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ANALYSIS OF THE MAGNETOHYDRODYNAMICS NANOVISCOUS FLUID BASED ON VOLUME FRACTION AND THERMOPHYSICAL PROPERTIES

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ABSTRACT

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Fluid flow control is applied in engineering and industry using computational fluid dynamics. Based on density, fluids are divided into two parts, namely non-viscous fluids and viscous fluids. Nanofluid is a fluid that has non-viscous and viscous characteristics. Nanoviscos fluid flow is interesting to study by considering the effect of volume fraction and thermophysical properties. Nanoviscous fluid flow models form dimensional equations that are then simplified into dimensionless equations. Dimensionless equations are converted into non-similar equations using flow functions and non-similar variables. Nanoviscous fluids with Cu particles and water-based fluids have higher temperatures and faster velocity. Based on the effect of volume fraction, the velocity of the nanoviscous fluid moves slower, while the temperature of the nanoviscous fluid increases.

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1. INTRODUCTION

Mathematical models can be used to create representations and constructs of real phenomena. Solution Mathematical modeling is done analytically and numerically. Fluid flow phenomena can be modeled and solved numerically using Computational-Fluid-Dynamic (CFD). The application of CFD is applied to the engineering and industrial fields in the form of cooling systems [1]. CFD can control the velocity and temperature of the fluid flow.

A fluid is a liquid or gas that changes state when pressure is applied [2]. Based on density, fluids are divided into two parts, namely non-viscous or non-viscous fluids and viscous or viscous fluids. Non-viscous fluids have high deformation rates due to the absence of inter-particle friction. On the other hand, in a viscous fluid, the deformation rate is hampered due to friction between particles. Nanofluid is a fluid that has non-viscous and viscous characteristics. This is because the nanofluid has two constituents, namely, the base fluid and nanoparticles. If the percentage of nanoparticles is greater than the base fluid, the nanofluid has a viscous character or a nanoviscous fluid. If the percentage of nanoparticles is smaller than the base fluid, the nanofluid has a non-viscous character or a nano non-viscous fluid. The percentage between the base fluid and the nanoparticles is expressed using the volume fraction. Research on nanofluids was first conducted by Choi et al in 1998, showing that nanofluids have a fairly high thermal conductivity [3]. Therefore, nanofluids that can conduct electric currents are called magnetic hydrodynamics [4]. The effect of magnetic hydrodynamics on nanofluid flow can increase the velocity and decrease the temperature of the fluid flow [5]. The results of the study conducted by Mahian et al showed that heat transfer could increase when given the addition of nanoparticles [6]. Fluid flow with metal nanoparticles has greater dynamics than metal oxide nanoparticles [7]. The effect of the Eckert number on nanofluid flow causes the fluid temperature to increase [8]. The effect of convection on fluid flow causes the fluid velocity to increase and the flow temperature decreases [9]. The effect of Al_2O_3 nanoparticles on fluid-particle friction capable of increasing by 67% has been carried out by Wei et al [10]. Further research on the effect of Cu-Ni nanoparticles on nanofluids can increase thermal conductivity [11].

Based on previous studies, there has not been much development on nanofluids with viscous characters. Therefore, nanoviscous fluids are interesting to be developed. In this study, the magnetohydrodynamics of nanoviscous fluids will be investigated based on volume fraction and thermophysical properties of nanoparticles concerning flow velocity and temperature. The volume fraction and thermophysical properties are considered variance parameters. The nanoparticles used were $Cu, CuO, Si, Mg, SiO_2, MgO$ nanoparticles with the base fluid being water and oil. The nanoviscous fluid equation that is formed is simplified into a dimensionless equation and a similarity equation. The numerical solution to the similarity equation uses the Euler finite difference method.

2. RESEARCH METHODS

This study uses a research method consisting of model development, model simplification, model completion, analysis, and discussion of the nanoviscous fluid mathematical model as follows.

