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IMPLEMENTATION OF GRAPH COLORING IN UMMUL MUKMININ HIGH SCHOOL STUDENT'S DORMITORY USING WELCH-POWELL ALGORITHM

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ABSTRACT

In Graph Theory, the concept of vertex coloring is an interesting topic because it can be implemented in various fields in everyday life. One of them is the distribution of dorm rooms at a school in Makassar. The placement of dorm rooms is made so that no students from the same class or region are in the same room. The data of region and class will be represented in an adjacency matrix with 137 rows and columns. Furthermore, the coloring will be solved by using the Welch-Powell algorithm. The coloring results obtained were 50 colors. That means, the rooms needed to place 137 students so that no one comes from the same region, and classes are 50 rooms with a maximum capacity of 4 people in each room.

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1. INTRODUCTION

Graphs serve as mathematical models to analyze many concrete real-world problems successfully. Certain problems in physics, chemistry, communication science, computer technology, genetics, psychology, sociology, and linguistics can be formulated as problems in graph theory. Also, many branches of mathematics, such as group theory, matrix theory, probability, and topology, have close connections with graph theory [1]. One of the studies in graph theory is graph coloring, either vertex coloring and edge coloring.

Graph coloring has become a very interesting topic, mainly because of its interesting history, various theoretical results, unsolved problems, and various applications. The problem related to graph coloring that has received the most attention is vertex coloring of a graph [2]. In the following, several previous studies involving the concept of vertex coloring in everyday life will be presented.

In 2013, Ana Mardiatus and Noor Saif conducted a vertex coloring study using the Welch-Powell algorithm on traffic lights in Yogyakarta. The solution to the traffic light problem can be viewed from a graph perspective, namely by representing intersections in graph form. Graph points indicate the direction of travel that is allowed from X to the Y, while the edges of the graph indicate the direction of travel that is not allowed to be carried out simultaneously. Then solve it with the vertex coloring method using the Welch-Powell algorithm. This solution will produce currents that can run simultaneously; besides that, it also obtains a new cycle duration alternative. Completion of calculating the duration of time on traffic light by coloring the nodes provides an alternative result that is more effective up to 78.64% than secondary data from the Yogyakarta Transportation Service in 2011 [3].

Furthermore, Lidia Lestari and Mulyono applied the Welch-Powell algorithm in mapping areas in the city of Medan and combined it with Location Quotient analysis to determine the most strategic sub-districts in Medan city and the potential of these sub-districts. By utilizing the Welch-Powell algorithm, the highest degree is located in the Medan Kota sub-district with degree 7, so based on this degree the most strategic sub-district is the Medan Kota sub-district and can be made a priority for the development of Medan City. Furthermore, based on Location Quotient analysis, the district that has superior potential is Medan Selayang District with potential in the fields of agriculture, mining, construction, trade, hotels and restaurants, finance, services, and many more [4].

Then, in 2020 Sri Efrinita conducted research on the application of graph coloring to the placement of student rooms at the Sumatra Institute of Technology using a data sample of 15 candidates of student dormitory. The results obtained were that only 6 rooms were needed to accommodate the 15 students so that no students from the same study program or UKT group were in one room, with a maximum capacity of 4 people in a room [2].

From some examples of these implementations, it can be concluded that graph coloring theory, especially with the Welch-Powell algorithm, can be used to solve various problems in society and make a major contribution in the field of research. In this study, the level of difficulty depends on the data used. The bigger and more data needed, the graph construction will be more difficult. In this study, the number of students who will be studied is more than 100 people, so the number of points on the graph to be colored is also more than 100 points. Because there is a condition that no students from the same area and class are allowed to be in one room, this will also affect the construction of the adjacency matrix.

2. RESEARCH METHODS

The research method is a literature review on the coloring graph. One of the most frequently implemented methods for vertex coloring on graphs is the Welch-Powell algorithm. This algorithm is used to color a graph based on the order of degrees of the vertices on the graph, from the highest to the lowest degree. The stages of the Welch-Powell algorithm are as follows:

1. Sort all the vertex on the graph to be examined, starting from the highest degree to the lowest degree;
2. Use one color to color the first vertex (which has the highest degree) and other vertices that are not neighboring to the first vertex;
3. Start again with another high-degree vertex that doesn't have color yet, give it a new color that is different from the first one and repeat the vertex coloring process using the second color;

4. Repeat coloring until all vertices have been colored.

The Welch-Powell algorithm does not always provide the minimum number of colors in a graph coloring, but this algorithm provides an upper limit on the number of colors that can be used to color a graph. In this research, we used the following stages.

