BAREKENG: Journal of Mathematics and Its Applications
June 2023 Volume 17 Issue 2 Page 0895–0902
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol17iss2pp0895-0902

ADAPTED PRESTON'S CURVE: A PROXY METHOD FOR LONGEVITY RISK ANALYSIS ON INDONESIAN PENSION PLAN

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Article History:

Received: 22th December 2022
Revised: 18th April 2023
Accepted: 20th April 2023

Keywords:

Mortality Model;
Social Security System; Longevity Risk;
Preston's Curve

ABSTRACT

Future lifetime will increase as the standard of living and the health insurance system develop. This increase will have an effect on financial contracts' actuarial present values, particularly the liabilities of pension funds. Longer-lived retirees will have more financial obligations to the pension plan in the future. Preston established a link between GDP and life expectancy at birth, which served as the inspiration for this paper's concept. We strive to advance Preston's work on longevity analysis, particularly how to create a proxy approach for capturing the dynamic of the mortality model with other data. In this case, we utilize the Lee-Carter model to capture the long-term dynamics of the mortality rate, and our GDP-related measure will be based on the model's parameters. We use the Human MortD data to gather the longevity parameter's estimate and fit the relationships using linear, local linear, and kernel regressions. Since the long-term goal of this study is longevity risk management in Indonesia, hence the model's applicability is assessed by how closely it resembles Indonesia's mortality models. We discovered that the linear model, which has an RMSE of 2.19234, has the lowest RMSE, then we conclude that the long-term relationships between longevity parameters and GDP can be explained by the linear model.

How to cite this article:

D. T. Qoyyimi and R. C. Utama., "ADAPTED PRESTON'S CURVE: A PROXY METHOD FOR LONGEVITY RISK ANALYSIS ON INDONESIAN PENSION PLAN," *BAREKENG: J. Math. & App.*, vol. 17, iss. 2, pp. 0895-0902, June, 2023.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article • **Open Access**

1. INTRODUCTION

The National Social Security System Law No. 45/2015 was adopted by the House of Representatives in June 2015. (SJSN). In addition to the pre-existing benefits under the prior law, such as health insurance, protection against work accidents, old age, and death, this law established a required pension program for workers in Indonesia. Additionally, this statute encouraged the establishment of BPJS Ketenagakerjaan and BPJS Kesehatan as SJSN providers. BPJS Kesehatan concentrates on providing health insurance, while BPJS Ketenagakerjaan regulates other benefits, including the mandatory pension program. According to an actuarial analysis of the reform of Indonesian Employment BPJS Ketenagakerjaan conducted by the International Labor Organization (ILO), based on various assumptions, the new pension system, when combined with severance benefits for pensioners, provides a replacement rate of roughly 50% for workers who are 56 years old and have 30 years of service. Their lump sum benefits may also be life annuitized. Only 30% of this is contributed by pension schemes alone [1]. The replacement rate can fall when the mortality rate rises since it is based on the annuitization of benefits paid in a lump sum. As pensioners live longer, their monthly annuity income decreases for fixed lump sum benefits [2]. This fact sheds light on how longevity risk affects participants in retirement plans. As a result, longevity risk, sometimes referred to as the risk of realizing a death rate that differs from the initial calculation, must be considered while paying pensions [3].

The population of Indonesia has a longer average life expectancy, according to data from the World Health Organization (WHO). Life expectancy at birth in Indonesia has increased dramatically in recent years, from just over 41 years in 1960 to more than 68 years in 2006. This raises the issue of how pension funds can be sustainable in the future, as it is anticipated that as infant life expectancy rises, pensioner life expectancy will as well.

Long-term mortality information is required to study lifespan. The rise in survival naturally takes place over a very long period. Short-time period information is unable to capture this improvement. Mortality tables with a lengthy time span are frequently used to observe the dynamics of survival or death rates in the same place over time. Future death rates are then predicted using this information. Indonesia is included in the longevity analysis, however, not all nations have long mortality data [4]. There are currently just four death tables available for Indonesia, notably 1993, 1996, 1999, and 2019 mortality tables [5]. To model mortality and predict long-term mortality, these four tables are not exploited to their full potential. To overcome data limitations, one possibility that can be done is to extract information from existing data, which is easier to obtain. The relationship with mortality is then analyzed to capture the pattern. This pattern was later reproduced for mortality elsewhere. This methodology will be applied in this paper.

Preston [6] shows the relationship between gross domestic product (GDP) and life expectancy. This relationship is represented in a graph that became known as the Preston curve. The Preston curve illustrates that individuals born in wealthier countries will have a longer life expectancy than those who live in poorer countries. Life expectancy is a summary of the mortality rate. This value is not enough to be used in pension fund risk management. Pension funds require a more detailed mortality rate to calculate the life annuity. The work Preston did for mortality models is modified in this publication. In this instance, the mortality model will be described using the Lee-Carter model, and its estimated parameters will be used to look for trends. An indicator of longevity is the annual variation in the death index parameter. The association between this parameter and the nation's GDP for the relevant year is then looked up.