2.1 Developing Mathematical Model for Nanoviscous Fluid

The nanoviscous fluid model is obtained from the applicable physical laws. The laws of physics used are related to the thermophysical properties and volume fraction of nanoparticles. The thermophysical property is in the form of density, so the model is associated with mass conservation law to form a continuity equation. The percentage of nanoparticles and base fluid is expressed by volume fraction. The volume fraction is related to the friction caused by nanoparticles and the basic fluid, related to Newton's 2nd law to form the momentum equation. The heat generated by the friction between the nanoparticles and the basic fluid forms an energy equation based on the Thermodynamics law. Therefore, developing a nanoviscous fluid flow model includes three parts: continuity, momentum, and energy equations.

2.2 Simplification of The Mathematical Model of Nanoviscous Fluid

The continuity, momentum, and energy equations that are built will then be simplified into dimensionless equations. The simplification of the equation uses a dimensionless variable as follows [12]. x, y, u, and v axes and gravity on the $-x$ and $-y$

$$x=\frac{\bar{x}}{a};\ u=\frac{\bar{u}}{U_\infty};y=Re^{1/2}\frac{\bar{y}}{a};\ v=Re^{1/2}\frac{\bar{v}}{U_\infty};g_{\bar{x}}=-g\sin\left(\frac{x}{a}\right);g_{\bar{y}}=g\cos\left(\frac{x}{a}\right)$$

time t, pressure p, temperature T [13]

$$t=\frac{U_\infty\bar{t}}{a};\ p=\frac{\bar{p}}{\rho_{fn}U_\infty{}^2};T=\frac{\bar{T}-T_\infty}{T_w-T_\infty} \tag{1}$$

The next step is to convert the dimensionless equation into a non-similar equation. Non-similar equations are used to simplify dimensionless equations into one variable by substituting stream functions and non-similar variables as follows [14].

Stream functions

$$u=\frac{\partial\psi}{\partial y};\ v=-\frac{\partial\psi}{\partial x}$$

Non-similar variable

$$\Psi=t^{\frac{1}{2}}u_e(x)f(x,\eta,t);\eta=\frac{y}{t^{\frac{1}{2}}};T=s(x,\eta,t) \tag{2}$$

2.3 Completion of The Mathematical Model of Nanoviscous Fluid

The nanoviscous fluid model was solved using Euler's finite difference numerical method with the following steps.

1. Discretization of non-similar equations using a central finite difference.
2. The discretized result is then converted into a tridiagonal matrix.
3. Determine the coefficient of the tridiagonal matrix formed.
4. Determine the solution at $t+\Delta t$.

3. RESULTS AND DISCUSSION

Mathematical modeling is an applied science that uses mathematics to represent real phenomena and problems. The flow modeling is obtained by forming the building equations by applying the law of conservation of mass, Newton's second law, and the first law of Thermodynamics. The builder equations of the nanoviscous fluid flow model with magneto-hydro-dynamic (MHD) influence the build of the dimensional equations in the form of continuity, momentum, and energy. This study uses a nanoviscous fluid that passes through a circular cylindrical surface. The flow of nanoviscous fluids is based on volume fraction and thermal properties.

3.1. Mathematical Modelling

The nanoviscous fluid model is obtained from the applicable physical laws as follows. Continuity equation based on mass conservation law

$$\frac{DMass}{Dt}=0$$

Thus, the continuity equation for the flow of nanoviscous fluids is stated as follows.

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0 \qquad (3)$$

Momentum equation based on Newton's 2nd law

$$\rho_{fnv}\left(\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}.\ \nabla\boldsymbol{V}\right)=F_P-F_M+F_A$$

Thus, the momentum equation for the flow of nanoviscous fluids is stated as follows.

