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ANALYZING EARTHQUAKE ACTIVITY LEVELS IN NORTH SULAWESI USING MAXIMUM LIKELIHOOD METHOD AND GUTENBERG – RICHTER LAW

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ABSTRACT

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Tectonic earthquakes are disasters that often cause damage to buildings and loss of life. The Province of North Sulawesi and its surroundings are earthquake-prone areas because they are located at the confluence of the major plates. This study aims to analyze the level of earthquake activity in the Province of North Sulawesi and its surroundings using the maximum likelihood method and the Guttenberg-Richter law. The distribution of the earthquake studied for the period 1941-2021 in the Province of North Sulawesi and its surroundings with a Mag ≥3.0 Mw and a depth ≤60 km. The data used comes from the Meteorological, Climatological, and Geophysical Agency Indonesia and ISC (International Seismological Center). The level of earthquake activity was reviewed based on nine groups of year intervals, three groups of depth levels, and the overall research area. The results show that the level of earthquake activity, rock fragility, and local stress at certain year intervals is sometimes high or low for the 1941-2021 earthquake period in North Sulawesi and its surroundings. The same condition for certain earthquake depths. On the other hand, the level of earthquake activity is quite high and the level of rock fragility is moderate in the research area as a whole.

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1. INTRODUCTION

A tectonic earthquake is one of the natural disasters that can cause large casualties and damage if the magnitude is large and occurs at shallow depths [1]–[4]. North Sulawesi Province is one of the regions of Indonesia and is categorized as earthquake-prone because it is located in the movement of the Eurasian, Indo-Australian, Philippine, and Pacific Plates [5], [6]. The province of North Sulawesi and its surroundings has been rocked several times by large earthquakes, especially in the Talaud Islands (23 October 1914, 24 September 1957, and 30 January 1969), which had an impact on the surrounding area. In addition, earthquake shocks in neighboring areas included destructive earthquakes in Davao, Philippines (14 April 1924, and 25 May 1943), and North Maluku (Halmahera (27 March 1949), and Morotai (26 May 2003)) which had an impact on damage in the Province of North Sulawesi and its surroundings.

The application of the Gutenberg-Richter law concept and the Maximum Likelihood method in analyzing the level of earthquake activity has been carried out by many researchers, including the application of the Guttenberg-Richter law in seeking to increase the value of seismic hazard, which can help earthquake prediction efforts by providing a process-based understanding of temporal changes in seismic activity *b*-value during the seismic cycle [7]. On the other hand, the application of the likelihood method in the analysis of the spatial and temporal distribution of seismotectonic is based on the *b*-value in Java [8]. Then the analysis of the spatial variation of *b*-values in the western part of the Tibetan Plateau and the surrounding area, including its relationship with fault blocks in the area, using the Gutenberg-Richter law approach and the maximum likelihood method [9]. The application of the Gutenberg-Richter law in the analysis of spatial variations of seismic *b* values, seismic moments, changes in Bouguer gravity, and the estimated fault plane solution in the Sumatra Region based on homogeneous earthquake data from June 2005 to March 2012 to investigate seismotectonic characteristics and tectonic activity between Sumatra earthquake 2004 and East Indian Ocean 2012 [10]. On the other hand, a maximum likelihood estimation approach in building a new algorithm to adapt non-local models for local applications is presented, including a case study assessing the probability of a major earthquake in Taipei [11].

This study is somewhat different from previous studies because in this study, the data was first described based on several groups of depth and magnitude so as to obtain an overview of the characteristics of the distribution of earthquakes in the study area. On the other hand, the data used is more than 50 years, so it is expected that the resulting *a* and *b* values will be more accurate and have a positive impact on the analysis of the level of earthquake activation in North Sulawesi Province and its surroundings. In this study, the earthquake level in North Sulawesi and its surroundings will be analyzed using the Gutenberg-Richter law approach and the maximum likelihood method. The values of *a* and *b* will be calculated and analyzed for certain year intervals, at certain depth groups, and the research area as a whole based on earthquake data for the 1941-2021 period in North Sulawesi Province and its surroundings.