2. RESEARCH METHODS

To clearly show the changes in financial liabilities of pension finance due to changes in population mortality, figures based on Indonesian data are presented below. Using four life tables AAUI [5] for 1993, 1999, 2011, and 2019, retiring at age 55, with $48 million in retirement income per year, and his retirement investment success rate Calculate the financial liability of a person's pension fund, which is 6% of 5% and 4%. In this case, we also compare the impact of interest rates on non-mortality liabilities using three interest rate assumptions. The calculation results are shown in Figure 1 and Figure 2. The two charts clearly show that the technical debt of pension funds has increased significantly over the years.

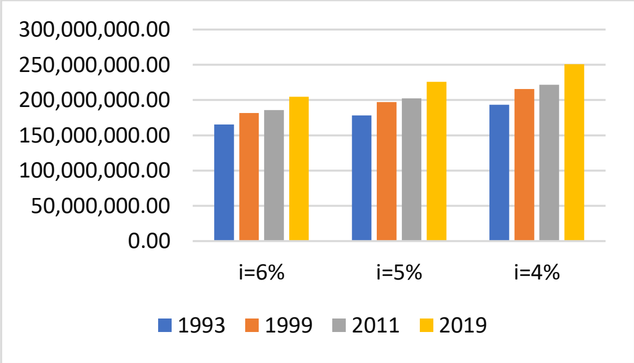

Figure 1. Pension fund's liabilities across mortality tables and interest rates (in millions of rupiah)

Actuarial liabilities will also rise because of the loss in investment performance, which is represented in the assumption of dropping interest rates. It is intended that pension fund managers would use this information in the future to begin taking longevity risk into account when determining their actuarial obligations.

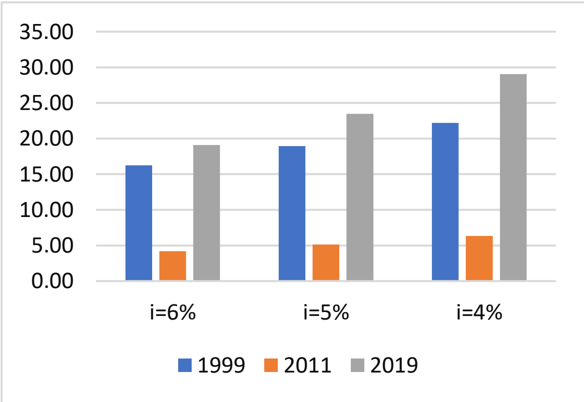

Figure 2. Differences in pension fund's actuarial liability valuations due to changes in mortality

The two examples presented above show how the death rate in the future will have a significant impact on the amount of debt owed to pension funds. Therefore, it is crucial to comprehend the evolution of the death rate shown in the life table. However, as we mentioned in the background, access to data is frequently a significant barrier. A solution to this issue will be provided by this research, which will look for a correlation between life tables and other, more easily obtainable metrics with a strong correlation to death rates, such as gross domestic product. The Lee-Carter model was selected as the mortality model to be employed first, after which we will extract one of the model's characteristics that can capture the impact of time. After that, we assess the link between this metric and GDP. Regression models such as linear, local linear, and kernel regression will be used to elaborate on the relationship between these two values. The year parameter in the model and GDP can be captured in a variety of ways, but for the purposes of this study, we only consider three in order to keep things simple. Using the RMSE as a metric of goodness, we evaluate the model's quality based on how well it performed when applied to data from Indonesia. The model with the least RMSE is selected.

The mortality data for this study were obtained from the Human Mortality Database (www.mortality.org). The research uses all available data in the HMD database. Gross domestic product data is obtained from the world bank website (http://data.worldbank.org/developers/api-overview).

The choice of reference measures is the first step in figuring out the pattern of the link between GDP and mortality. Unlike this study, Preston [6] used the mortality table and calculated the average age at death (life expectancy at birth). Determine the mortality model that will be utilized as a reference for this at the outset of the investigation. The Lee-Carter model was applied in this investigation. After choosing a mortality model, choose the model's reference parameters and look for patterns depending on how they relate to GDP. Researchers will use the established trend to forecast upcoming mortality tables based on GDP forecasts. We incorporate the RMSE (Root Mean Squared Error) as a measure of goodness of our study.

2.1. Lee-Carter Model

A stochastic model of mortality is necessary to capture the dynamics in mortality. Age and time are typically included in the model. The Lee-Carter model is one of the most frequently used mortality models. Lee and Carter [7] suggested a model to integrate trends in death prediction in their landmark study. This model blends time series models with demographic models to estimate death rates. The following relationship describes Lee and Carter's model of central mortality for age x in year t.

$$ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t} \tag{1}$$

where $m_{x,t}$ denotes the central mortality rate for age x at time t. a_x indicates the mean log mortality at age x. b_x measures the change in mortality at age x due to changes in the parameter k_t. k_t denotes the all-cause mortality rate at time t. and $\epsilon_{x,t}$ is the error rate at age x and time t. This model has conditions $\sum_t k_t = 0$ dan $\sum_x b_x = 1$. One of the things that makes this model appealing is how straightforward it is. The model's three parameters offer the flexibility needed to precisely estimate mortality over the long period.

