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MODEL OF TRANSMISSION COVID-19 USING SIQRD MODEL WITH THE EFFECT OF VACCINATION IN MATARAM

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ABSTRACT

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Mathematical modeling is considered an effective tool for analyzing real-life problems. In this research, we analyze the dynamics of the COVID-19 spread in Mataram city using the SIQRD model with the influence of the vaccination. The analysis is based on varying some parameter values of the model, i.e., the transmission rate (β), the recovery rate for COVID-19 (γ), and the death rate (δ), before and after vaccination, respectively. Our chosen methodology involves parameter estimation using the Euler method. The result shows that the model has an endemic equilibrium point that remains stable before and after vaccination. Furthermore, the basic reproduction number (R0), which states the number of secondary cases that occur if there are infected people in a population, has a value of more than 1 before the vaccination but equal to 1 after the vaccination. This suggests that prior to COVID-19 vaccination, infected individuals could potentially infect more than one person, but after vaccination, each infected person tends to only infect one other individual. This shift is attributed to the subsidence of COVID-19 symptoms following vaccination.

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1. INTRODUCTION

Mathematical modeling is a particular type of problem-solving on science or real-life issues that use mathematical methods. Mathematical modeling of real-life issues is regarded as an effective tool to analyze problems. One example is describing and providing dynamic insight into a disease or virus, called the epidemic model [1]–[4]. The first and classical epidemic model is the SIR (Susceptible, Infective, Recovery) model, which was modified into several models, one of them being the SIQRD model. The SIQRD (Susceptible Infectious Quarantine Recovered Death) model consists of 5 variables. The variable S describes a group that can be infected by the virus, I describes the number of the group that has been confirmed positive, Q describes the number of the group that is quarantined, R indicates the group that has recovered, and D indicates the number of deaths due to the virus [5]–[7].

There are several types of viruses that have been studied in mathematics to be constructed in epidemic models. The research on viruses currently being carried out is research on the SARS-CoV-2 virus. The disease caused by this virus is called Coronavirus Disease 2019 (COVID-19). There are several epidemic models for COVID-19, such as the SIR epidemic model and the SIQRD epidemic model. Based on the current state of COVID-19, the susceptible group will become an infectious group when it has been infected with the virus through direct contact with COVID-19 patients [8]–[10]. If someone is infected or infectious, the patient will quarantine either independently or under the supervision of medical personnel. During the quarantine period, there are two possibilities that will happen to COVID-19 patients: patients will recover, or the worst possibility will die. Based on this explanation, the more appropriate model used to describe the state of COVID-19 is to use the SIQRD model [4], [11]–[14].

Prevention of the spread of COVID-19 with efforts to provide vaccines has been carried out. Several studies were conducted to review the effectiveness of vaccine administration, such as the effect of the vaccine in the SARS-COV-2 [15], the effectiveness of the vaccine in Asian countries [16], and the effectiveness of the vaccine in the US [17]. According to this description, a model of transmission of COVID-19 using the SIQRD model with the effect of vaccination in the city of Mataram was made.

2. RESEARCH METHODS

The data used in this study is secondary data, which is COVID-19 data from November 2020 - April 2021, which data was obtained from the website [18]. SIQRD model on the spread of COVID-19 with the influence of vaccination in Mataram City. From this data, parameter values for the SIQRD model on the spread of COVID-19 with the influence of vaccines in Mataram City are obtained. The research analysis steps can be explained as follows:

1. Explain the SIQRD model for the spread of COVID-19 with the influence of vaccination in Mataram City.
2. Specifies the parameter value. To estimate the parameters $\beta, \gamma,$ and δ used Euler's method:

$$y_{n+1} = y_n + hf(x_n, y_n) \tag{1}$$

   With $y_n \equiv y(n)$ and step size $h = x_n + 1 - x_n$ [19]. However, to determine the value of other parameters using assumptions according to existing data and previous research.

3. Doing numerical simulation and determining the equilibrium point. To obtain the equilibrium point of a differential equation system, the first derivative of the differential equation is equal to zero. Definition [20]:

   The point $x^* \in \mathbb{R}^n$ is called the equilibrium point of

$$\dot{x} = f(x) \tag{2}$$

   if $f(x^*) = 0$.