2. RESEARCH METHODS.

North Sulawesi Province, with Manado as its capital city, is one part of the territory of Indonesia and consists of 15 districts/cities and has an area of 13,892 km2. It is bordered by the Sulawesi Sea, the Pacific Sea to the north, and the Maluku Sea to the east. The southern part is bordered by Tomini Bay and the West is bordered by Gorontalo Province [12]. This study uses 12,212 earthquake data for the period years 1941-2021 in the Province of North Sulawesi and its surroundings and sourced from the International Seismological Center (ISC) and the Indonesian Meteorological, Climatological, and Geophysical Agency (BMKG) with a magnitude $\geq$ 3.8 Mw and a depth $\leq$ 60 km.

The method used in this study is an applied statistical research method related to earthquakes with the maximum likelihood method approach and the Guttenberg-Richter law. Before applying the maximum likelihood method and the Guttenberg-Richter law, the distribution of earthquakes for the 1941-2021 period will be mapped. Then, it was continued by mapping the distribution of earthquakes for the period 1941-2021 based on magnitude and depth, which was previously divided into several groups. The results of the map analyzed the distribution of earthquakes in the province of North Sulawesi and its surroundings. Furthermore, the values of *a* and *b* are calculated using the maximum likelihood method approach and the Guttenberg-Richter law. The calculation of the *a*-value and *b*-value is reviewed in three parts, namely the calculation based on the year interval group, the depth group, and as a whole for the North Sulawesi Province. The results of calculations a and b are analyzed related to the level of an earthquake and tectonic earthquake activation depending on the nature of the rock and local stress activity in the North Sulawesi Region and its surroundings.

The maximum likelihood method is a method for estimating the parameters of a probability distribution based on the likelihood function. If a probability distribution function depends on the parameter, it corresponds to the likelihood function [13], [14] which is functioned as:

$$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta) \tag{1}$$

The maximum estimate of θ is the value of the maximum function $L(\theta)$ for the corresponding calculation, the derivation of the log $L(\theta)$is generally to get the maximum value of θ, namely:

$$\frac{\partial \log L(\theta)}{\partial \theta} = 0$$

Next, suppose the probability density function of M [10], [11],

$$f(M, \alpha) = \alpha e^{-\alpha(M - M_0)},\ M_0 \leq M \tag{2}$$

where

$$\alpha = \frac{b}{log_{10} e}, \tag{3}$$

Furthermore, if there are n earthquakes with a magnitude M_1, M_2, …, M_n, then according to the likelihood function of **Equations (1-2)**, the relationship is obtained

$$P(\alpha) = \prod_{i=1}^{n} f(M_i, \alpha)$$
$$= f(M_1, \alpha).f(M_2, \alpha) \dots f(M_n, \alpha)$$
$$= \alpha^n e^{-\alpha \sum_{i=1}^{n}(M_i - M_0)}, \tag{4}$$

Furthermore, the estimated maximum likelihood of b is obtained from the relationship of **Equations (3-4)** [15]–[17]:

$$\hat{b} = \frac{\log e}{\bar{M} - M_0}, \tag{5}$$

where

$$\bar{M} = \frac{\sum_{i=1}^{n} M_i}{n},$$

and $\bar{M}$ is the average magnitude of the earthquake, and M_0 is the minimum magnitude of the earthquake data, and $\log e = 0.434$. To find out the deviation of the b-value calculation using the Maximum Likelihood method, the standard deviation formula (**Equation (6)**) [17], [18] is used, namely:

$$\sigma_b = 2.30\, b^2 \sqrt{\sum_{i=1}^{n} \frac{(M_i - \bar{M})^2}{n(n-1)}} \tag{6}$$

Corresponding to the value of a calculated from the cumulative frequency relationship for $M \geq M_0$ is:

$$\hat{a} = \log N\ (M \geq M_0) + \log(\hat{b} \ln 10) + M_0 \hat{b} \tag{7}$$

On the other hand, the Gutenberg-Richter Law (**Equation (8)**) which is a statistical analysis method that describes the relationship between frequency and magnitude [15], [19],