Momentum equation on x axes: $\rho_{fnv}\left(\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right)=-\nabla p+\mu_{fnv}\nabla^2\boldsymbol{V}+\sigma(B_0)^2\bar{u}+(\rho_{fnv}-\rho_\infty)g_{\bar{x}}$

Momentum equation on y axes: $\rho_{fnv}\left(\frac{\partial\bar{v}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right)=-\nabla p+\mu_{fnv}\nabla^2\boldsymbol{V}+\sigma(B_0)^2\bar{v}+(\rho_{fnv}-\rho_\infty)g_{\bar{y}}$ (4)

Energy equation based on Thermodynamics law

$$\rho_{fnv}\left(\frac{\partial e}{\partial t}+\nabla.(e\boldsymbol{V})\right)=Q$$

Thus, the energy equation for the flow of nanoviscous fluids is stated as follows.

$$\frac{\partial\bar{T}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}=\alpha_{fnv}\left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right) \qquad (5)$$

The builder equation was obtained, then simplified first using dimensionless variables and dimensionless parameters as follows.

Reynold number and Granshof number [15]

$$Gr=\frac{g\beta(T_w-T_\infty)a^3}{\upsilon^3} \text{ and } Re=\frac{U_\infty a}{\upsilon} \qquad (6)$$

by substituting Equation (2) and Equation (6), we get

The continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

The momentum equation

Momentum equation on x axes: $\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{v_{fnv}}{Re\, v_{fd}}\frac{\partial^2 u}{\partial x^2}+\frac{v_{fnv}}{v_{fd}}\frac{\partial^2 u}{\partial y^2}+Mu+\alpha_{fnv}T\sin x$

Momentum equation on y axes: $\frac{1}{Re}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{v_{fnv}}{v_{fd}}\frac{1}{Re^2}\frac{\partial^2 v}{\partial x^2}+\frac{v_{fnv}}{v_{fd}}\frac{1}{Re}\frac{\partial^2 v}{\partial y^2}+\frac{M}{Re}v-\frac{\alpha_{fnv}}{Re^{\frac{1}{2}}}T\cos x$

The energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\alpha_{fnv}}{\alpha_{fd}}\frac{\partial^2 T}{\partial y^2} \qquad (7)$$

The obtained equations were then simplified using the flow function and non-similar variables and the thermophysical parameters of nanofluid as follows.

Prandtl number [16]

$$Pr=\frac{v_{fnv}}{\alpha_{fnv}}$$

Density nanoviscous fluid [6]

$$\rho_{fnv}=(1-\chi)\rho_{fd}+\chi\rho_s$$

Viscosity nanoviscous fluid [6]

$$\mu_{fnv}=\mu_{fd}\frac{1}{(1-\chi)^{2.5}}$$

Specific heat of nannoviscous fluid [17]

$$(\rho C_p)_{fnv}=(1-\chi)(\rho C_p)_{fd}+\chi(\rho C_p)_s$$

Thermal conductivity [6]

$$k_{fnv}=\frac{k_s+2k_{fd}-2\chi(k_s-k_{fd})}{k_s+2k_{fd}+\chi(k_s-k_{fd})}k_{fd} \tag{8}$$

Using **Equation (2)** and **Equation (8)**, we obtained the following non-similar equation.

The momentum equation

$$\left[\frac{1}{(1-\chi)^{2.5}\left[(1-\chi)+\left(\frac{\rho_s}{\rho_{fd}}\right)\right]}\right]\frac{\partial^3 f}{\partial\eta^3}+\frac{\eta\partial^2 f}{2\partial\eta^2}+\frac{3}{2}t\cos x\left[1-\left(\frac{\partial f}{\partial\eta}\right)^2+2f\frac{\partial^2 f}{\partial\eta^2})\right]=t\frac{\partial^2 f}{\partial\eta\partial t}+\frac{3}{2}\ t\sin x\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial x\partial\eta}-\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial\eta^2}\right)+Mt\left(1-\frac{\partial f}{\partial\eta}\right)-\frac{2}{3}\alpha_{fnv}st$$

