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A STUDY OF SMALL AREA ESTIMATION TO MEASURE MULTIDIMENSIONAL POVERTY WITH MIXED MODEL POISSON, ZIP, AND ZINB

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ABSTRACT

The research began with calculating the value of multidimensional poverty at the district level in West Java Province from SUSENAS 2021. The calculation of multidimensional poverty was based on individuals in each district or city household. The dimensional weights are weighed the same, and the indicators in the dimensions are also weighed the same. Furthermore, the simulation study used the Poisson, ZIP, and ZINB mixed models to examine the model's performance on data with cases of excess zero values and overdispersion. The simulation was by generating data without overdispersion and with overdispersion. Overdispersion data was generated with parameters of ω (0.1, 0.3, 0.5, and 0.7), and the model was evaluated from the AIC value. The best method in the simulation study was used to estimate multidimensional poverty in sub-districts in West Java Province using PODES 2021. Simulation studies on data without overdispersion and ZINB are better than Mixed Model Poisson. The percentage of the multidimensional poverty population at the sub-district level in West Java Province is quite diverse, from 0.04% to 75.54%.

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1. INTRODUCTION

Poverty is a condition where a person cannot fulfill basic needs, seen from various aspects or dimensions. Poverty data released by the Statistics Central Bureau of Indonesia is monetary poverty, calculated based on meeting basic food and non-food needs based on income dimensions [1]. Apart from monetary poverty, poverty also needs to be seen from other dimensions, such as health, education, and standard of living [2]. Poverty seen from various dimensions is called multidimensional poverty. Multidimensional poverty is in line with the goals of the SDGs. Of the 17 goals, three are closely related to multidimensionality, including the first goal of no poverty and the third goal of ensuring a healthy life and promoting well-being for all people of all ages. The fourth goal ensures quality education that is inclusive and fair and promotes lifelong learning opportunities for all. According to the indicators in the goal, there is a target in the first goal of eradicating poverty. According to the national definition, the target is to reduce at least half the proportion of men, women, and children of all ages living in poverty in all dimensions by 2030.

The resulting multidimensional poverty describes the average percentage of the population experiencing multidimensional deficiencies. Many methods and indicators can be used to form multidimensional poverty scores. According to UNDP, one of them is launching the Global Multidimensional Poverty Index (MPI), a multidimensional poverty index formed from three dimensions: education, health, and a decent standard of living. The constituent indicators consist of 10 indicators [3]. This multidimensional poverty can be an alternative to measure the level of poverty. It is just that the government does not routinely release this multidimensional poverty like monetary poverty.

Direct poverty calculation using SUSENAS data from BPS can only estimate well down to the district level, while at lower levels, it needs more samples to estimate appropriately. In many cases, survey data is limited and can only represent a larger area or domain. It is due to a smaller area. The samples obtained from surveys are relatively small [4], even though local governments can use the poverty rate at a smaller level to evaluate or make a policy at a smaller level. For example, the district government wants to know the number of poor people by the district. Therefore, we need a method to estimate the value of poverty at a smaller level, such as the district level. Small Area Estimation (SAE) is essential for inferences from survey data [5].

Small area estimation can be done using direct (direct-based) and indirect (indirect-based/model-based) methods. Direct estimation is an estimation based on the design of conducting sampling. This method will produce estimates with a significant variance due to the small sample size. Indirect estimation is carried out based on a certain model [6]. One of the models for estimating small areas is the Generalized Linear Mixed Model (GLMM). MLTC modeling can include the accompanying variable as a fixed effect and the area as a random effect [7].

Analysis of the data obtained from surveys often finds data whose range of observations is greater than the range of expectations. This situation is known as overdispersion. Another possible source of overdispersion is zero excesses. In this case, the Poisson method for dealing with overdispersion may not be appropriate [8]. Lambert uses the Zero-Inflated Poisson (ZIP) regression technique to deal with zero-inflated data originating from error applications in manufacturing [9]. Based on Lambert's results, other studies highlight some of the main contributions in the literature to Zero-Inflated (ZI) modeling regression calculations and show that the ZI and Zero-Inflated Negative Binomial (ZINB) models have undoubtedly been widely used to deal with data dispersion because they are more flexible [10]. Other research compares the estimation performance of the Poisson, Generalized Poisson, ZIP, ZIGP, and ZINB models. Those propose a new estimator called the probability estimator (PE) for the inflation parameter of the ZIP distribution based on the moment estimation (ME) of the average parameter and compares its performance with ME and the maximum likelihood estimator (MLE) through simulation studies [11].