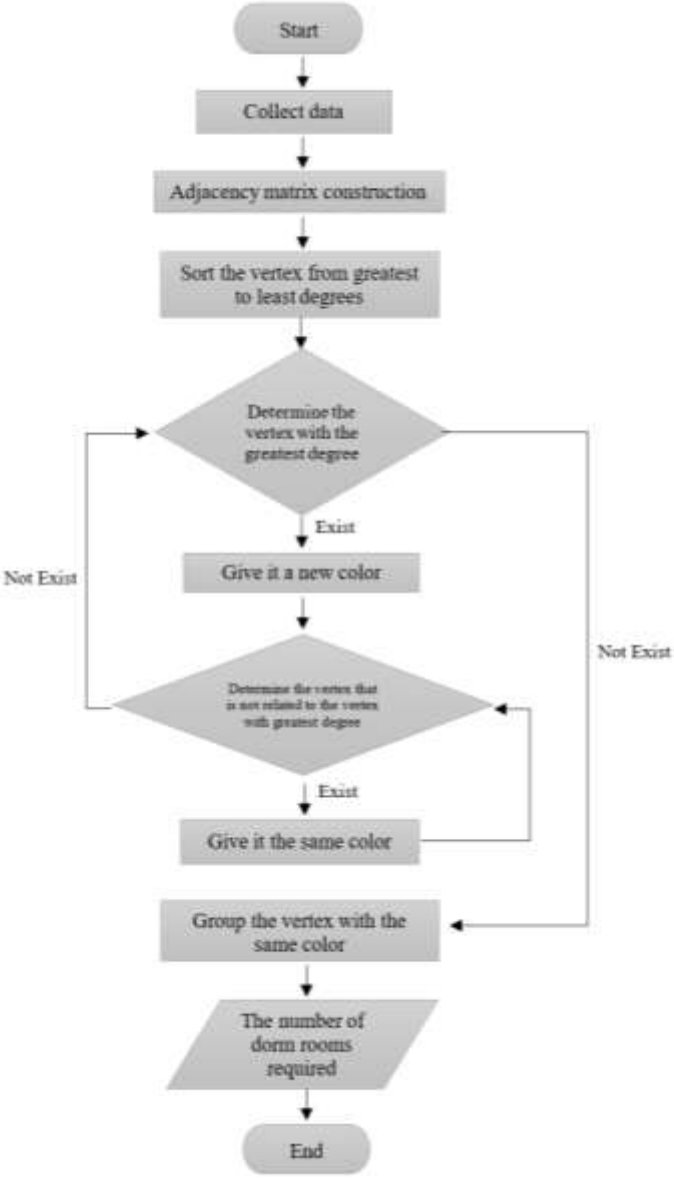

Figure 1. Research Flow Chart

3. RESULTS AND DISCUSSION

3.1. Basic Terminology

There are several definitions and terms that are important to know and will be used in graph coloring, as follows.

Definition 1. Graph is defined as a pair of sets (V, E), where V is a discrete set whose members are called vertex or vertices and E is a set of pairs of members of V which are called edges [5].

If uv is an edge of G, then u and v are adjacent vertex. Two adjacent vertices are reffered to as neighbors of each other. If uv and vw are distinct edges in G, then uv and vw are adjacent edges. The vertex u and the edge uv are said to be incident with each other. The degree of a vertex v in a graph G is the number of vertices in G that are adjacent to v. Thus, the degree of v is the number of vertices in its neighborhood.

Equivalently, the degree of v is the number of edges incident with v. The degree of a vertex v is denoted by $\deg_G v$ or $\deg v$ [6].

A graph that has 2 vertex of degree 1 and another vertex of degree 2 is called a path. If for every pair of vertices u and v on a graph, there is a path from u to v, then the graph is called a connected graph. A graph can also be presented in matrix, it can make it easier to analyze and calculate on a graph. There are two types of matrices, namely the incidence matrix and the adjacency matrix. The incidence matrix is defined as follows.

Definition 2. The incidence matrix $K = (k_{i,j})$ of a graph with p vertex and q edge is a matrix with $q \times p$ size, and $k_{i,j} = 1$ if e_i is related to v_j and $k_{i,j} = 0$ for others [5].

And then, the adjacency matrix is defined as follows.

Definition 3. [6] Suppose that G is a graph of order n, where $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$. The adjacency matrix of G is the $n \times n$ zero-one matrix $A(G) = [a_{ij}]$, or simply $A = [a_{ij}]$, were

$$a_{ij} = \begin{cases} 1, & if\ v_i v_j \in E(G) \\ 0, & if\ v_i v_j \notin E(G) \end{cases}.$$

The matrix that will be used in this study is the adjacency matrix. The student's data will be represented in the form of a graph, then it will be constructed in the form of an adjacency matrix to make the calculations easier in the next stage.