4. Analyze the stability of the model. To analyze the equilibrium point under certain conditions, a nonlinear differential equation system can be used, the Jacobian matrix.

   Given function $f = (f_1, f_2, \dots, f_n)$ on the system $\dot{x} = f(x)$. Matrix

$$Jf(x^*) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x^*) & \frac{\partial f_1}{\partial x_2}(x^*) & \dots & \frac{\partial f_1}{\partial x_n}(x^*) \\ \frac{\partial f_2}{\partial x_1}(x^*) & \frac{\partial f_2}{\partial x_2}(x^*) & \dots & \frac{\partial f_2}{\partial x_n}(x^*) \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1}(x^*) & \frac{\partial f_n}{\partial x_2}(x^*) & \dots & \frac{\partial f_n}{\partial x_n}(x^*) \end{bmatrix}$$

   is called the Jacobian matrix of f at point x^*.

   Theorem 1:
   Given the Jacobian matrix $\boldsymbol{Jf(x^*)}$ of a nonlinear system $\dot{x} = f(x)$ with eigenvalues (λ) [21].
   a. Stable asymptotic local, if all the real parts of the eigenvalues of the matrix $\boldsymbol{Jf(x^*)}$ are negative.
   b. Unstable, if there is at least one eigenvalue of the matrix $\boldsymbol{Jf(x^*)}$ whose real part is positive.

5. Find the value of the basic reproduction number. The basic reproductive number or R_0 defined as the number of secondary cases produced by infectious cases entering the susceptible population. R_0 searched by assuming $I(t) = i^*$ where $i^* > 0$ [22]. The conditions that will occur for each R_0 value are:
   1. If $R_0 < 1$, then the disease or virus will disappear
   2. If $R_0 = 1$, then the disease or virus will persist
   3. If $R_0 > 1$, then the disease or virus will become an epidemic.

6. Make conclusions based on research results.

3. RESULTS AND DISCUSSION

In the SIQRD model on the spread of COVID-19 with the effect of vaccination in Mataram City, there are 5 variables used in this study, namely variable $S(t)$ is the number of susceptible groups, variable $I(t)$ is the number of infectious groups, variable $Q(t)$ is the number of quarantined groups, variable $R(t)$ is the number of recovered groups, and variable $D(t)$ is the number of death groups. The SIQRD model can be described in Figure 1.

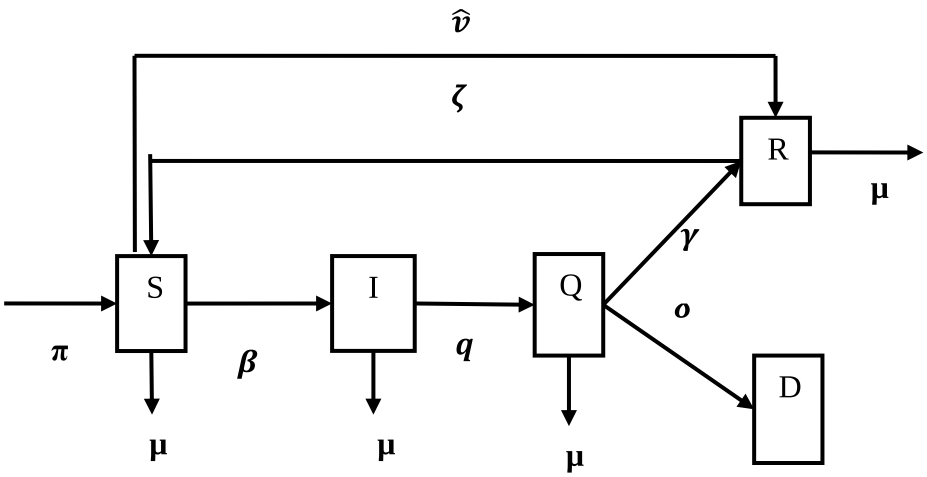


Figure 2. SIQRD model flow chart

$$\begin{aligned}
\frac{dS(t)}{dt} &= \pi\hat{N}(t) + \zeta R(t) - \left(\beta\frac{I(t)}{N} + \eta v + \mu\right)S(t),\\
\frac{dI(t)}{dt} &= \frac{\beta S(t)I(t)}{N} - (q+\mu)I(t),\\
\frac{dQ(t)}{dt} &= qI(t) - (\gamma+\delta+\mu)Q(t),\\
\frac{dR(t)}{dt} &= \gamma Q(t) + \eta v S(t) - (\zeta+\mu)R(t),\\
\frac{dD(t)}{dt} &= \delta Q(t).
\end{aligned} \tag{3}$$

the equations system above satisfies

$$N = S(t) + I(t) + Q(t) + R(t) + D(t), \tag{4}$$

define $\hat{N}(t) = N - D(t)$ and assumption to be equal to μ [6]. The following is a description of each variable and parameter