This research begins by calculating multidimensional poverty values from SUSENAS 2021 data. Furthermore, in the simulation study, the MLTC Poisson, MLTC ZIP, and MLTC ZINB models were used to examine how these models perform on data with data that has excess zero values and experiences overdispersion. Application of the best methods in simulation studies was used to estimate multidimensional poverty in sub-districts in West Java Province using SUSENAS and PODES data for 2021.

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2. RESEARCH METHODS

2.1 Data

The data used were actual data and simulation data. Real data were used to calculate the percentage of poor people and the multidimensional poverty index at the district/city level and to apply a small area estimator for the multidimensional poor population at the district level using a mixed model. Simulation data was used to compare model performance.

2.1.1 Real Data

The actual data used in this research was secondary data from SUSENAS 2021 and PODES 2021 with the scope of West Java Province. SUSENAS data used to compile multidimensional poverty is:

Table 1. Indicators for Constructing Multidimensional Poverty

Dimension	Indicators
Education	Do not have family members who have completed 9 years of education (Junior High School)
	Have at least one school-aged child (up to grade 9) but has dropped out of school
Health	Having at least one family member who is malnourished
	Have one or more children under five who do not receive complete basic immunization
Decent Standard of Living	Have no electricity
	Do not have access to clean drinking water
	Do not have access to adequate sanitation
	Using cooking fuel from charcoal, coal or firewood
	Owns a house with dirt floors
	Does not own a motorized vehicle and only owns one of the following items (bicycle, motorcycle, radio, refrigerator, telephone or television)

Data from the 10 indicators calculated the number of people experiencing multidimensional poverty in households. Meanwhile, the accompanying data for the small area estimator comes from PODES 2021. The following variables were used in the model:

Table 2. Variables in the Small Area Estimator Model

Variables	Meaning
Y	The number of people with multidimensional poverty
X1	The proportion of Junior High School to total population
X2	The proportion of health centers and hospitals per thousand population
X3	The proportion of shops/minimarkets per thousand population
X4	The proportion of households with the main source of income in agriculture to the total population

2.1.2 Data Simulation

Simulation data were generated with two conditions: without overdispersion and with overdispersion. Data without overdispersion was generated by Poisson distribution, and data overdispersion was generated by a negative binomial. The data was generated with the following equation:

$$\ln(\mu_i) = \beta_0 + \beta_1 x_i + v_i \qquad (1)$$

Where μ_i is the median of the response variable, x_i is the explanatory variable, and v_i is the small area random effect. Setting the parameter ω (the proportion of the value 0 on the response variable) is given to data with an overdispersion of (0.1, 0.3, 0.5, and 0.7). The number of data generated was 30 and 100. The simulation was carried out in line with the simulation of Yang et al. and Amalia et al. who model the various parameters ω on negative binomial data [12] [13].

2.2. Data Analysis

2.2.1. Calculation of multidimensional poverty

1. Do weighting the multidimensional poverty index using the weighted weights of the dimensions and indicators. The indicator weights for each dimension are education and health 16.7 and a decent standard of living 5.6.
2. Everyone assessed in the IKM is seen from the indicators being assessed. The rating consists of a range of 0 and 1. The value that indicates a poor or non-poor household is calculated based on the following formula:
$$c_i = w_1 I_1 + W_2 I_2 + \cdots + W_d I_d \quad (2)$$
where $I_i = 1$ if the individual experiences a deprivation in indicator i and $I_i = 0$ if he does not experience a deprivation. W_i is the weight of indicator I with $\sum_{i=1}^{d} W_i = 100$
3. Sum up the assessment of 10 individual indicators and identifying multidimensional household poverty with the *cut-off point*(c) of 33.3%.
4. Calculate the value of the Multidimensional Poverty Index (IKM), Multidimensional Poverty Rate (AKM), and Intensity in each district/city with the following formula:
$$\textit{Headcount Ratio (H)}: \frac{q}{n} \quad (3)$$
where, H is the proportion of the poor population, q is the number of people with multidimensional poverty, and n is the total population (households).
$$\text{AKM}: \textit{Headcount Ratio} \times 100 \quad (4)$$
$$\text{Intensity } (A): \frac{\sum_{i=1}^{n} c_i}{q} \quad (5)$$
$$\text{IKM}: H \times \text{A} \quad (6)$$
where A is the severity of poverty, q is the number of individuals with multidimensional poverty, and c is the score of the individuals

