OPTIMIZATION OF RICE INVENTORY USING FUZZY INVENTORY MODEL AND LAGRANGE INTERPOLATION METHOD

Eka Susanti, Fitri Maya Puspita, Evi Yuliza, Siti Suzlin Supadi, Oki Dwipurwani, Novi Rustiana Dewi, Ahmad Farhan Ramadan, Ahmad Rindarto

ABSTRACT

Interpolation is a method to determine the value that is between two values and is known from the data. In some cases, the data obtained is incomplete due to limitations in data collection. Interpolation techniques can be used to obtain approximate data. In this study, the Lagrange interpolation method of degree 2 and degree 3 is used to interpolate the data on rice demand. A trapezoidal fuzzy number expresses the demand data obtained from the interpolation. The other parameters are obtained from company data related to rice supplies and are expressed as trapezoidal fuzzy numbers. The interpolation accuracy rate is calculated using Mean Absolute Percentage Error (MAPE). The second-degree interpolation method produces a MAPE value of 30.76 percent, while the third-degree interpolation has a MAPE of 32.92 percent. The quantity of order $Q_1, Q_2, Q_3, Q_4$ respectively 202677 kg, 384610 kg, 1012357 kg, 1447963 kg, and a total inventory cost of Rp. 129231797951.

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1. INTRODUCTION

The concept of inventory can be applied to the problem of optimizing inventory, inventory levels, optimal order time, and the optimal amount of inventory for reordering to minimize the total costs incurred due to inventory. Research related to inventory and methods has been developed and applied in various fields. Inventory problems with various payment systems are discussed by [1]. The algebraic method is used by [2] to solve the inventory model. The modified fruit fly algorithm method was introduced by [3] to solve the optimization problem of allocation and inventory. In [4], the problem of inventory for perishable products. The application of the concept of inventory in the health sector is discussed in [5]. Optimization of the red chili inventory system using the fuzzy inventory probabilistic model is discussed by [6].

The EOQ inventory model can be used in optimization problems that aim to obtain the optimal amount of inventory and reorder time by considering demand parameters. The following is a research on inventory problems using the EOQ model. Inventory problems using the EOQ model with nonlinear constraints are given by [7]. Research related to inventory problems using the EOQ model by considering the level of damage and inventory depending on demand is discussed [8]. The application of the EOQ model with demand assumed to be a nonlinear function is discussed by [9].

In some cases, the inventory model parameters are uncertain. For example, the demand parameter for an item for each period is not always the same and fluctuates. So, to determine the value of these parameters, the interpolation technique can be done. Interpolation is a method to get a value that is between range values. Several interpolation techniques have been developed. The following is research related to the application of the interpolation method. The study of Spline interpolation is given by [10]. [11] used Kriging interpolation on a nonlinear model to predict rock shear strength. The use of the Cubic-B-Spline interpolation method on the boundary value problem was introduced by [12]. The multi-level quadratic spline interpolation technique was introduced by [13]. Newton's polynomial interpolation was used by [14] on underwater ROV systems. [15] introduced Newton’s multi-variable polynomial interpolation method. In addition to the interpolation methods already mentioned, fractal interpolation methods have been developed and applied in various fields. The use of fractal interpolation in time series data was introduced by [16]. Fractal interpolation on Covid-19 data is discussed by [17]. Fractal interpolation is also applied by [18] to seismic data by introducing a scale factor.

The interpolated data is the result of the approach, not the actual data. Therefore, the deterministic approach is inappropriate. Fuzzy, probabilistic, and stochastic approaches can be used for uncertain parameters. The EOQ model with fuzzy parameters is called the fuzzy EOQ model (FOQ). The following is research related to the application of the fuzzy EOQ model. The fuzzy EOQ model was developed by [19] with the request parameter expressed as cloudy fuzzy. Fuzzy numbers express the fuzzy EOQ inventory model with demand parameters, and the defuzzification technique using Graded Mean Integration is discussed by [20]. The fuzzy EOQ model, considering the discount proportion factor, is discussed by [21]. The procedure for solving the fuzzy EOQ model is given by [22] and [23]. This study will apply the Lagrange interpolation technique to the rice demand data. The interpolation technique is carried out to determine the value of the rice demand parameter approach based on price. Rice is one of the primary needs of Indonesian people. The demand for rice in each ordering period is different. Therefore, a technical approach is needed to determine the number of rice orders from suppliers to fulfill customer demand. The demand for interpolation results is approximate so that the fuzzy approach can be applied. Generally, the maximum and minimum limits for rice demand can be determined. Demand is constant at certain price intervals, and at certain prices, it increases or decreases. Trapezoidal fuzzy numbers can be used to express rice demand parameters. Data interpolated are expressed as trapezoidal fuzzy numbers and as fuzzy parameters in the fuzzy EOQ model introduced by [22] and [23]. This study developed a fuzzy EOQ model by considering the level of demand, holding costs, and inventory procurement capabilities.