Table 1. Variables and Parameters

Notation	Description	Unit	Value
$S(t)$	Number of susceptible populations at time t	Person	-
$I(t)$	The total infectious population at the time t	Person	-
$Q(t)$	Number of the quarantined population at time t	Person	-
$R(t)$	Number of the recovered population at time of t	Person	-
$D(t)$	Total population death at the time of t	Person	-
β	COVID-19 transmission rate	1 day	Estimate
γ	Cure rate against COVID-19	1 day	Estimate
δ	Death rate against COVID-19	1 day	Estimate
q	Quarantine rate	1 day	Estimate
v	Vaccination rate	1 day	Estimate
η	Vaccine efficiency	-	Assumption
$\hat{v} = \eta v$	Vaccine potency level	1 day	Estimate
ζ	Reinfection rate	1 day	Assumption
π	Pure birth rate	1 day	Assumption
μ	Pure death rate	1 day	Assumption

The obtained data were divided into two groups, namely data before the vaccination and after the vaccination. The total population used in this study is $N = 495.000$. There are several parameters whose values use assumptions such as parameters $\pi, \mu, q, \eta, \zeta,$ and v. For reinfection (ζ) and quarantine parameters (q) is difficult to determine the value [6], so in this research was made two conditions for the value of ζ and q.

3.1 Simulation of transmission COVID-19 in Mataram

Table 2. Parameter Assumptions for The First Condition

No	Parameter	Parameter value	
		Before vaccine	After vaccination
1	π	4×10^{-5}	4×10^{-5}
2	μ	4×10^{-5}	4×10^{-5}
3	η	0.7	0.7
4	v	0	0.001131
5	ζ	0.023	0.023
6	q	0.4	0.4

This parameter assumption is used for before-vaccination and after-vaccination data to obtain values of parameters $\beta, \gamma,$ and δ. As the initial condition, we use the value $\zeta = 0.023$ and $q = 0.4$.

Table 3. Values of parameters β, γ, and δ for the first condition before and after the vaccine

No	Parameter	Parameter value	
		Before vaccine	After vaccination
1	β	0.42	0.41
2	γ	0.38	0.39
3	δ	0.003	0.002

In Table 3, we show the calculation results (estimates) for the respective values of parameters β, γ, and δ before and after vaccination. From the table, it is evident that all three of these parameters have decreased after vaccination. This indicates that the quarantine process, recovery rate, and death rate have all reduced following vaccination.

For the second condition is assumed as follows.

Table 4. Parameter assumptions for the second condition

No	Parameter	Parameter value	
		Before vaccine	After vaccination
1	π	4×10^{-5}	4×10^{-5}
2	μ	4×10^{-5}	4×10^{-5}
3	η	0.7	0.7
4	v	0	0.001131
5	ζ	0.007	0.007
6	q	0.08	0.08

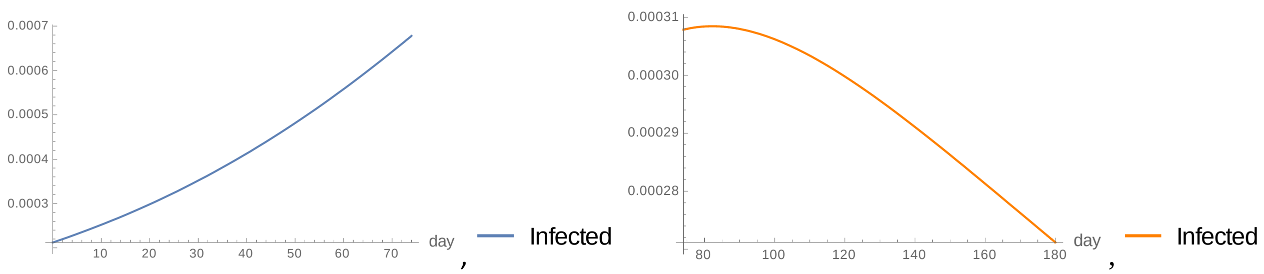
These parameter assumptions were used before vaccination and after-vaccination data to obtain values of parameters β, γ, and δ. In this second condition we setting the initial value $\zeta = 0.007$ and $q = 0.08$.