There are 2 well-known types of graph coloring concepts, namely vertex coloring and edge coloring. Meanwhile, there is also a type of coloring-f. However, because f-coloring is less common or rarely used, only vertex coloring and edge coloring will be explained in this review. Vertex coloring is the assignment of color to a set of vertices $V(G)$ where each vertex is only given one color and for every two-neighboring vertex, it has a different color [6].

While the colors used can be elements of any set, we typically use positive integers, say $1, 2, \dots, k$, for some positive integer k. Thus, a (proper) coloring can be considered as a function $c : V(G) \to N$ (where N is the set of positive integers) such that $c(u) \neq c(v)$ if u and v are adjacent in G. If each color used is one of k given colors, then we refer to the coloring as a k-coloring. In a k-coloring, we may then assume that it is the colors $1, 2, \dots, k$ that are being used. A graph G is k-colorable if there exists a coloring of G from a set of k colors. In other words, G is k-colorable if there exists a k-coloring of G. The minimum positive integer k for which G is k-colorable is the chromatic number of G and is denoted by $\chi(G)$. The chromatic number of a graph G is therefore the minimum number of independent sets into which $V(G)$ can be partitioned. A graph G with chromatic number k is a k-chromatic graph. Therefore, if $\chi(G) = k$, then there exists a k-coloring of G but not a $(k - 1)$-coloring [6].

For example, given the image of a wheel graph W_7, W_6, and S_7 as follows.

Figure 2. Vertex Coloring on W_7,W_6, and S_7

In **Figure 2**, the number of colors on W_7 is 5, W_6 is 3, and S_7 is 2. Or it can be written in notation $\chi(W_7) = 5, \chi(W_6) = 3$, and $\chi(S_7) = 2$.

An edge coloring of a graph G is an assignment of colors to the edges of G, one color to each edge. If adjacent edges are assigned distinct colors, then the edge coloring is a proper edge coloring. An edge coloring that uses colors from a set of k colors is a k-edge coloring. Thus, a k-edge coloring of a graph G can be described as a function $c : E(G) \to \{1, 2, \dots, k\}$ such that $c(e) \neq c(f)$ for every two adjacent edges e and f in G. A graph G is k-edge colorable if there exists a k-edge coloring of G [6].

3.2. Implementation of Graph Coloring in Ummul Mukminin High School Student's Dormitory

The placement of dorm rooms is made so that there are no students from the same class or region in one room. The data containing region and class will be represented in an adjacency matrix which will then be solved using the Welch-Powell algorithm. Student data is represented in graph G with each student representing one vertex on the graph so that a graph consisting of 137 points is obtained or can be written in the set as follows $V(G) = \{v_1, v_2, v_3, \ldots, v_{137}\}$. Next, a neighborhood matrix will be constructed with entries containing the number 1 for each student who has the same regional and/or class, and 0 for students who have a different regional and/or class. The matrix has the order of 137 x 137 or consists of 137 columns and 137 rows. The following figure is the adjacency matrix that is formed.

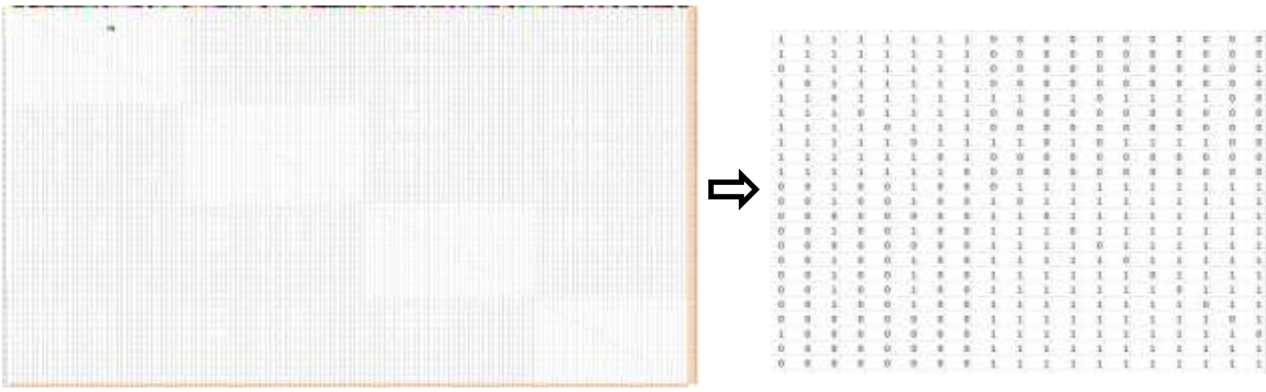

Figure 3. The Adjacency Matrix with The Order of 137 x 137

The sum of each row or column entry i is the degree of the vertex v_i. So that the degree of each vertex on the graph G is obtained. Then, the vertices are sorted from the vertex with the highest degree to the vertex with the lowest degree. Then the sequence of vertices is obtained as follows.