2.2.2. Simulation

1. Generating variable X randomly with Normal distribution (μ = 0.6, σ = 0.2) as many as n (30 and 100).
2. Generating random variables with Normal distribution (μ = 0, σ = 0.03)
3. Generating n variables Y in the following way
   a. Data without overdispersion.
      - Calculating the mean value (μ) for each observation using the formula $\mu_i = \exp\,(\beta_0 + \beta_1 x_i + v_i)$, β_0 and β_1 set to 1,8 and -0.3.
      - For each μ_i Poisson spreading data is generated (μ_i).

   b. Data with overdispersion
      - Setting parameters ω = (0,1, 0,3, 0,5 and 0,7) and τ=2
      - Dividing explanatory variables and random variables into two parts, namely for Y=0 as many as $n\omega$ and Y > 0 as many as $n - n\omega$
      - Generating a zero response variable $n\omega$
      - Calculating the mean value (μ) for each observation with the formula $\mu_i = \exp\,(\beta_0 + \beta_1 x_i + v_i)$ for non-zero response variables. β_0 and β_1 are set at 6.8 and 2.1 respectively.
      - Generating a non-zero response variable as many as n-nω following the Negative Binomial distribution($\mu_i, \tau = 2$)
4. Performing MLTC Poisson modeling, MLTC ZIP and MLTC ZINB [14]
5. Repeating each model 500 times
6. Evaluating the model from the value of the Akaike Information Criterion [15] with the formula
$$AIC = -2\ell + 2p \quad (7)$$
where ℓ is the value of the log-likelihood model, and p is the number of parameters.

2.2.3. The estimation of multidimensional poverty at the sub-district level

1. Take raw data on the number of poor people in a household when calculating q at the multidimensional poverty calculation stage.
2. Do a direct estimate.
3. Look at the correlation between the accompanying variables.
4. Check for excess zero and overdispersion.

5. Model using the best model based on the simulation results.
6. In general, the model used is.

$$\ln(\mu_i) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + v_i \quad (8)$$

MLTC ZIP and MLTC ZINB has two models, namely the log model like equation 7 and the logit model for all data. Logit model equations:

$$\ln\left(\frac{p_i}{1-p_i}\right) = \delta_0 + \delta_1 z_1 + \delta_2 z_2 + \delta_3 z_3 + \delta_4 z_4 + u_i \quad (9)$$

7. Evaluate the model from the AIC value.
8. Predict the number of people with multidimensional poverty based on the model
9. Calculate the total percentage of people with multidimensional poverty

3. RESULTS AND DISCUSSION

3.1 Multidimensional Poverty Calculation

The 2021 SUSENAS data for West Java Province totaled 25,813 household samples from 27 regions/cities in West Java Province. The smallest sample is Banjar City, with 567 households with an average sample of 956 households. The percentage, intensity, and multidimensional poverty index were calculated from these data. The percentage results of the people with multidimensional poverty at the region/city level are shown in **Figure 1**. The district/city with the lowest percentage of people with multidimensional poverty is Bekasi City at 1.59%. Several regencies/cities with a high percentage of people with multidimensional poverty, above 10%, namely Sukabumi Regency, Cianjur Regency, Garut Regency, Tasikmalaya Regency, Karawang Regency, West Bandung Regency, Pangandaran Regency, and Sukabumi City. The region/city with the most significant percentage of people with multidimensional poverty is Garut Regency at 20.14%.