Table 5. Parameter values β, γ,and δ for the second condition before and after the vaccine

No	Parameter	Parameter value	
		Before vaccine	After vaccination
1	β	0.0977	0.0922
2	γ	0.0596	0.0664
3	δ	0.003	0.002

Based on Table 5, vaccination has an impact on reducing the quarantine rate, increasing the recovery rate, and decreasing the death rate.

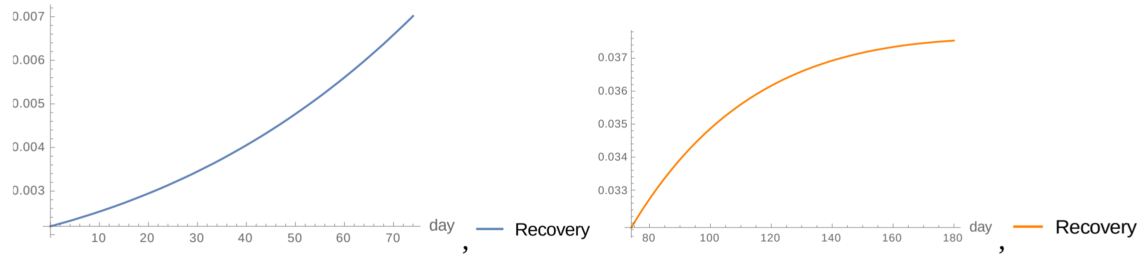
The results of iterating numerical solutions for simulating the spread of COVID-19 using the SIQRD model before and after vaccination are shown in graphic plots. The iteration results for the rate of the infected population (I) and the rate of the recovered (R) population are shown in each graph plot as shown in the following Figure 2.



(a) (b)

Figure 3. Rate of infected cases (I) for the first condition (a) before vaccination, (b) after vaccination

Figure 2**(a)** The graph of the rate of infected cases, which continued to rise before vaccination in Mataram City. However, something different is shown in Figure 2**(b)**, where after the vaccination the graph began to decline to start from day to day.

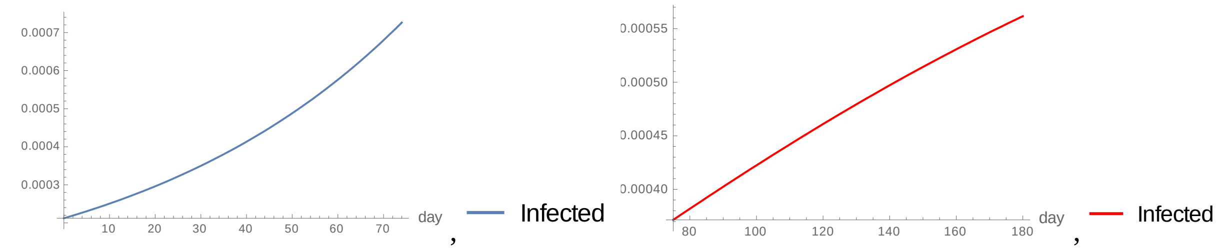


(a) **(b)**

Figure 3. Recovery rate (R) for the first condition (a) before vaccination, (b) after vaccination

From **(a)** and **(b)**, it can be said that before vaccination and after the vaccination, the recovered cases of COVID-19 in Mataram City always increased from day to day.