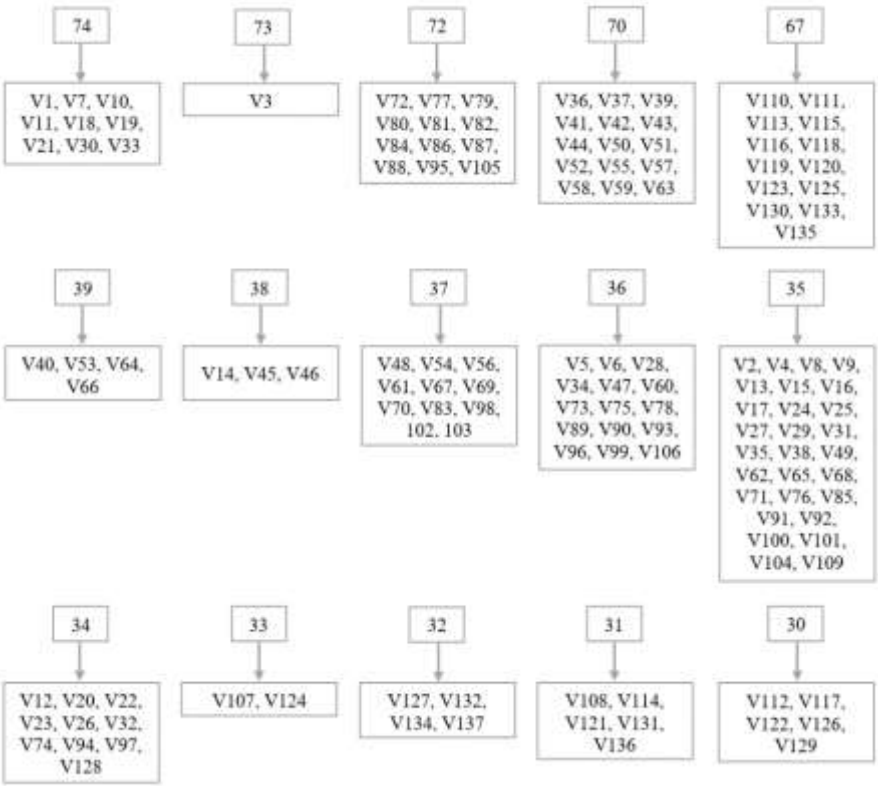

Figure 4. The Order of Vertex's Degrees

The following **Figure 4** is a graph representation that formed from the adjacency matrix above. It created using Gephi software. We get a graph consisting of 137 vertices.

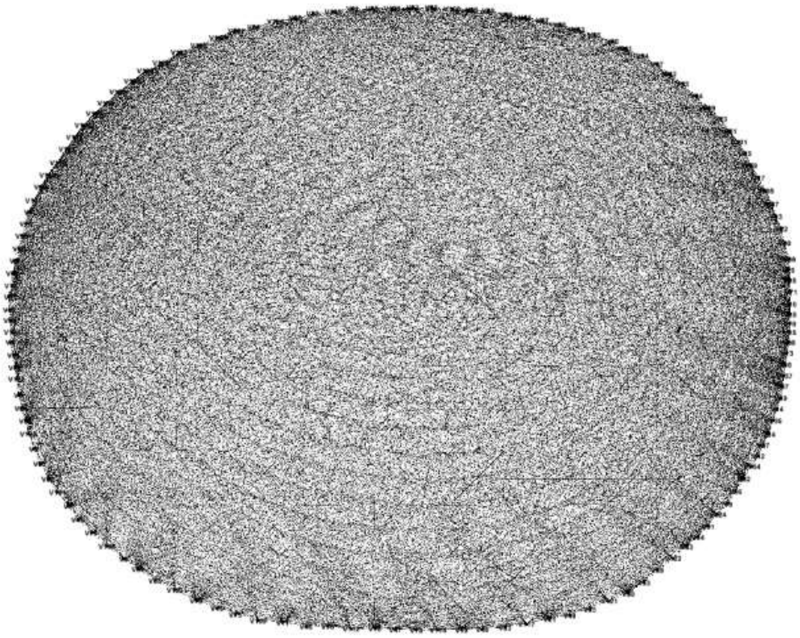

Figure 5. Graph Representation of 137 x 137 Adjacency Matrix

The vertex coloring begins by coloring the highest degree of vertex first, for example, we starting with V_1 which is given color 1. Next, by looking at the adjacency matrix, look for another vertex that is not related to V_1, for example V_{38}, give it the same color as V_1. After that, find another vertex that is not related to V_1 and V_{38}, and color it with color 1 as well. And so on until there are no other vertex that are neighbors to the vertex that has been colored with color 1. If there isn't one, then we can move to another point with a high degree to be colored with the next color. The following figure is the result of coloring that has been done using the Welch-Powell algorithm.