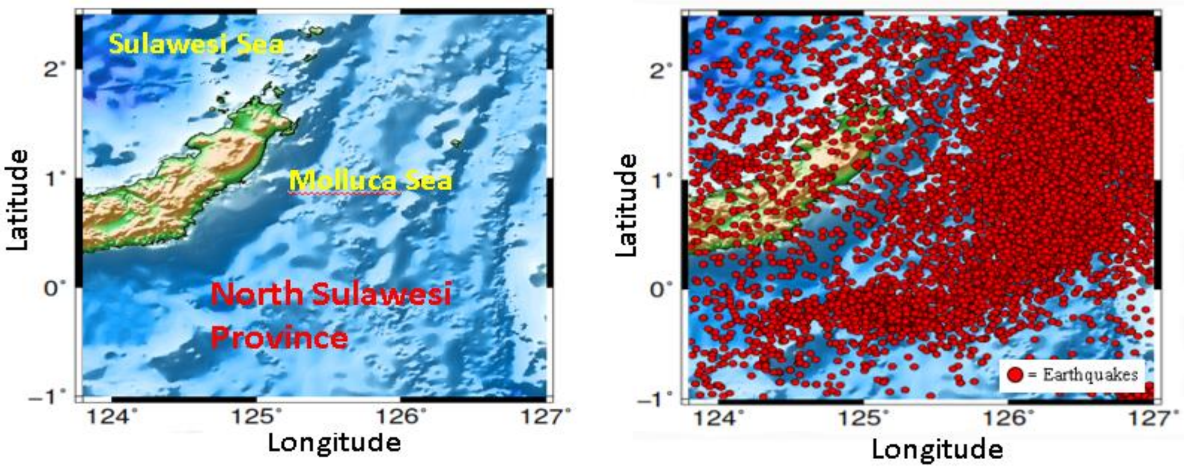
$$LogN(M) = a - bM, \tag{8}$$

where $N(M)$ is the number of earthquakes with a magnitude $\geq M$, a is a spatial characteristic parameter, b is a regional characteristic parameter, and M is a magnitude.

3. RESULTS AND DISCUSSION

3.1 Earthquake Characteristics of North Sulawesi Province and Surrounding Areas

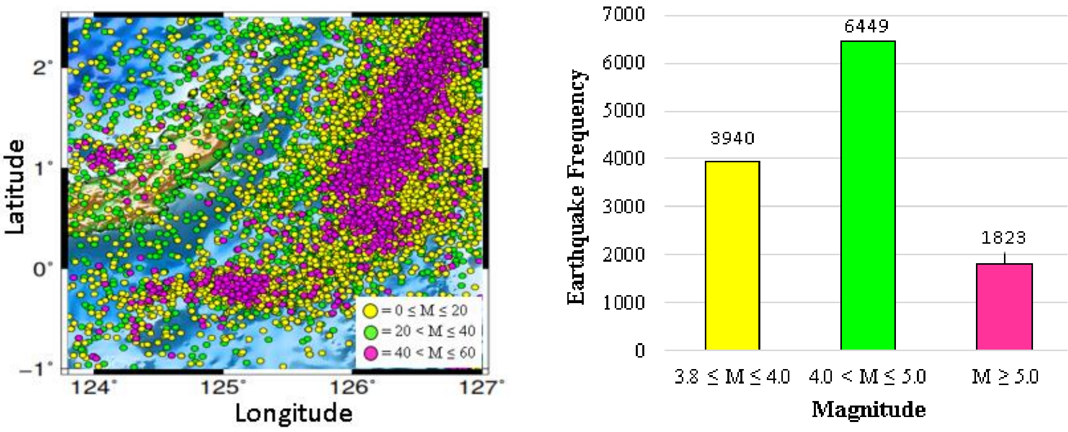
The boundaries of the research area are in the Province of North Sulawesi and its surroundings, covering several areas located at coordinates 123.8°-127° LE and 1° LS–2.5° NL (Figure 1a). Based on data obtained from the ISC (International Seismological Center) and BMKG online for the period years 1941-2021, or about 80 years with a magnitude ≥3Mw and a depth ≤ 60 km, there were 12,212 earthquake events (Figure 1b).