The energy equation

$$\left(\frac{k_s+2k_{fd}-2\chi(k_s-k_{fd})}{k_s+2k_{fd}+\chi(k_s-k_{fd})}\frac{1}{(1-\chi)+\chi\left(\frac{(\rho C_p)_s}{(\rho C_p)_{fd}}\right)}\right)\frac{\partial^2 s}{\partial\eta^2}+\Pr\frac{\eta}{2}\frac{\partial s}{\partial\eta}+3\cos x\Pr t f\frac{\partial s}{\partial\eta}=\Pr t\frac{\partial s}{\partial\eta}+\Pr t\frac{3}{2}\sin x\left(\frac{\partial f}{\partial\eta}\frac{\partial s}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial s}{\partial\eta}\right) \tag{9}$$

based on the following boundary conditions:

$t=0: f=\frac{\partial f}{\partial\eta}=s=0$ for every x,η

$t>0: f=\frac{\partial f}{\partial\eta}=0,\ s=1$ when $\eta=0$

$\frac{\partial f}{\partial y}=1,\ s=0$ when $\eta\rightarrow\infty$

Detailed Description

M : Magnetic parameter
F_P : The surface force
F_M : The magnetic force
F_A : The buoyant force
Q : Heat
B_0 : Magnetic field
ρ_{fnv} : Density of nanoviscous fluid
μ_{fnv} : Dynamic-viscosities of nanoviscous fluid
k_{fnv} : Thermal conductivity of nanoviscous fluid
C_p : Specific heat
T : Temperature
Gr : Granshof number
Re : Reynold number
Pr : Prandtl number

3.2. Analysis and Discussion

Analysis of the magnetohydrodynamic nanoviscous fluid was completed using the Euler finite difference method. MATLAB software is used to solve numerical results. Parameters related to the flow characteristics of the MHD nanoviscous fluid are thermophysical parameters and volume fraction parameters. Thermophysics on basic fluids and nanoparticles density (ρ), specific heat (c_p), and thermal conductivity (k). Analysis of flow characteristics was carried out on the magnetohydrodynamic temperature and velocity of the nanoviscous fluid. Based on the thermophysical property of the metal nanoparticles, the metal oxide particles, and the base fluid, the following data is provided.

Table 1. Thermophysical Property of Base Fluid

Property	Base Fluid	
	Water	Oil
$\rho(\frac{kg}{m^3})$	999.1	1670
$C_p(\frac{J}{kg.K})$	4179	920
$k(\frac{W}{m.K})$	0.613	0.138

Data source:[9],[18]

Table 2. Thermophysical Property of Nanoparticles

Property	Nanoparticles							
	Si	Mg	Cu	Fe	SiO_2	MgO	CuO	Fe_3O_4
$\rho(\frac{kg}{m^3})$	2329	1738	8933	7874	2220	3580	6000	5180
$C_p(\frac{J}{kg.K})$	700	1020	397	440	745	877	551	670
$k(\frac{W}{m.K})$	130	156	393	80.2	1.4	42	33	6.9

Data source: [18],[19], and [20]

The velocity and temperature with variations nanoparticles of magnetohydrodynamic nanoviscous fluid are shown in **Figure 1** and **Figure 2**. Metal particles are namely Cu, Si, Mg, Fe and metal oxide nanoparticles are CuO, SiO_2, MgO, Fe_3O_4. In the variation of nanoparticles, the base fluid used is water with the Prandtl number Pr=5. Figure 1 shows that the nano-viscous fluid of Cu metal nanoparticles moves faster than other nanoparticles. Cu metal nanoparticles have a higher density (ρ=8933) than metal nanoparticles and other metal oxide particles. As a result, the velocity of the nano-viscous fluid of Cu metal particles moves faster.

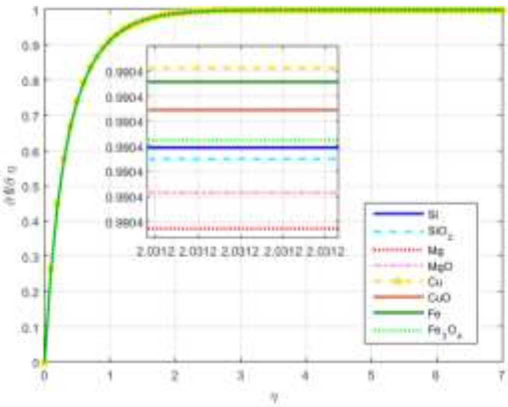

Figure 1. The Velocity with Nanoparticles Variations of Magnetohydrodynamic Nanoviscous Fluid