2. RESEARCH METHODS

This study applies the Lagrange interpolation method to determine demand data in the inventory model. Interpolation of demand data is determined based on the selling price. A Trapezoidal fuzzy number expresses the interpolated demand data. The FEOQ model is used to determine the optimal inventory. The stages of solving inventory optimization problems are given as follows.
2.1 Lagrange Interpolation Method

The Lagrange polynomial interpolation function is given by [24]. In the \( j \)-th dimensional search space, the Lagrange interpolation polynomial \( P(x_j) \) of degree \( d \) pass through \( d + 1 \) different given points \((x_0^j, f(x_0^j)), (x_1^j, f(x_1^j)), \ldots, (x_d^j, f(x_d^j))\), \( j = 1, \ldots, D \), where \( x_k^j \) is the \( j \)-th parameter in the vector \( x^k = [x_1^k, x_2^k, \ldots, x_n^k] \), \( k = 0, \ldots, d \). \( f(x^k) \) is the corresponding fitness value of \( x^k \) and \( f(x^k) = f([x_1^k, x_2^k, \ldots, x_n^k]) \). It Satisfies \( P(x_j^k) = f(x^k) \), as shown in Equation (1).

\[
P(x_j) = \sum_{k=0}^{d} \left[ \prod_{m=0, m \neq k}^{d} \frac{x_j^m - x_j^k}{x_m^j - x_j^k} \right] f(x^k), j = 1, \ldots, D \tag{1}
\]

\( P(x_j) \) is the polynomial of fitness \( f(x) \). Equation (1) for an interpolation of degree 2 can be rewritten as follows.

\[
P(x_2) = \frac{(x-x_0)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_2)}{(x_2-x_0)(x_2-x_1)} f(x_2) \tag{2}
\]

Equation (1) for an interpolation of degree 3 can be rewritten as follows.

\[
P(x_3) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3) \tag{3}
\]

2.2 Trapezoidal Fuzzy Number and Mean Integration Method

Trapezoidal fuzzy number \((n_1, n_2, n_3, n_4)\) is given in Figure 1.

![Figure 1. Trapezoidal fuzzy number](image)

The mean integration method is used for the defuzzification stage [23].

Let \( \tilde{B} = (n_1, n_2, n_3, n_4) \), \( P(\tilde{B}) = \frac{n_1 + 2n_2 + 2n_3 + n_4}{6} \)

2.3 Fuzzy Economic Order Quantity (FOQ) Model

Given the total cost, one cycle is as follows.

\[
TC(Q, P) = \frac{K(a - bP)}{Q} + c(a - bP) + \frac{gcP}{2} \tag{4}
\]

A partial differential of Equation (4) with respect to \( Q \) and the result equal to zero is obtained.

\[
Q = \sqrt[3]{\frac{2K(a - bP)}{gc}} \tag{5}
\]

By using Trapezoidal fuzzy numbers \( Q_l, K_l, P_l, c_l, a_l, b_l \) and \( g_l \), the following is given a FEOQ model introduced by [23].

\[
\frac{JTC(Q)}{Q} = 1/6 \left[ \frac{K_l(a_1 - b_1P_1)}{Q_4} + c_1(a_1 - b_1P_1) + \frac{g_1c_1Q_1}{2} + 2 \left( \frac{K_2(a_2 - b_2P_2)}{Q_3} + c_2(a_2 - b_2P_2) + \frac{g_2c_2Q_2}{2} \right) + 2 \left( \frac{K_3(a_3 - b_3P_3)}{Q_2} + c_3(a_3 - b_3P_3) + \frac{g_3c_3Q_3}{2} \right) + \frac{K_4(a_4 - b_4P_4)}{Q_1} + \frac{g_4c_4Q_4}{2} \right] \tag{6}
\]
A partial differential of Equation (6) with respect to $Q_i$, $i = 1, 2, 3, 4$, and the result equal to zero is obtained.

$$Q_1 = \sqrt{\frac{2K_4(a_4-b_4P_4)}{g_1c_1}}$$  \hspace{1cm} (7)

$$Q_2 = \sqrt{\frac{2K_3(a_3-b_3P_3)}{g_2c_2}}$$  \hspace{1cm} (8)

$$Q_3 = \sqrt{\frac{2K_2(a_2-b_2P_2)}{g_3c_3}}$$  \hspace{1cm} (9)

$$Q_4 = \sqrt{\frac{2K_1(a_1-b_1P_1)}{g_4c_4}}$$  \hspace{1cm} (10)

where:

- $K_i$: ordering cost
- $a_i$: constant demand rate coefficient
- $b_i$: price-dependent demand rate coefficient
- $P_i$: selling price
- $Q_i$: ordering size
- $c_i$: unit purchasing cost
- $g_i$: constant holding cost coefficient
- $i = 1, 2, 3, 4$