(a) (b)

Figure 1. a) Research area of North Sulawesi Province and its surroundings, and b). Earthquake distribution in the research area

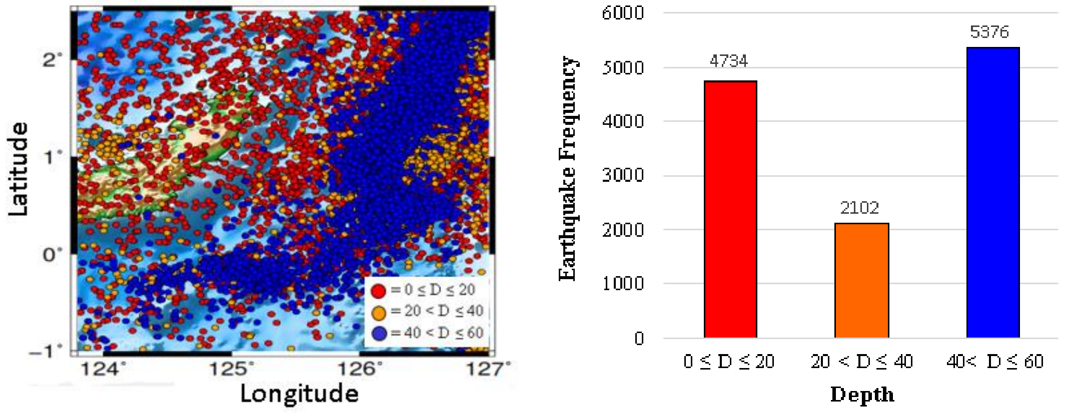
Based on Figure 1b, the epicenter distribution of the earthquake was 12,212 in the study area. The distribution of the earthquake is more dominant in the sea area when compared to the mainland area of North Sulawesi Province and its surroundings. The Maluku Sea is the area with the most earthquakes. This is because this area is one of the collision zones. Furthermore, the 1941-2021 earthquake period was reviewed based on the magnitude and grouped into three parts, namely 3.8 ≤ M ≤ 4.0, 4.0 < M ≤ 5.0, and M > 5.0. Epicenter distribution can be shown in Figure 2a. The distribution of earthquakes with a magnitude at intervals of 4.0 < M ≤5.0 has the highest number of earthquakes compared to the other two groups of magnitude intervals (Figure 2b).



(a) (b)

Figure 2. The Earthquakes for the period 1941-2021 were based on a) the distribution of the magnitude group, and b) the graph of the earthquake frequency for each magnitude group

The distribution of the earthquakes was also viewed in terms of depth (km) and was divided into three groups, namely 0 ≤ D ≤ 20, 20 < D≤ 40, and 40 < D ≤ 60 (Figure 3a). The depth group (km) with an interval of 40 < D ≤ 60 had the most earthquakes compared to the other depth groups (Figure 3b).



(a) (b)

Figure 3. Earthquake period 1941-2021 based on a) distribution of depth groups by epicenter, and b) graph of earthquake frequency for each depth group

Based on Figure 3a, it can be seen that an earthquake with a depth (km) of $0 \leq D \leq 20$ is more dominant in the land area of North Sulawesi Province and its surroundings, while an earthquake with a depth of (km) $40 < D \leq 60$ is more dominant in the sea area.

3.2 Earthquake Activity Level in North Sulawesi and Surrounding Areas

In this study, the level of earthquake activity in North Sulawesi Province and its surroundings will be reviewed using the Guttenberg-Richer Law approach and the maximum likelihood method. Based on period data, the level of earthquake activity related to rock fragility will be reviewed from two sides, namely based on the year interval group and the depth for the 1941-2021 earthquake period. Earthquake data in North Sulawesi Province and its surroundings are grouped into nine-year interval groups with an interval of nine years for each group. In detail, the grouping and the frequency of earthquakes in each group at year intervals can be shown graphically (Figure 4).