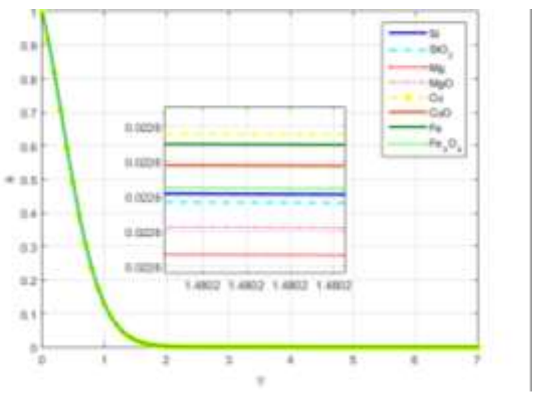

Figure 2. The Velocity with Nanoparticles Variations of Magnetohydrodynamic Nanoviscous Fluid

Figure 2 shows the temperature of the nanoviscous fluid with variations in metal particles and variations in non-metallic particles. It can be seen in the figure that the temperature of Cu metal nanoparticles is higher than other nanoparticles. This is because Cu, metal nanoparticles have a higher thermal conductivity (k=393) than other nanoparticles.

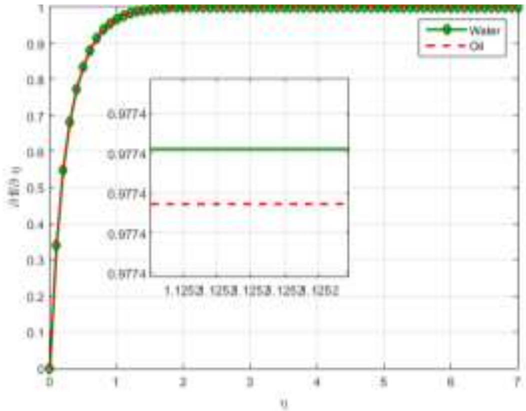

Figure 3. The Velocity with Base Fluid Variations of Magnetohydrodynamic Nanoviscous Fluid

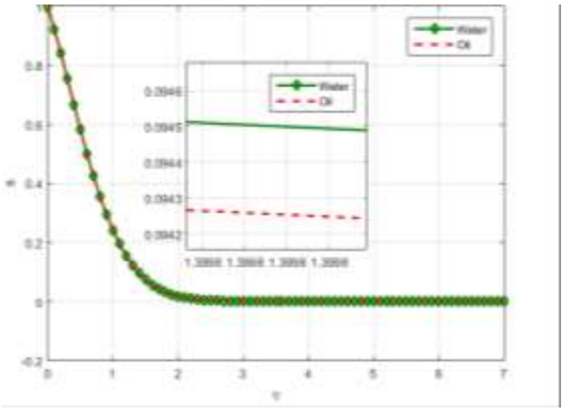

Figure 4. The Velocity with Base Fluid Variations of Magnetohydrodynamic Nanoviscous Fluid

The velocity and temperature with variations in base fluids of magnetohydrodynamic nanoviscous fluid are shown in **Figure 3** and **Figure 4**. The base fluids used are water base fluid and oil base fluid. In the variation of base fluid, the nanoparticle used is Cu. **Figure 3** and **Figure 4** show that the velocity and temperature of the water base fluid are higher than that of the oil base fluid. The density of the water base fluid is smaller than that of the oil base fluid. The large density of the base fluid causes the fluid to move slower because of its slow velocity. The smaller the density of the base fluid, the higher the fluid velocity. The temperature of the nanoviscous fluid in the water base fluid is higher than in the oil base fluid. This is due to the specific heat and thermal conductivity of the water base fluid which is higher than that of the oil base fluid. The specific heat and thermal conductivity of the water base fluid are 4179 and 0.613, respectively. Water base fluid has 920 specific heat and 0.138 thermal conductivity. Therefore, the temperature of the water base fluid is higher than the oil base fluid.