### 3. RESULTS AND DISCUSSION

In this research, interpolation was carried out using Lagrange interpolation techniques to determine rice demand based on price. Initial conditions are selected based on prices at the highest and lowest price intervals. The highest and lowest prices are 12000 and 10000 respectively. Using Equation (2) and Equation (3), we obtain 2nd and 3rd-order Lagrange interpolation functions with initial conditions (10000, 10800, 12000) and (10000, 10800, 11400, 12000).

$$P_2(x) = \left(\frac{(x-12000)(x-10000)}{(10800-12000)(10800-10000)}\right)(1177733) + \left(\frac{(x-10800)(x-10000)}{(12000-10800)(12000-10000)}\right)(362400) + \left(\frac{(x-10800)(x-12000)}{(10000-10800)(10000-12000)}\right)(3138678.5)$$

The following are the 2nd-degree and 3rd-degree Lagrange interpolation results. Variable $x$ can be replaced with the price of rice, which will determine the value of the demand approach. Interpolation calculations using Python programming.

Table 1. Lagrange Interpolation 2nd Degree and 3rd degree

<table>
<thead>
<tr>
<th>Time</th>
<th>Price (Rupiah)</th>
<th>Demand (Kg)</th>
<th>2nd Interpolation</th>
<th>3rd Interpolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>10500</td>
<td>1877819</td>
<td>1780207.23</td>
<td>901673.62</td>
</tr>
<tr>
<td>February</td>
<td>10150</td>
<td>1512299</td>
<td>2684629.01</td>
<td>1980337.89</td>
</tr>
<tr>
<td>March</td>
<td>10800</td>
<td>1177733</td>
<td>1177733.00</td>
<td>1177733.00</td>
</tr>
<tr>
<td>April</td>
<td>11550</td>
<td>1405435</td>
<td>369169.14</td>
<td>2411759.78</td>
</tr>
<tr>
<td>May</td>
<td>11400</td>
<td>2419069</td>
<td>451153.71</td>
<td>2419069.00</td>
</tr>
<tr>
<td>June</td>
<td>12000</td>
<td>362400</td>
<td>362400.00</td>
<td>362400.00</td>
</tr>
<tr>
<td>July</td>
<td>10250</td>
<td>1966169</td>
<td>2404076.06</td>
<td>1464533.17</td>
</tr>
<tr>
<td>August</td>
<td>10500</td>
<td>1848084</td>
<td>1780207.23</td>
<td>901673.62</td>
</tr>
<tr>
<td>September</td>
<td>10400</td>
<td>1449570</td>
<td>2016466.73</td>
<td>1016890.71</td>
</tr>
<tr>
<td>October</td>
<td>11800</td>
<td>923049</td>
<td>321115.06</td>
<td>1726768.83</td>
</tr>
<tr>
<td>November</td>
<td>10350</td>
<td>2119598</td>
<td>2141240.50</td>
<td>1126534.17</td>
</tr>
<tr>
<td>December</td>
<td>10000</td>
<td>3138678.5</td>
<td>3138678.50</td>
<td>3138678.50</td>
</tr>
</tbody>
</table>

The level of accuracy is calculated using the Mean Absolute Percentage Error (MAPE). MAPE interpolation 2nd Degree is 30.76 percent, and degree 3 is 32.92 percent. Rice inventory optimization calculation using 2nd degree interpolated demand data.

Table 2. Parameter Value

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fuzzy Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_i$</td>
<td>(250; 450; 650; 1050)</td>
</tr>
<tr>
<td>$a_i$</td>
<td>(321115; 600295; 2504218; 3138679)</td>
</tr>
<tr>
<td>$b_i$</td>
<td>(11.287; 12.108; 26.604; 50)</td>
</tr>
<tr>
<td>$P_i$</td>
<td>(1.181; 1.148; 1.245; 1.249)</td>
</tr>
<tr>
<td>$c_i$</td>
<td>(9200; 9400; 9600; 9800)</td>
</tr>
<tr>
<td>$g_i$</td>
<td>(28.92; 29.86; 35.06; 38.30)</td>
</tr>
</tbody>
</table>

The fuzzy parameter constant demand rate coefficient $a_i$ is determined based on the results of 2nd-degree interpolated demand data by considering the lowest, highest, and average demand. Other parameters are determined based on actual data. Based on the fuzzy parameter values in Table 2 and Model (6) to (10) obtained

1. The quantity of order $Q_4, Q_5, Q_2, Q_1$, respectively 202677 kg, 384610 kg, 1012357 kg, 1447963 kg.
2. Total inventory cost of Rp 12923179751.

4. CONCLUSIONS

Based on the results of calculations using rice inventory data in one company, it can be concluded that

1. The interpolated MAPE value of degree 2 is smaller than degree 3.
2. Trapezoidal fuzzy parameters provide four optimal inventory solutions. The larger the trapezoidal fuzzy number, the smaller the optimal inventory amount.
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