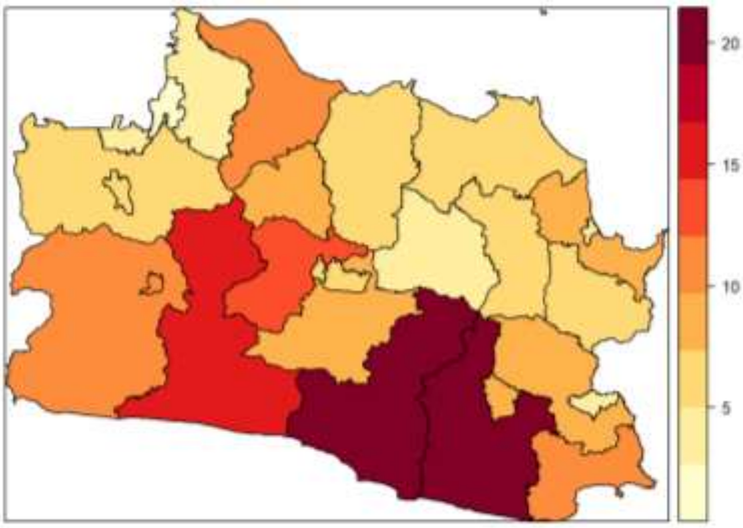
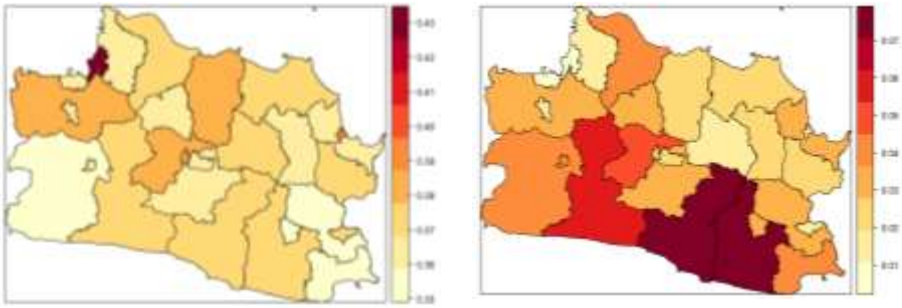


Figure 1. Percentage of population with multidimensional poverty based on regions/cities in West Java Province

Bekasi City has a small percentage of poor people, as shown in **Figure 2a**. Bekasi City has the most multidimensional poverty intensity/severity compared to other districts/cities, with a value of 0.42. A high-intensity value means that poverty in Bekasi City is more severe because every poor person in that city is poorer regarding the number of forming indicators. Overall, when viewed from the multidimensional poverty index in **Figure 2b**, the index for each urban district is relatively small, namely in the range of 0.006 to 0.07.



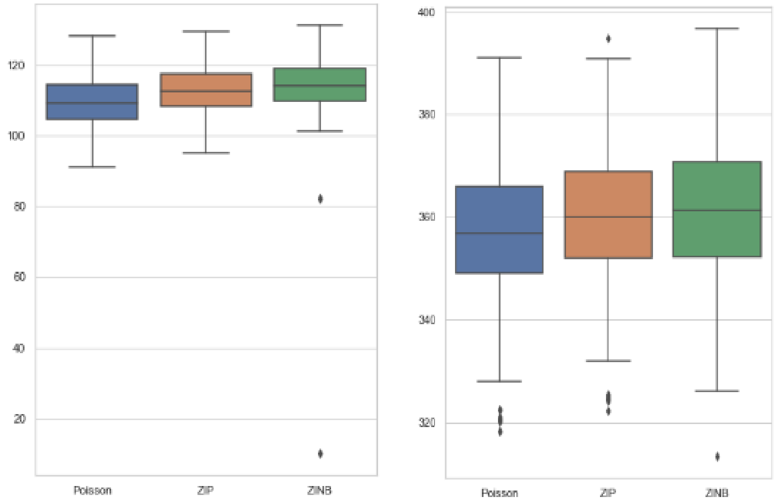
(a) (b)

Figure 2. Multidimensional poverty intensity and index by district/city in West Java Province (a) intensity, (b) Index

3.2. Simulation of MLTC Poisson, MLTC ZIP, and MLTC ZINB

3.2.1. Data Without Overdispersion

The evaluation was done by looking at the AIC value. The smaller the AIC, the better the model. The simulation results in **Figure 3** show no difference in the modeling results for data 30 and 100. The boxplots of the MLTC Poisson, ZIP, and ZINB models overlap and are almost in the same position.