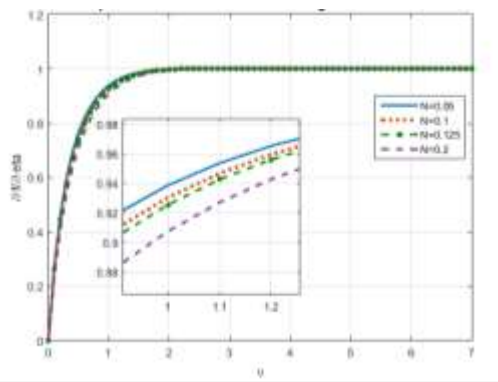

Figure 5. The Velocity with Volume Fraction of Si-Water Nanoviscous Fluid

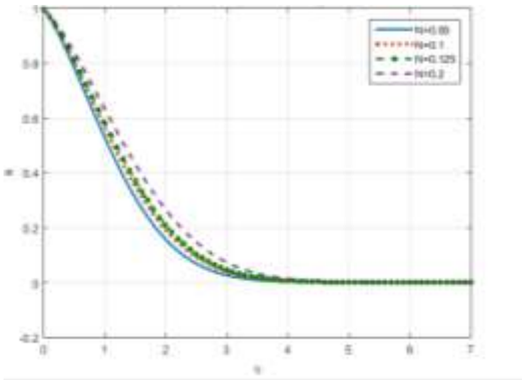

Figure 6. The Velocity with Base Fluid Variations of Si-Water Nanoviscous Fluid

The velocity and temperature of nanoviscous fluid with Si nanoparticles (ρ=2329) and water-base fluid (ρ=997.1) are shown in **Figure 5** and **Figure 6**. Si nanoparticles were observed based on volume fraction variations because Si nanoparticles had an average velocity and temperature based on the results of **Figure 1** and **Figure 2**. In the variation of volume fraction parameters, the base fluid used is water. The velocity and temperature of the Si-Water nano-viscous fluid using the Prandtl number Pr=5 and volume fraction varies by N=0.05,0.1,0.125, and 0.2. In **Figures 5** and **Figure 6**, the velocity of the Si-Water nanoviscous fluid is varied by varying its volume fraction. It can be seen that as the volume fraction increases, the velocity of Si-Water nanoviscous becomes slower. As the volume fraction increases, the nanoparticles increase as well. As the nanoparticles in the fluid increase, the movement of the nano-viscous fluid slows down. The increase in volume fraction also affects the fluid temperature. The more nanoparticles increase, the greater the friction between the fluids. Thus, the nanoviscous fluid's temperature increases due to heat generation.

