, the author examine the risk value backtesting on the estimated Gumbel Copula using a stock portfolio closing price of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk period 7 January 2019 – 3 January 2020.

2. RESEARCH METHODS

2.1 Data Exploration

Calculating the return from each closing price of investment shares that will be used as observational data by using Equation (1) [5]

$$return = \frac{[P_t - P_{t-1}]}{P_{t-1}} \tag{1}$$

where

P_t : Close price of shares on day t,
P_{t-1}: Close price of shares on day $t-1$

2.2 Data Identification

The data used will be tested first with several tests, including a normality test, heteroscedasticity test, and autocorrelation test. The stages are as follows:

a. By examining the histogram of each variable and running the Kolmogorov-Smirnov test, one can identify the distribution of the data and whether it is normally distributed or not. The interpretation of the Kolmogorov-Smirnov test result is that if the value is above 0,05 then the data distribution is declared to meet the assumption of normality, and if the value is below 0,05 then it is interpreted as abnormal; [9]

b. Then examine the nature of the autocorrelation to see whether or not there is a correlation effect on the data by using the Ljung-Box test and seeing the ACF plot formed from the observational data;

c. Perform a heteroscedasticity test to find out whether the data has very diverse variances so that it can result in an unstable residue by looking at the data squared ACF plot and the ARCM LM test.

2.3 Copula

Copula is a common distribution function of several marginal distribution functions [10]. One of the most popular copula families used is the Archimedean Copula [11]. Archimedean Copula is divided into several parts, including Clayton, Gumbel, Frank [12]. Finding the value of the correlation coefficient using the Kendall's Tau Correlation [13] for copula Archimedean in Equation (2)

$$\tau = 1 + 4\int_0^1 \frac{\varphi(t)}{\varphi'(t)}dt \tag{2}$$

where $\varphi(t)$ is the generating function of the Archimedean Family Copula

2.4 Gumbel Copula

In this study, Gumbel copula will be used. Gumbel Copula has a tail link at the top [5]. This shows that when the independent variable only has a relationship with the dependent variable when the independent variable is very high, while when the independent variable is low, the correlation between the two is also low, and sometimes there is no relationship. The correlation coefficient value that has been obtained in the previous step is then used to calculate the estimated value of the Gumbel Copula parameter.

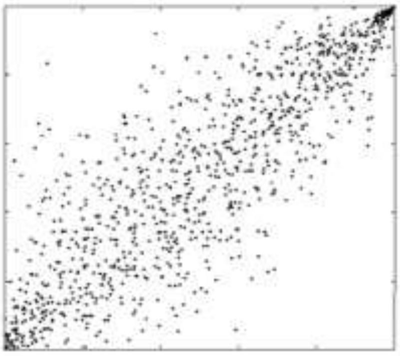

Figure 1. Simulated data with Gumbel Copula [14]

2.5 Value at Risk

VaR is a measuring tool that can calculate the magnitude of the worst losses that may occur by knowing the position of assets, the level of confidence in the occurrence of risk, and the period of asset placement (time horizon) [15]. Risk value or commonly referred to as VaR is a concept used for risk measurement in risk management [16]. In simple terms, VaR wants to answer the question "how much (in percent or a certain amount of money) investors can lose during investment time t with an error rate of α [17]. From these questions, it can be seen that there are three important variables, namely the amount of loss, the time-lapse, and the error rate. The stages are as follows:

a. Simulating the return value by generating random return data with the Gumbel Copula function based on the estimated parameter values obtained as many as n data.

b. Finding the maximum loss at the confidence level $(1-\alpha)$ which is the quantile value of the return obtained in step (a) then denoted by R^*.

c. Calculating VaR with a certain level of confidence using **Equation (3)**

$$VaR_{(1-\alpha)} = W_0 R^* \quad (3)$$

where

$VaR_{(1-\alpha)}$: VaR with confidence level $(1-\alpha)$

W_0 : Initial investment

R^* : The worst possible value of return

d. Repeat steps (a) to step (c) m times so as to reflect the various possible VaR as $VaR_1, VaR_2, \ldots, VaR_m$

e. Calculate the average of the results from step (d) to stabilize the VaR because the VaR obtained from each simulation is different.

2.6 Backtesting

The main problem in building a risk model is to validate the model. When a model is formed, it is important to validate it beforehand. Backtesting is a statistical procedure in which actual returns are systematically compared to corresponding VaR estimates **[18]**. The method used in validating risk models is known as the backtesting method **[19]**. The resulting model is compared with the actual results in a given time. The stages are as follows:

a. Determine the failure rate value which is denoted by 1, while the return value which does not exceed VaR is denoted by 0.

b. Perform data backtesting with a certain level of confidence using **Equation (4)**.

$$LR_{uc} = -2ln[(1-p)^{T-N}p^N] + 2ln\left\{\left[1-\left(\frac{N}{T}\right)\right]^{T-N}\left(\frac{N}{T}\right)^N\right\} \quad (4)$$

where

T : Number of observation data

N : Number of failure rates

ρ : Probability (1-level of confidence)

The value of LR_{uc} is compared with the critical value of the Chi-Square distribution at a certain degree of freedom, using a certain level of confidence. For example, if at the 95% confidence level the value of $LR_{uc} <$ 3,841 then the calculation of Value at Risk (VaR) is accepted (valid). On the other side, if the value of $LR_{UC} >$ 3,841 then the Value at Risk (VaR) is rejected (invalid).