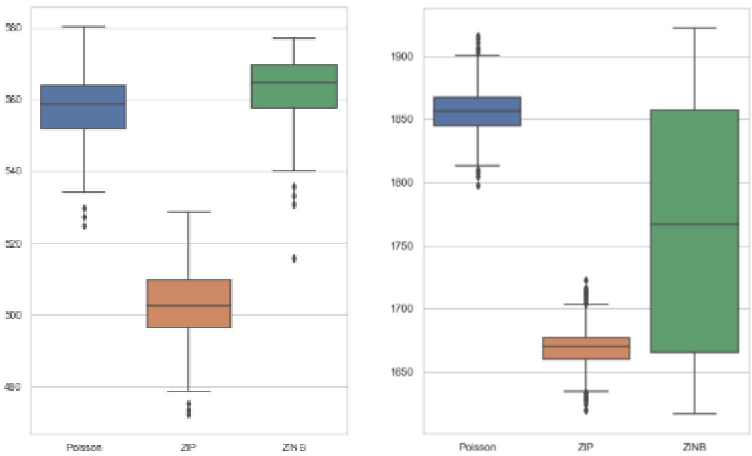


(a) (b)

Figure 3. AIC Boxplot of MLTC Poisson, MLTC ZIP, and MLTC ZINB simulations with data without overdispersion in (a) n = 30, (b) n = 100.

3.2.2. Data Overdispersion

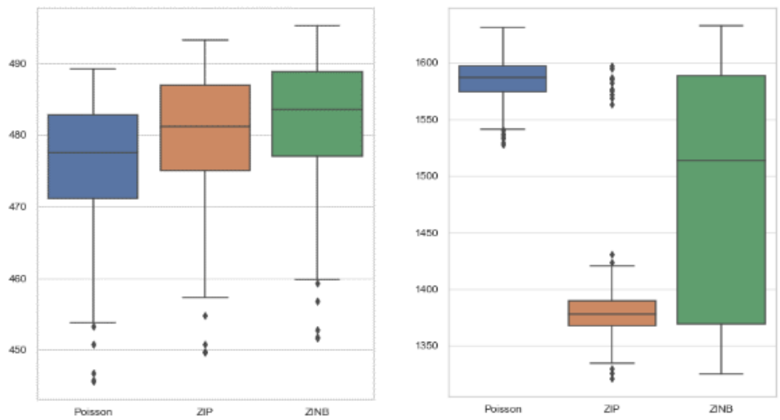
Overdispersion data was generated following a negative binomial distribution plus excess zero values. The parameter ω (the proportion of the value 0 on the response variable) is given to the data with an overdispersion of 0.1, 0.3, 0.5, and 0.7. The AIC value from **Figure 4a** is smaller in MLTC ZIP when compared to the other two models, while in data 100, it can be seen from Figure 4b that the AIC values of MLTC ZIP and ZINB are better than Poisson.



(a) (b)

Figure 4. Boxplot AIC simulation MLTC Poisson, MLTC ZIP, and MLTC ZINB with overdispersion data and $\omega = 0,1$ on (a) n= 30, (b) n= 100

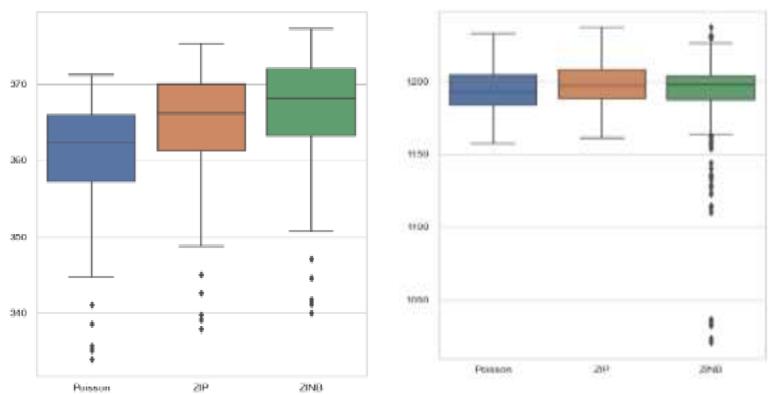
The simulation results on the parameter ω=0.3 are shown in **Figure 5**. When the data totals 30, there is no difference between the three models. When the data is 100, MLTC ZIP and ZINB are better than the Poisson model. However, no conclusion is reached between MLTC ZIP and MLTC ZINB. In some data, it gives smaller ZIP AIC, but some data gives bigger ZINB AIC.