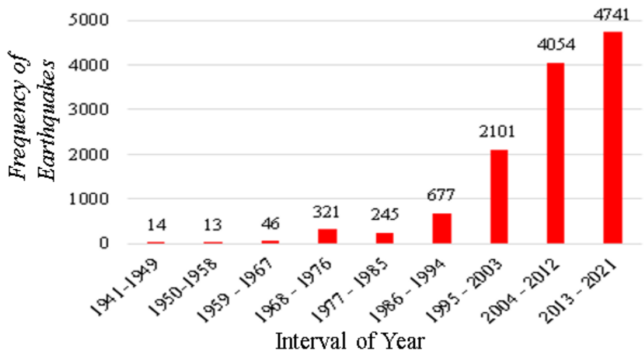


Figure 4. Graph of earthquake frequency for each year interval group

The division based on year intervals from the 1941-2021 earthquake period in the North Sulawesi Province aims to carry out an analysis of earthquake activity for certain periods of the year on an ongoing basis in North Sulawesi Province and its surroundings. The approach taken is to analyze the *a*-value and the *b*-value. Furthermore, the calculation of *a*-value, *b*-value, and standard deviation using Equations (5-7). The results of the calculation of the *a*-value and *b*-value are associated with the Gutenberg-Richter law (Equation (8)) and in detail can be shown in Table 1.

Table 1. *a*-value, *b*-value, standard deviation, and Gutenberg-Richter Law for each year interval

Interval of year	*a*	*b*	Standard Deviation	*LogN*(*M*) = *a*-*bM*
1941 - 1949	10.946	0.585	0.081	10.946 - 0.585*M*
1950 - 1958	16.996	1.737	0.430	16.996 - 1.737*M*
1959 - 1967	9.174	0.444	0.028	9.174 - 0.444*M*
1968 - 1976	12.705	0.418	0.009	12.705 - 0.418*M*
1977 - 1985	10.693	0.344	0.008	10.693 - 0.344*M*
1986 - 1994	12.982	0.387	0.006	12.982 - 0.387*M*
1995 - 2003	15.277	0.697	0.013	15.277 - 0.697*M*
2004 - 2012	16.878	0.819	0.012	16.878 - 0.819*M*
2013 - 2021	16.683	0.715	0.008	16.683 - 0.715*M*

Based on Table 1, it can be seen that the 1950-1958 interval has the largest *b*-value when compared to the other eight-year intervals. While the smallest *b*-value is in the interval 1977-1985. A large *b*-value indicates a tendency for earthquakes with a small magnitude to occur with a higher frequency, while a small *b*-value indicates the rock's resistance to large stresses, which means that the tendency for an earthquake with a small magnitude to have a smaller frequency of occurrence can lead to an earthquake with a large magnitude. The data in Table 1 for the *b*-value can be shown in a graph (Figure 5). The Gutenberg-Richter law shows the relationship between the value of *a*-value and *b*-value related to the seismic activity of the North Sulawesi Province and its surroundings, which is influenced by the level of rock fragility.