3. RESULTS AND DISCUSSION

The data used in this study is secondary data in the form of closing price data from daily stock investments in Rupiah. Data obtained from http://finance.yahoo.com, namely historical data of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk period 7 January 2019 – 3 January 2020. The program used to support this research process is the Rstudio version 3.6.3 software with 255 data.

The first step in the data processing process is to perform a normality test. Normality tests need to be carried out to anticipate the occurrence of price instability which is feared to experience a price decline so that it can harm investors. In this study, the normality test used was the Kolmogorov-Smirnov test. The

Kolmogorov-Smirnov test is one of the tests that can be used in determining a suitable distribution for a data [14]. This test was conducted to determine the distribution of the stock return variable of PT. Telkomunications Indonesia tbk and PT. XL Axiata tbk with the following hypothesis:

H_0: return is normally distributed

H_1: return Not Normal Distribution

Table 1. Normal Distribution Test

Stock Return	D_c	*p-value*	α
PT. XL Axiata tbk	0.15294	0.0000139	0.05
PT. Telkomunikasi Indonsia tbk	0.12941	0.0003905	0.05

The results in Table 1 show the $D_c$values of the two stock returns, namely 0.12941 and 0.15294, which are greater than the value of Kolmogorov-Smirnov's Table which is worth 0.08517. Likewise, when compared with the p-value, the value $\alpha = 0{,}05$ is greater than the p-value (0.0003905 and 0.0000139). So reject H_0which means that the stock return variable of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk. not normal distribution.

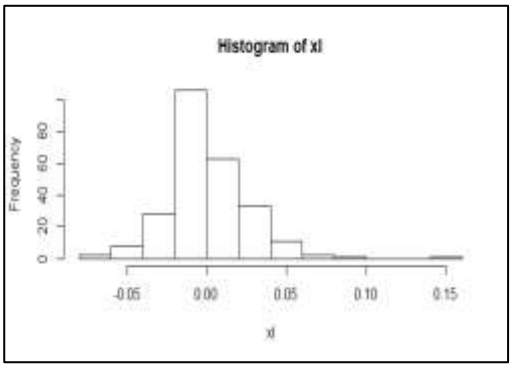

Figure 2. Histogram Returns PT. XL Axiata tbk

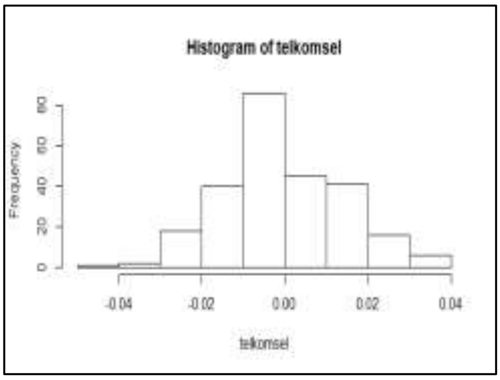

Figure 3. Histogram Returns PT. Telkomunikasi Indonesia tbk

After conducting the normality test, the next step is to perform an autocorrelation test. The autocorrelation test is a test that describes the relationship that occurs between the residuals of one observation with another observation. Strong autocorrelation can cause two seemingly unrelated variables to be related, so to detect the presence or absence of autocorrelation, in this study, the Ljung Box test was used with the following hypothesis:

H_0: return no autocorrelation.

H_1: return has autocorrelation

Table 2. Ljung Box *Return* PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk

Stock Return	**Ljung Box**		**α**
	Lag	***p-value***	
PT. XL Axiata tbk	1	0.1232	
	5	0.1022	
	10	0.1195	0.05
	15	0.1535	
	20	0.2142	
	1	0.1275	
	5	0.4078	
PT. Telkomunikasi Indonsia tbk	10	0.3658	0.05
	15	0.4883	
	20	0.1275	

From **Table 2**, it can be seen that each p-value is greater than $\alpha = 0.05$, then it can be concluded that accepting H_0 which means that the stock return variable is not autocorrelated.

The next test is the heteroscedasticity test. The heteroscedasticity test is used to determine whether there is a variance inequality between the errors of one observation and another. The heteroscedasticity test used in this study is the Lagrange Multiplayer (LM) test or commonly referred to as ARCH LM, which is a test for the presence of heteroscedasticity elements with the following hypothesis:

H_0: there is no ARCH/GARCH effect on data return

H_1: there is an ARCH/GARCH effect on the data return.