(a) (b)

Figure 5. Boxplot AIC simulation MLTC Poisson, MLTC ZIP, and MLTC ZINB with data overdispersion and $\omega = 0,3$ on (a) n=30, (b) n=100

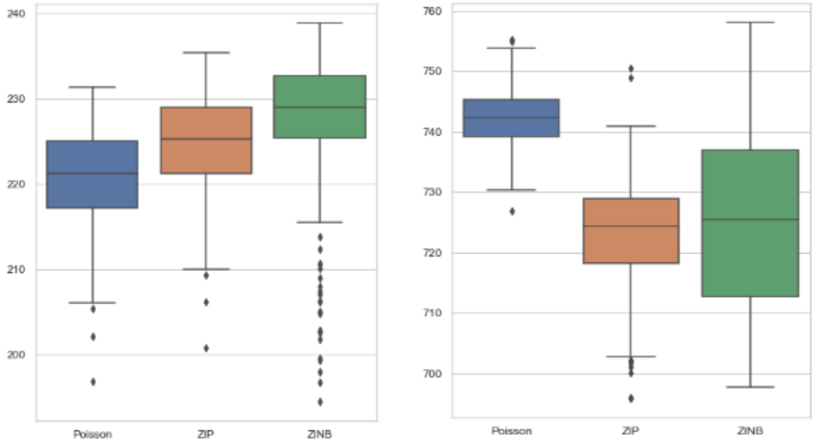
The simulation results on the $\omega = 0.5$ are shown in **Figure 6**. Both data 30 and data 100 cannot provide conclusions about the three models. However, the 100 data shows that MLTC ZINB has some small AIC results.



(a) (b)

Figure 6. Boxplot AIC simulation MLTC Poisson, MLTC ZIP, and MLTC ZINB with data overdispersion and $\omega = 0,5$ on (a) n=30, (b) n=100

The simulation results on the parameter ω=0.7 are shown in Figure 7. In data 30, it can be seen that several AIC values are almost the same as shown from the boxplots that intersect, so no conclusions can be drawn. In the amount of 100 data, the AIC value of MLTC ZIP and MLTC ZINB is better than MLTC Poisson.

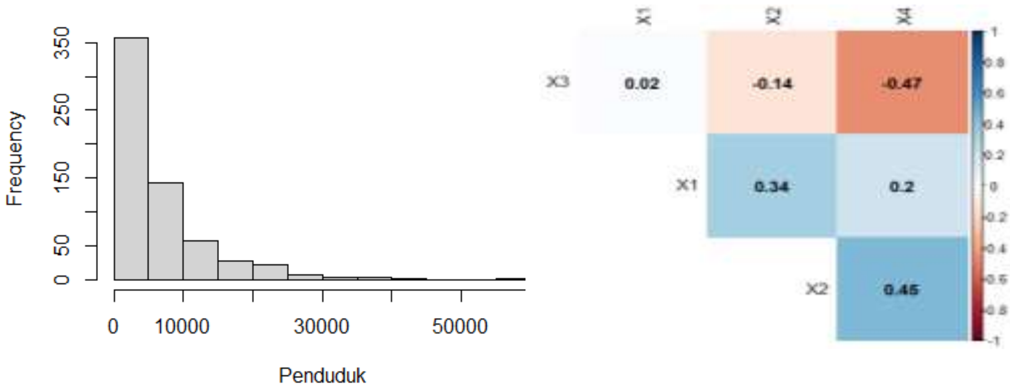


(a) (b)

Figure 7. Boxplot AIC simulation MLTC Poisson, MLTC ZIP, and MLTC ZINB with data overdispersion and $\omega = 0,7$ on (a) n=30, (b) n=100

3.3. Estimation of Multidimensional Poverty at the District Level

Estimation of the number of poor people used direct estimation results in the number and distribution of people with multidimensional poverty based on Figure 8a. There are many zero values, such as 87 out of 626 sub-districts in West Java Province. This value is 13.89% of the total data or parameter ω = 0.1389. Overdispersion testing was carried out with the qcc package in the R program with the function qcc.overdispersion.test. The null hypothesis in this test is that the data follows the Poisson distribution, and the alternative hypothesis is that it violates the Poisson distribution. The resulting P-value is 0, so it is concluded that there is overdispersion.