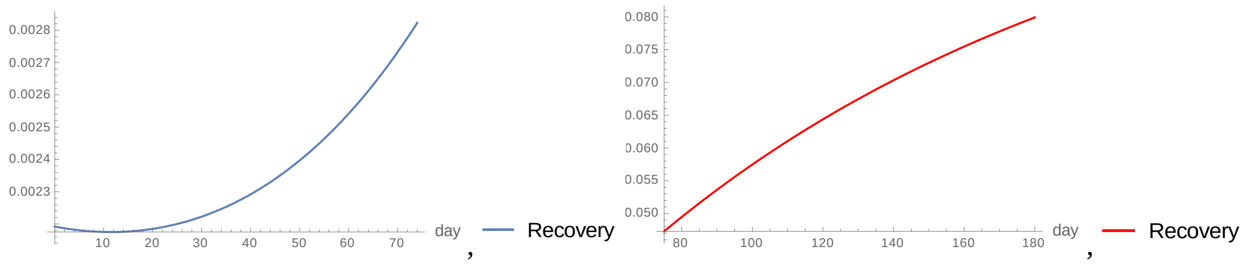
If it is reviewed based on the parameter values for the second condition against the data before the vaccine and after the vaccine, the graph plot is obtained as follows:



(a) **(b)**

Figure 4. Rate of infected cases (I) for the first condition (a) before vaccination, (b) after vaccination

Figure 4 **(a)** and **Figure 4** **(b)** present graphs of infected rate cases for the second condition where both before and after vaccination, the rate continues to increase. However, the increase in infection rates after the vaccine was not as significant as before the vaccine.



(a) **(b)**

Figure 5. Recovery rate (R) for the first condition (a) before vaccination, (b) after vaccination

Based on **Figure 5(a)** and **Figure 5(b)**, it can be said that before vaccination, and after the vaccination the recovered cases of COVID-19 in Mataram City always increased from day to day.

After obtaining a graphic description of the first and second conditions both before and after the vaccination, the next step is to analyze the equilibrium point of the SIQRD model for each condition. Based on **Table 2** and **Table 3**, before the vaccination was obtained, the equilibrium point was obtained:

$$E_1 = (1, 0, 0, 0\,)$$

and

$$E_2 = (0.956,\ \ 0.000455,\ \ 0.000476,\ \ 0.00783).$$

E_1 is the equilibrium point when free of disease, and E_2 is the equilibrium point when endemic. E_1 based interpretation means that COVID-19 will disappear in the city of Mataram when no one is infected, quarantined, or died from COVID-19. Viewed from the value E_2, if there are 455 infected people out of $100{,}000$ people, there are 476 people who are quarantined out of $100{,}000$ people, there are 783 people who have recovered from 1000 people, and there are 956 vulnerable people out of 1000 people in Mataram, then COVID-19 will become endemic in Mataram City. Based on the values E_1 obtained λ for $J(E_1)$ namely:

$$(-0.382628, \quad 0.118563, \quad -0.02304, \quad -0.00004)$$

while λ to $J(E_2)$ get:

$$(-0.38264, -0.0187, -0.004339, -0.000210).$$

Because there is one λ positive value on $J(E_1)$ based on Theorem 1, it can be concluded that the equilibrium point at E_1 is unstable, while on $J(E_2)$ the real part all λ is negative, the equilibrium point at E_2 is stable asymptotically local.

Then, if based on the parameter values in the first condition after the vaccination, the equilibrium point results are obtained, namely:

$$E_1 = (0.967, \quad 0, \quad 0, \quad 0.033)$$

and

$$E_2 = (0.966, \; 0.00001, \; 0.00001, \; 0.033).$$

From the value E_1 obtained λ for $J(E_1)$ namely:

$$(-0.388, \quad -0.0238, \quad 0.00027, \quad -0.00004)$$

while from E_2 is obtained λ for $J(\mathrm{E}_2)$:

$$(-0.388 + 0.i, -0.0238 + 0.i, -0.000059 + 0.000085i, -0.000059 - 0.000085i).$$

Since there is one λ with positive value, so $J(E_1)$ based on Theorem 1, it can be concluded that the equilibrium point at E_1 is unstable, while on $J(E_2)$ the real part λ is negative, so the equilibrium point at E_2 is stable asymptotically local.

Based on the parameter values in the second condition before the vaccine, the equilibrium point results are obtained, namely:

$$E_1 = (1, \quad 0, \quad 0, \quad 0)$$

and

$$E_2 = (0.825, \quad 0.0016, \quad 0.0020, \quad 0.0172)$$

Based on the values E_1obtained λ for$J(E_1)$ namely:

$$(-0.062, \quad 0.01696, \quad -0.007758, \quad 0.000678)$$

whereas λ for $J(E_2)$ obtained:

$$(-0.062 + 0.i, -0.0057 + 0.i, -0.00077 + 0.0005i, -0.00077 - 0.0005i).$$

Because there are two λ positive values on $J(E_1)$, it can be concluded that the equilibrium point at E_1 is unstable, while $J(E_2)$ the real part λ is negative, so the equilibrium point E_2 is stable.