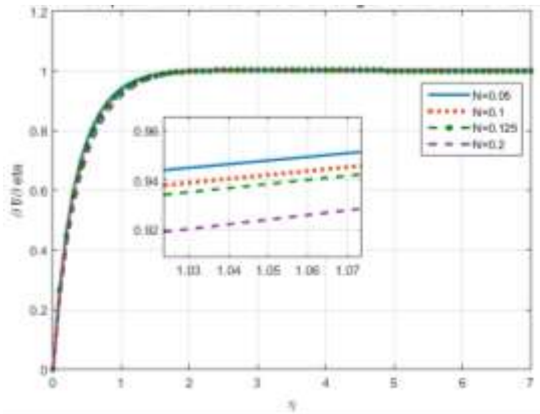

Figure 7. The Velocity with Volume Fraction of Si-Water Nanoviscous Fluid

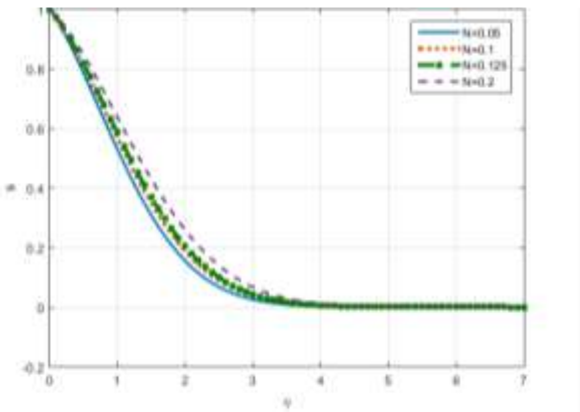

Figure 8. The Velocity with Base Fluid Variations of Si-Water Nanoviscous Fluid

The velocity and temperatur of the nano-viscous fluid with Si nanoparticles and the oil base fluid are shown in **Figure 7** and **Figure 8**. The velocity and temperature of the Si-Oil nanoviscous fluid using the Prandtl number Pr=10. The input volume fraction varies by N=0.05,0.1,0.125, and 0.2. The velocity and temperature of the Si-Oil nanoviscous fluid is carried out by increasing the variance of the parameters in the volume fraction. The nanoparticles used are Si with oil as base fluid. As the volume fraction parameter increases, the velocity becomes slower and the temperature increases. With increasing volume fraction, more nanoparticles are contained in the nanoviscous fluid. More nanoparticles in the fluid, the movement of the nano-viscous fluid will slow down. The friction between the fluids then increases, increasing the temperature of the nanoviscous fluid.

4. CONCLUSIONS

The results showed that the larger the volume fraction, the lower the velocity of the nanoviscous fluid and the higher the temperature of the nanoviscous fluid. The greater the convection parameter, the faster the velocity and temperature of the nanoviscous fluid will increase. When compared to other nanoparticles, Cu nanoparticles have the fastest speed and temperature compared to other nanoparticles. This is due to Cu nanoparticles (ρ=8933) having a density compared to other nanoparticles. Meanwhile, the water base fluid has a greater velocity and temperature than the oil base fluid. This is because the water base fluid has a high specific heat (C_p = 4179) and high thermal conductivity (k = 0.613).