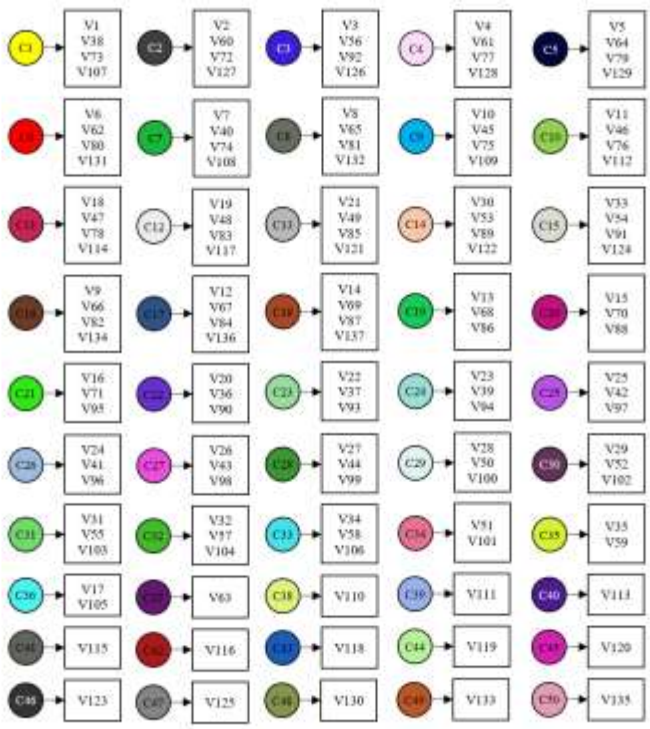

Figure 6. Vertex Coloring on Graph

The number of colors from the coloring results of 137 vertices is 50 colors. With a maximum of 4 vertices and a minimum of 1 vertex colored in one color.

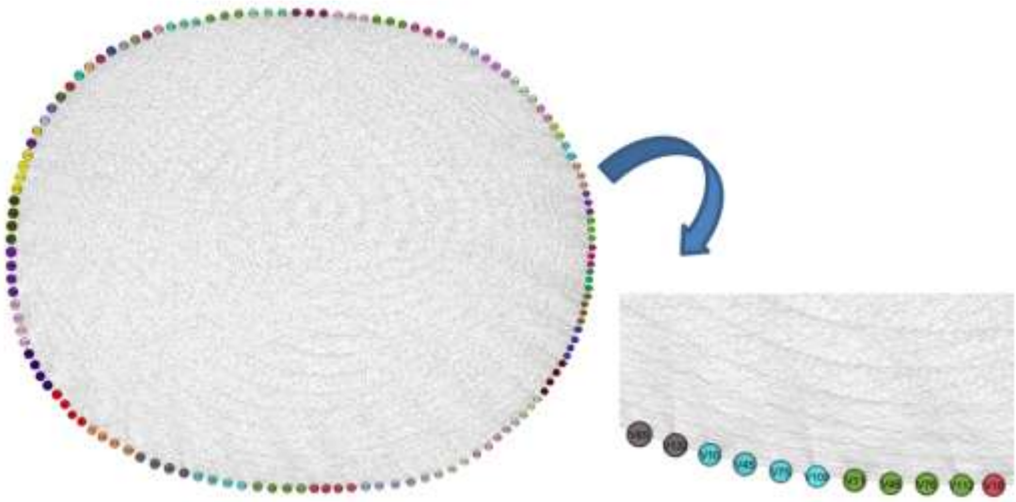

Figure 7. Representation of Graph Coloring

Each vertex that has the same color is not related to the other, which means that in one room there are no students from the same region or class. This can be seen from **Figure 7** above.

4. CONCLUSIONS

Based on the explanation above, it can be concluded that the minimum number of coloring obtained from the data of SMA Ummul Mukminin students is 50 colors. Which means the number of rooms needed to accommodate 137 students so that there are no students from the same region and class in one room is 50 rooms with a maximum capacity of 4 students in each room.

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