Then when viewed from the parameter value of the second condition after the vaccine, the equilibrium point results are obtained, namely:

$$E_1 = (0.899, \quad 0, \quad 0, \quad 0.101)$$

and

$$E_2 = (0.889, \; 0.00016, \; 0.00019, \; 0.102).$$

From the value E_1obtained λ for $J(E_1)$ namely:

$$(-0.068, -0.0078, 0.000862, -0.00004)$$

while from E_2 is obtained λ for $J(\mathrm{E}_2)$:

$$(-0.068 + 0.i, -0.0077 + 0.i, -0.000107 + 0.00015i, -0.000107 - 0.00015i).$$

It means that the equilibrium point E_1 is unstable since $J(E_2)$ the real part λ is negative, while the equilibrium point E_2 is stable asymptotically local.

Based on the results above, it can be concluded that the equilibrium point E_1 for the first and second conditions before and after the vaccine is unstable, and the equilibrium point E_2 is always stable asymptotically local. The consequences that arise from the equilibrium point at E_2 the stable asymptotically local are those that go to t infinity, the system solution towards the endemic equilibrium point or in other words COVID-19 in Mataram City before the vaccine is available for a long time will still exist in the community.

3.2 Analyze of equilibrium point

After obtaining a graphic description of the first and second conditions both before and after the vaccination, the next step is to analyze the equilibrium point of the SIQRD model for each condition. Based on Table 2 and Table 3 before the vaccination was obtained, the equilibrium point was obtained:

$$E_1 = (1, 0, 0, 0\ )$$

and

$$E_2 = (0.956,\ \ 0.000455,\ \ 0.000476,\ \ 0.00783).$$

E_1 is the equilibrium point when free of disease and E_2 is the equilibrium point when endemic E_1 based interpretation means that COVID-19 will disappear in the city of Mataram when no one is infected, quarantined, or died from COVID-19. Viewed from the value E_2, if there are 455 infected people out of $100{,}000$ people, there are 476 people who are quarantined out of $100{,}000$ people, there are 783 people who have recovered from 1000 people and there are 956 vulnerable people out of 1000 people in Mataram, then COVID-19 will become endemic in Mataram City. Based on the values E_1 obtained λ for $J(E_1)$ namely:

$$(-0.382628, \qquad 0.118563, \qquad -0.02304, \qquad -0.00004)$$

while λ to $J(E_2)$ get:

$$(-0.38264, -0.0187, -0.004339, -0.000210).$$

Because there is one λ positive value on $J(E_1)$ based on **Theorem 1**, it can be concluded that the equilibrium point at E_1 is unstable while on $J(E_2)$ the real part all λ is negative, the equilibrium point at E_2 is stable asymptotically local.

Then if based on the parameter values in the first condition after the vaccination, the equilibrium point results are obtained, namely:

$$E_1 = (0.967, \qquad 0, \qquad 0, \qquad 0.033\ )$$

and

$$E_2 = (0.966,\ \ 0.00001,\ \ 0.00001,\ \ 0.033).$$

From the value E_1 obtained λ for $J(E_1)$ namely:

$$(-0.388, \qquad -0.0238, \qquad 0.00027, \qquad -0.00004)$$

while from E_2 is obtained λ for $J(\mathrm{E}_2)$:

$$(-0.388 + 0.\mathrm{i}, -0.0238 + 0.\mathrm{i}, -0.000059 + 0.000085\mathrm{i}, -0.000059 - 0.000085\mathrm{i}).$$

Since there is one λ positive value, so $J(E_1)$ based on Theorem 1, it can be concluded that the equilibrium point at E_1 is unstable, while on $J(E_2)$ the real part λ is negative, so the equilibrium point at E_2 is stable asymptotically local.