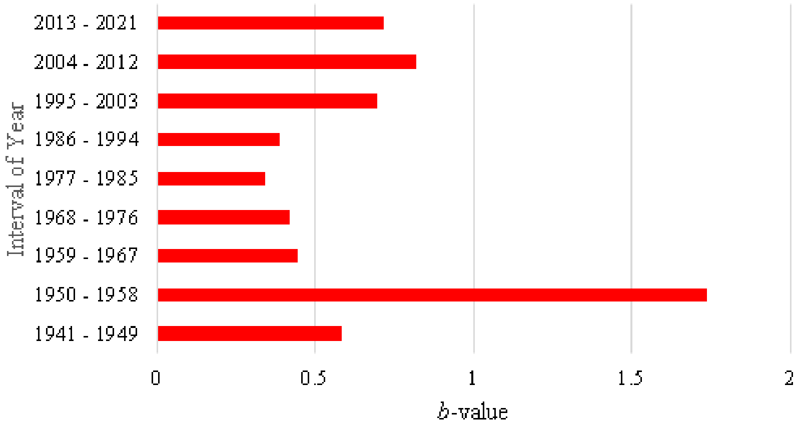


Figure 5. Graph of *b*-value for each year interval

Based on Figure 5, it can be seen that the *b*-value from the 1941-1949 interval to the 1950-1958 interval experienced a very significant increase. Then the *b*-value decreased very significantly from the interval 1950-1958 to 1959-1967. The decrease in the *b*-value continued, although not significantly from the interval 1959-1967 to 1977-1985 and increased even though it was not significant to the interval from 1986-1994. The increase in the *b*-value continued significantly until the 2004-2012 interval and experienced a significant decrease in the 2013-2021 interval. The decrease and increase in the *b*-value indicate that rock fragility and local stress at certain year intervals are sometimes high or low at a depth (km) of $0 \leq D \leq 60$. On the other hand, the standard deviation value in the 1950-1958 interval is high when compared to eight intervals of year another. The high standard deviation value for the calculation of the b-value is due to the very small amount of earthquake data in the 1950-1958 interval. This also happened between 1941-1949 and 1959-1967. While the other six-year intervals have a small standard deviation value for the calculation of the *b*-value because the amount of earthquake data is quite large. In detail, the standard deviation values for nine-year intervals can be shown by graphs (Figure 6).

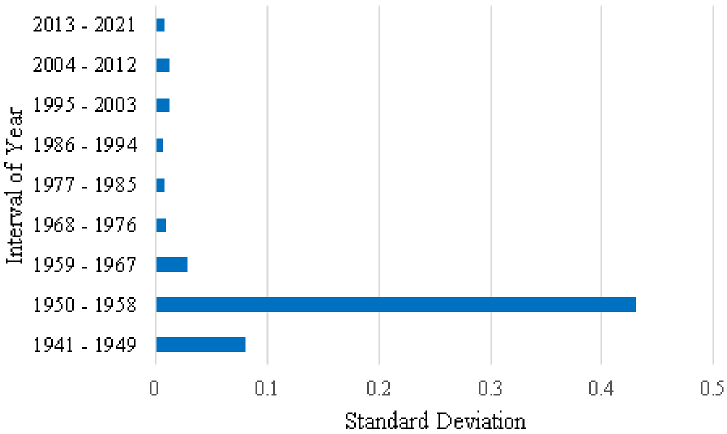


Figure 6. Graph of standard deviation values for each year interval

On the other hand, the largest a-value is in the 1950-1958 interval, while the smallest is in the 1959-1967 interval. This means that in the 1950-1958 interval, the North Sulawesi Province region had high seismic activity whereas in 1959-1967 it had low seismic activity. The North Sulawesi Provinces region and its surroundings have a fairly high earthquake activation which also occurred in the interval from 1986-1994 to 2013-2021. This means that at certain year intervals, earthquake activity is sometimes high or low in the province of North Sulawesi and its surroundings with a depth (km) of $0 \leq D \leq 60$. Graphically, a-value can be shown in Figure 7.

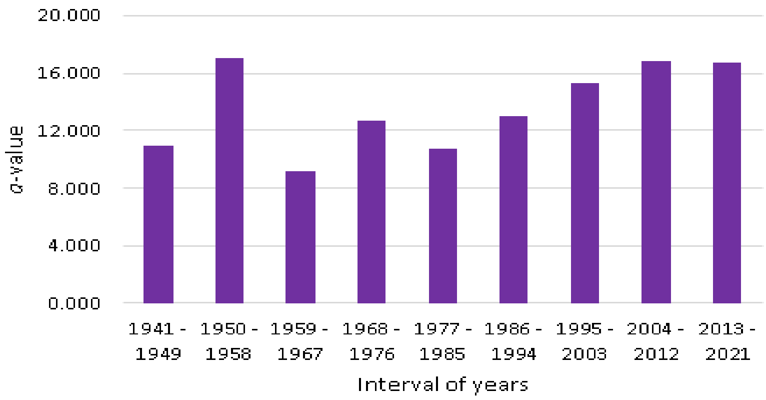


Figure 7. Graph of *a*-value graph for each interval of years

Furthermore, the a-value and b-value based on the depth group were evaluated, and the a-value, b-value, and standard deviation for the depth group (km) $0 \leq D \leq 20$, $20 < D \leq 40$, and $40 < D \leq 60$ were calculated based on earthquake data. The period 1941-2021 using Equations (5-7). The depth of the earthquake data for the period 1941-2021 is included in the shallow category ($\leq$ 60km). The results of the calculation of the a-value and b-value are associated with the Guttenberg-Richter law (Equation (8)) and in detail can be shown in Table 2.