(a) (b)

Figure 8. Data distribution and relationships (a) Y distribution, (b) explanatory variable correlation

Then, the relationship between the accompanying variables was checked. Modeling does not allow for multicollinearity conditions. The correlation results in Figure 8b show that the correlation value is not too large, so there is no suspicion of multicollinearity. Further checking of data Y is carried out with an excess zero tests. The vcd extra package was used in the R program for the excess zero tests. The test results produce a p-value of 2.22e-16. It means that the Poisson model cannot correctly handle zero values. In other words, there is an excess of zeros in the data. When the equidispersion assumption in Poisson modeling is violated, and an excess zero occurs, the parameter estimation becomes biased, and the statistical test derived from the model becomes incorrect. [16].

Table 3. Result of Model Parameter Estimation

Parameter	MLTC ZIP			MLTC ZINB		
	Estimator	*p-value*		Estimator	*p-value*	
β_0	8.5141	<2e-16	***	8.3685	<2e-16	***
β_1	3.9501	1.36e-05	***	3.7467	3.71e-05	***
β_2	-5.3343	7.41e-07	***	-5.4385	4.79e-07	***
β_3	-362.2871	0.0913	.	-5.9956	0.972	
β_4	0.0316	0.8937		0.2128	0.353	
δ_0	-11.685	7.57e-06	***	-1.316	0.0004	***
δ_1	-3.072	0.861		-2.509	0.3110	
δ_2	-5.861	0.694		5.2615	0.05364	
δ_3	-1.609	1.000		0.5652	0.9991	
δ_4	2.239	0.516		-2.1340	0.0012	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Based on the simulation results with the extensive data (n = 100), both parameters ω = 0.1 and 0.3 show that MLTC ZIP and ZINB are better than MLTC Poisson. Meanwhile, the difference in the AIC values cannot be concluded in the MLTC ZIP and MLTC ZINB. Therefore, the modeling of multidimensional poverty data at the sub-district level was continued with the two models. Table 3 shows the parameters of the MLTC ZIP and ZINB estimation results.

Then, evaluation was done based on AIC. The MLTC ZIP value is 10.950, and MLTC ZINB is N/A or divergent. The value prediction was carried out using the MLTC ZIP from the evaluation results. The prediction result is estimating a small area of the population with multidimensional poverty using a mixed model. The prediction results were divided by the population of each sub-district and multiplied by 100 to get the percentage of the population with multidimensional poverty. The calculation results are shown in Figure 9. The percentage of people with multidimensional poverty at the sub-district level in West Java Province is quite diverse, ranging from the lowest 0.04% to 75 .54%.

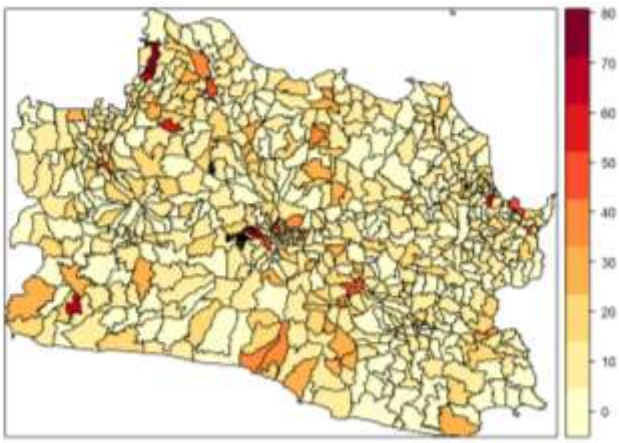


Figure 9. Percentage of population with multidimensional poverty by sub-district in West Java Province

4. CONCLUSIONS

The results of the calculation of multidimensional poverty at the region/city level show that the percentage of the population with multidimensional poverty varies quite widely between regions/cities. There are eight regions/cities where the percentage of people with multidimensional poverty is relatively high. The value of the multidimensional poverty index at the region/city level is relatively low. Simulation studies on data without overdispersion did not show differences in the model's goodness, meaning the three models could predict well. However, the MLTC Poisson was deemed sufficient for the modeling. In the data with overdispersion, it can be seen that MLTC ZIP and MLTC ZINB are better than the MLTC Poisson model.

However, the two models do not show significant differences. The best model for estimating small areas in sub-district level multidimensional poor population data is the MLTC ZIP. The estimated results are diverse, from the lowest 0.04% to 75.54%. From the modeling results, there are three influential variables, namely the proportion of the number of junior high schools per thousand population, the proportion of the number of health centers and hospitals per thousand population, and the proportion of shops/minimarkets per thousand population.

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