If based on the parameter values in the second condition before the vaccine, the equilibrium point results are obtained, namely:

$$E_1 = (1, \quad 0, \quad 0, \quad 0\,)$$

and

$$E_2 = (0.825, \quad 0.0016, \quad 0.0020, \quad 0.0172)$$

Based on the values E_1obtained λ for$J(E_1)$ namely:

$$(-0.062, \quad 0.01696, \quad -0.007758, \quad 0.000678)$$

whereas λ for $J(E_2)$obtained:

$$(-0.062 + 0.\text{i}, -0.0057 + 0.\text{i}, -0.00077 + 0.0005\text{i}, -0.00077 - 0.0005\text{i}).$$

Because there are two λ positive values on $J(E_1)$, it can be concluded that the equilibrium point at E_1 is unstable, while $J(E_2)$ the real part. λ is negative, so the equilibrium point E_2 is stable.

Then when viewed from the parameter value of the second condition after the vaccine, the equilibrium point results are obtained, namely:

$$E_1 = (0.899, \quad 0, \quad 0, \quad 0.101\,)$$

and

$$E_2 = (0.889,\ 0.00016,\ 0.00019,\ 0.102).$$

From the value E_1obtained λ for $J(E_1)$ namely:

$$(-0.068, -0.0078, 0.000862, -0.00004)$$

while from E_2 is obtained λ for $J(\text{E}_2)$:

$$(-0.068 + 0.\text{i}, -0.0077 + 0.\text{i}, -0.000107 + 0.00015\text{i}, -0.000107 - 0.00015\text{i}).$$

So, it can be concluded that the equilibrium point at E_1 is unstable while $J(E_2)$ the real part λ is negative, so the equilibrium point E_2 is stable asymptotically local.

Based on the results above, it can be concluded that the equilibrium point E_1 for the first and second conditions before and after the vaccine is unstable, and the equilibrium point E_2 is always stable asymptotically local. The consequences that arise from the equilibrium point at E_2 the stable asymptotically local are those that go to t infinity, the system solution towards the endemic equilibrium point or in other words COVID-19 in Mataram City before the vaccine is available for a long time will still exist in the community.

3.3 Analyze The basic reproductive number (R_0)

After analyzing the equilibrium point, then reviewing the value of R_0. To determine the value R_0 in this system, the following formula can be used:

$$\frac{(\,\pi N + \zeta R)}{(\eta v + \mu)S} \tag{5}$$

Based on the above formula, the value R_0 for the first condition before the vaccine is 41.11. For the first condition after the existence of the vaccine, the value R_0 obtained is 1. In the second condition before the vaccine, the value R_0 obtained is 4.87, whereas if using the parameter value for the second condition after the vaccine, the value R_0 is obtained, namely 1. The analysis that can be taken from this condition is that for the first condition and the second condition, it can be stated that before the COVID-19 vaccine will be an epidemic in Mataram City, and after the COVID-19 vaccine is still available in Mataram City, it will not become an epidemic. This means that giving the vaccine affects reducing the rate of infection in Mataram City with the assumptions in the controlled SIQRD model.

4. CONCLUSIONS

The conclusions obtained based on the results and discussion are as follows:

1. The variations of parameter values for the quarantine rate (β) under the first condition, both before and after vaccination, are 0.42 and 0.41, respectively. The recovery rate (γ) before and after vaccination in the same condition are 0.38 and 0.39, respectively, while the death rate (δ) before and after vaccination in this condition are 0.003 and 0.002, respectively. Moreover, under the second condition, the parameter values for β before and after vaccination are 0.0977 and 0.0922, respectively. The values for γ before and after vaccination in this condition are 0.0596 and 0.0664, respectively, and the parameter values for δ before and after vaccination are 0.003 and 0.002, respectively.
2. Before the vaccine and after the vaccine, the equilibrium point for the SIQRD model is stable towards the endemic point. This means that in the long term, COVID-19 in Mataram City will always exist.
3. The value of the basic reproduction number (R_0) against the SIQRD model in Mataram City
    - R_0 before vaccination for the first and second conditions worth more than 1, which means that one infected person can infect more than 1 person who is susceptible to infection so COVID-19 in Mataram City will become an epidemic.
    - R_0 after vaccination for the first and second conditions worth 1, which means if there is an infected person in Mataram City, then that person can infect one other person so that COVID-19 in Mataram City remains but does not become an epidemic, this will happen as long as the assumption is controlled.

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