Table 2. *a*-value, *b*-value, standard deviation, and Gutenberg-Richter Law for each depth group

Depth (km)	*a*	*b*	Standard Deviation	*LogN(M)=a-bM*
$0 \leq D \leq 20$	19.935	1.434	0.027	19.935 - 1.434M
$20 < D \leq 40$	14.692	0.522	0.008	14.692 - 0.522M
$40 < D \leq 60$	16.203	0.514	0.005	16.203 - 0.514M

Based on Table 2, it can be seen that the b-value in the depth interval group (km) $0 \leq D \leq 20$ has the highest value of 1,434 when compared to the other two groups (Figure 8). This shows that the North Sulawesi Province region and its surroundings at a depth (km) of $0 \leq D \leq 20$ have a high level of fragility and resistance to small rocks and low stress. This means that at a certain depth during the 1941-2021 earthquake period, rock fragility and local stress were low or high. On the other hand, the value of the standard deviation of depth is small to the calculation of the b-value because the amount of data studied is quite large.

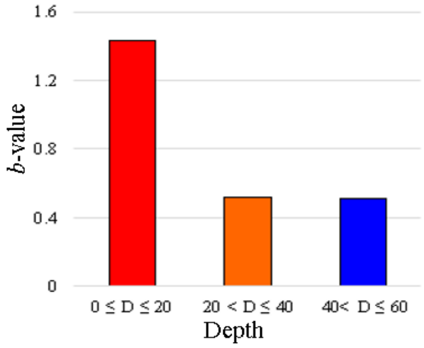


Figure 8. Graph of *b*-value for each depth group

On the other hand, the highest a-value is also in the depth group (km) $0 \leq D \leq 20$ when compared to other depth groups (**Figure 9**). This means that at a depth (km) of $0 \leq D \leq 20$, North Sulawesi and its surroundings have higher seismic activity while the other two groups have lower seismic activity. This means that the level of earthquake activity at a certain depth is sometimes low or high during the 1941-2021 earthquake period in the Province of North Sulawesi and its surroundings.

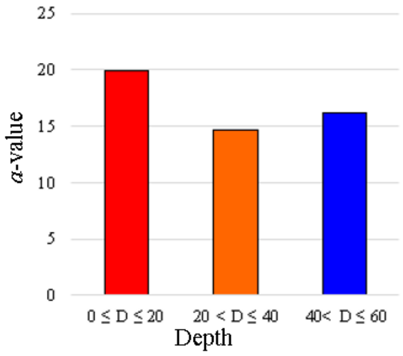


Figure 9. Graph of *a*-value for each depth group

If overall calculated the *a*-value, *b*-value, and standard deviation without grouping based on earthquake data for the period 1941-2021 and depth (km) $0 \leq D \leq 60$ in the Province of North Sulawesi and its surroundings using **Equation (5-7)** then the *a*-value is 6.640 and the *b*-value is 0.636 while the standard deviation is 0.005. The overall *a*-value is smaller when compared to the year interval group and the depth group. On the other hand, the overall b-value is smaller than the $0 \leq D \leq 20$ depth group, while from the nine-year interval groups, several-year groups are greater than the overall *b*-value. On the other hand, the standard deviation value for the calculation of the *b*-value is very small because the observation data is very large.

4. CONCLUSIONS

The results of the study shows that the overall level of earthquake activity in North Sulawesi Province and its surroundings for the earthquake period of 1941-2021 with a depth (km) of $0 \leq D \leq 60$ is quite high and the level of rock fragility and the level of local stress is low. However, if based on nine groups of year intervals divided from the 1941-2021 earthquake period, it shows that at certain year intervals, the *a*-value has increased and decreased significantly, which means the level of earthquake activity at certain year intervals in that year has increased or decreased. Then the *b*-value also increases and decreases significantly at certain intervals, which means that the level of rock fragility and local stress changes from high to low or vice versa. This also applies to different depth levels, the *a*-value decreases and increases quite significantly, which means that at a certain depth level, earthquake activity changes from high to low or vice versa. Furthermore, the *b*-value decreased to a certain depth ($\leq$ 60km), which means that at a certain depth, the rock fragility and local stress are higher.

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