Where H_0 rejected when $p - value < \alpha$ with $\alpha = 0.05$ which means there is a heteroscedasticity effect on the return data PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk.

Table 3. ARCH LM *Return* PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk.

Stock Return	***p-value***	**α**
PT. XL Axiata	0.3153	0.05
PT. Telkomunikasi Indonsia tbk	0.9008	0.05

The results of the ARCH LM test can be seen in **Table 3**. for each $p - value$ of each stock return is greater than $\alpha = 0.05$. This shows that accept H_0 which means that the stock return variable of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk has no heteroscedasticity effect. This shows that there is no heteroscedasticity effect on the tested return data.

3.1. Gumbel Copula

The parameter used is parameter $\hat{\theta}$ obtained using the Kendall's tau correlation method. The first thing to do in estimating the parameters is to find the correlation coefficient or the value of τ for the return variable of daily stock investment closing price of PT. XL Axiata tbk and the return variable of daily stock investment closing price of PT. Telkomunikasi Indonesia tbk obtained the value of the correlation coefficient, is:

$$\tau = 0.2112325$$

This correlation coefficient value is used to estimate $\hat{\theta}$ parameter through equation as follows:

$$\hat{\theta} = \frac{1}{1-\tau} = \frac{1}{1 - 0.2112325} = 1.2678.$$

Then a simulation will be carried out. This simulation is carried out by generating random numbers using the Gumbel Copula model using $\hat{\theta}$ parameter as follows:

$$C(u_1, u_2) = \exp\left(-[(-\ln u_1)^{1.2678} + (-\ln u_2)^{1.2678}]^{\frac{1}{1.2678}}\right)$$

$$C(u_1, u_2) = \exp(-[(-\ln u_1)^{1.2678} + (-\ln u_2)^{1.2678}]^{0.788767})$$

The relationship pattern that follows the Gumbel Copula illustrates that there are extreme events at high values. The relationship that occurs between the two variables when the value of both is high, the lower the value of the observations on the variable, the weaker the relationship between the two. This is because Gumbel Copula has a close relationship if there is a pile at the top. This relationship pattern indicates that when there is an increase in stock returns, that is, when PT. Telkomunikasi Indonesia tbk stock returns go up then the stock returns of PT. XL Axiata tbk also go up.

3.2. Value at Risk

Calculation of Risk Value or often referred to as VaR is carried out with a confidence level of 90% and 99%. VaR is always different in each simulation. This is caused by differences in the random data generated. However, basically, it gives results that are not much different from one another because returns are generated with the same parameters. One way to reduce this problem is to run many simulations and then take the average value. VaR at several levels of confidence can be seen in Table 4.

Table 4. VaR Estimation of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk

Level of confidence $(1-\alpha)$	α	VaR
90%	0.05	3.28%
99%	0.01	6.47%

The estimation results in Table 4. state that the VaR for PT. XL Axiata tbk. and PT. Telkomunikasi Indonesia tbk at 3.28% for a 90% confidence level indicating there is a 10% probability that the loss will be lower than the expected VaR, for a 99% confidence level the risk value is 6.47% indicating there is a 1% probability that the loss will be lower than the VaR suspected.

3.3. Backtesting

The results of the backtesting test can be seen in Table 5. It is known that for each LR of each level of confidence is smaller than the critical value with the hypothesis below:

H_0 : VaR is accurate

H_1 : VaR is not accurate

This shows that the risk value obtained from the calculation of VaR using a Monte Carlo simulation on the stock data of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk valid.

Table 5. Test Backtesting VaR

Level of confidence $(1-\alpha)$	N	α	T	LR_{uc}	CV
90%	255	0.05	21	0.93	2.71
99%	255	0.01	1	1.23	6.64

The results of backtesting with the loglikelihood ratio approach on the stock portfolio of PT. XL Axiata tbk and PT. Telkomunikasi Indonesia tbk stated that the Value at Risk for the 95% and 99% confidence levels for each value $LR_{uc} > \chi^2{}_{(1;\alpha)}$ so rejecting H_0. This indicates that the VaR obtained is accurate.

4. CONCLUSIONS

Based on the VaR calculation for the initial stock investment of Rp. 1,000,000 at a 90% confidence level of Rp. 32,800 and at a 99% confidence level of Rp. 64,700. The greater the level of trust taken, the greater the risk that must be borne and the allocation of capital used to cover these losses. The backtesting test performed shows that the resulting VaR value is accurate to be used as an estimate of the risk value in the analyzed stock investment data. This shows that the VaR estimation carried out with the Gumbel Copula parameter estimator can be used as a model reference if the investment return data is not normally distributed, but by paying attention to the stock return pattern of each data used.

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