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REFERENCES

[1] A. S. Jenifer, P. Saikrishnan, and R. W. Lewis, “Unsteady MHD Mixed Convection Flow of Water over a Sphere with Mass Transfer,” *J. Appl. Comput. Mech.*, vol. 7, no. 2, pp. 935–943, 2021, doi: 10.22055/jacm.2021.35920.2761.
[2] B. Widodo, *Mathematical Modeling*. ITSpress, 2012.
[3] S. Choi, “Nanofluid technology: current status and future research.,” *Energy*, p. 26, 1998, [Online]. Available: http://www.ncbi.nlm.nih.gov/pubmed/11048%5Cnhttp://www.osti.gov/energycitations/product.biblio.jsp?osti_id=11048
[4] A. Khan, M. Ashraf, A. M. Rashad, and H. A. Nabwey, “Impact of heat generation on magneto-nanofluid free convection flow about sphere in the plume region,” *Mathematics*, vol. 8, no. 11. pp. 1–18, 2020. doi: 10.3390/math8112010.
[5] B. Widodo, C. Widodo, B., Anggriani, I., Imron, D. Aryany, N. Asiyah, F. A. Widjajati, and K. Kamiran, “The effect of magnetohydrodynamic nano fluid flow through porous cylinder,” *AIP Conf. Proc.*, vol. 1867, no. August, 2017, doi: 10.1063/1.4994472.
[6] O. Mahian *et al.*, “Recent advances in modeling and simulation of nanofluid flows-Part I: Fundamentals and theory,” *Phys. Rep.*, vol. 790, pp. 1–48, 2019, doi: 10.1016/j.physrep.2018.11.004.
[7] Y. Norasia and Z. Zulaikha, “Pengaruh Partikel Nano Zn dan ZnO terhadap Aliran MHD Fluida Nano Pada Lapisan Batas Bola Bermagnet,” *Sq. J. Math. Math. Educ.*, vol. 1, no. 2, p. 133, 2019, doi: 10.21580/square.2019.1.2.4792.
[8] T. Gul *et al.*, “Magneto hydrodynamic and dissipated nanofluid flow over an unsteady turning disk,” *Adv. Mech. Eng.*, vol. 13, no. 7, pp. 1–11, 2021, doi: 10.1177/16878140211034392.
[9] Y. Norasia, B. Widodo, and D. Adzkiya, “Pergerakan Aliran MHD Ag-AIR Melewati Bola Pejal,” *Limits J. Math. Its Appl.*, vol. 18, no. 1, p. 15, 2021, doi: 10.12962/limits.v18i1.7888.
[10] H. Wei *et al.*, “Effect of volume fraction and size of Al2O3nanoparticles in thermal, frictional and economic performance of circumferential corrugated helical tube,” *Case Stud. Therm. Eng.*, vol. 25, no. February, p. 100948, 2021, doi: 10.1016/j.csite.2021.100948.
[11] M. Z. Abdullah, K. H. Yu, H. Y. Loh, R. Kamarudin, P. Gunnasegaran, and A. Alkhwaji, “Influence of Nanoparticles on Thermophysical Properties of Hybrid Nanofluids of Different Volume Fractions,” *Nanomaterials*, vol. 12, no. 15, p. 2570, 2022, doi: 10.3390/nano12152570.
[12] Adnan, U. Khan, N. Ahmed, S. T. Mohyud-Din, M. D. Alsulami, and I. Khan, “A novel analysis of heat transfer in the nanofluid composed by nanodimaond and silver nanomaterials: numerical investigation,” *Sci. Rep.*, vol. 12, no. 1, pp. 1–11, 2022, doi: 10.1038/s41598-021-04658-x.
[13] Y. Norasia *et al.*, “Laminar Viscous Fluid Flow with Micro-rotation Capabilities through Cylindrical Surface,” vol. 6, no. 4, pp. 865–875, 2022.
[14] B. Widodo, D. K. Arif, D. Aryany, N. Asiyah, F. A. Widjajati, and K. Kamiran, “The effect of magnetohydrodynamic nano fluid flow through porous cylinder,” *AIP Conf. Proc.*, vol. 1867, no. August 2017, 2017, doi: 10.1063/1.4994472.
[15] S. Mojumder, S. Saha, S. Saha, and M. A. H. Mamun, “Combined effect of Reynolds and Grashof numbers on mixed convection in a lid-driven T-shaped cavity filled with water-Al2O3 nanofluid,” *J. Hydrodyn.*, vol. 27, no. 5, pp. 782–794,

2015, doi: 10.1016/S1001-6058(15)60540-6.

[16] C. Renon, M. Fénot, M. Girault, S. Guilain, and B. Assaad, "An experimental study of local heat transfer using high Prandtl number liquid jets," *Int. J. Heat Mass Transf.*, vol. 180, p. 121727, 2021, doi: 10.1016/j.ijheatmasstransfer.2021.121727.

[17] R. A. Arul Raja, J. Sunil, and R. Maheswaran, "Estimation of Thermo-Physical Properties of Nanofluids using Theoretical Correlations," *Int. J. Appl. Eng. Res.*, vol. 13, no. 10, pp. 7950–7953, 2018, [Online]. Available: http://www.ripublication.com

[18] C. J. Etwire, I. Y. Seini, R. Musah, and O. D. Makinde, "Combined Effects of Variable Viscosity and Thermal Conductivity on Dissipative Flow of Oil-Based Nanofluid over a Permeable Vertical Surface," *Diffus. Found.*, vol. 16, no. June, pp. 158–176, 2018, doi: 10.4028/www.scientific.net/df.16.158.

[19] M. N. Bello *et al.*, "Reaction Dynamics of Rocket Propellant with Magnesium Oxide Nanoparticles," *Energy and Fuels*, vol. 29, no. 9, pp. 6111–6117, 2015, doi: 10.1021/acs.energyfuels.5b00905.

[20] K. A. Mohammed, A. R. A. Talib, N. A. Aziz, and K. A. Ahmed, "Three dimensional numerical investigations on the heat transfer enhancement in a triangular facing step channels using nanofluid," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 152, no. 1, 2016, doi: 10.1088/1757-899X/152/1/012057.