The a_x parameters are estimated by minimizing the error value for a given t. The estimated value of the parameter a_x is $\hat{a}_x = \frac{1}{T}\sum_{t=1}^{T} \ln m_{x,t}$. The parameters b_x and k_t are estimated using the singular value decomposition method [7]. To perform the estimation a Z matrix is formed

$$Z_{x,t} = \ln(m_{x,t}) - a_x = b_x k_t. \tag{2}$$

By using the Singular Value Decomposition (SVD), the matrix Z can be decomposed into

$$USV' = SVD(Z_{x,t}) = S_1 U_{x,1} V_{t,1} + S_2 U_{x,2} V_{t,2} + \cdots + S_r U_{x,r} V_{t,r} = \sum_{i=1}^{r} S_i U_{x,i} V_{x,i} \tag{3}$$

where U is the age ratio x, V is the time ratio t, and S is a single value with $S = diag(\sigma_1, \sigma_2, \dots, \sigma_r)$ and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r$, where $r = 1$ is the rank based on the model constraints of the matrix $Z_{x,t}$ of the Lee-Carter model, the estimated parameters b_x and k_t are obtained

$$\hat{b}_x = \frac{1}{\sum_x u_{x,1}}(u_{1,1}, u_{2,1}, \dots, u_{x,1}) \tag{4}$$

and

$$\hat{k}_t = \sum_x u_{x,1}\sigma_1(v_{1,1}, v_{2,1}, \dots, v_{t,1}). \tag{5}$$

Researchers, actuaries, statisticians, and even economists utilize the Lee-Carter model to generate estimates that suggest how well businesses, financial markets, and even governments might handle the risks associated with mortality and its implications [8]. The Lee-Carter model was created with the goal of simulating and forecasting mortality for a nation or region. It was first used to model mortality in the United States [7], then later in Iran [9], Hungary [10], Malaysia [11], and Indonesia [12]. In addition, as demonstrated by Nantwi et al. [13] and Scognamiglio [14], the Lee-Carter model is also utilized to examine lifespan risk.

2.2. Local Linear Regression

In contrast to regular linear regression, local linear regression is a non-parametric regression. The relationship pattern between the response variable and the predictor variable is estimated using this method. Based on the features of each pair of response variables and predictors, local linear regression parameters are chosen [15].

Given a pair of variables (X_i, Y_i), where $i = 1, 2, \dots, n$, and n is the total number of data points, X_i and Y_i stand for the i-th predictor and i-th response variable, respectively. The following non-parametric regression model describes the relationship between these variables:

$$y_i = \phi(x_i) + e_i, i = 1,2, \dots n. \tag{6}$$

The function $\phi(x_i)$ is called the regression function of unknown form and $e_i \sim N(0, \sigma^2)$ is an error [16]. It is known that X is a predictor variable so that the ϕ function is estimated using the local polynomial regression curve approach with the expansion of the Taylor series. The function $\phi(x_i)$ is approximated by a polynomial of degree p as follows:

$$\phi(x_i) \approx \phi(x) + (x_i - x)\phi^{(1)}(x) + \cdots + \frac{(x_i - x)^{(p)}\phi^{(p)}(x)}{p!}, \tag{7}$$

$$\phi(x_i) \approx \beta_0 + \beta_0(x_i - x) + \cdots + \beta_p(x_i - x)^p = \sum_{k=0}^{p} \beta_k (x_i - x)^k, \tag{8}$$

where $\beta_k = \frac{\phi^{(k)}(x)}{k!}, x_i \in [x - h, x + h]$ and h is the bandwidth. The β_i parameter depends on the x point which is called the local point. These parameters can be estimated using a weighted least square (WLS) by minimizing the function

$$L = \sum_{i=1}^{n} \left(y_i - \sum_{k=0}^{p} \beta_k (x_i - x)^k\right)^2 K\left(\frac{x_i - x}{h}\right). \tag{9}$$

2.3. Kernel Regression

This method, known as kernel regression, estimates conditional expectancies from random variables by performing a local linear regression that only includes local constants. The objective is to use the weight of the kernel function to determine the relationship between two random variables [16]. If a function $K(.)$ is continuous, real-valued, symmetrical, finite, and $\int_{-\infty}^{\infty} K(z)\, dz = 1$, it is referred to as a kernel function. Just local variables are contained in $\phi(x_i)$, hence there is only argument β_0. Therefore, by minimizing the function

$$L = \sum_{i=1}^{n} (y_i - \beta_0)^2 K\left(\frac{x_i - x}{h}\right), \tag{10}$$

we get

$$\beta_0 = \sum_{i=1}^{n} \frac{K\left(\frac{x - x_i}{h}\right)}{\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)} y_i. \tag{11}$$

In this study, the Gaussian kernel function is used, namely

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} z^2\right). \tag{12}$$

3. RESULTS AND DISCUSSION

The algorithm of the approach we propose is as follows, and it aims to identify patterns of the relationship between Gross Domestic Product (GDP) and mortality models in a country:

1. Calculate the mortality rate using the proper technique; in this example, the Lee-Carter Model is used.
2. Establish the parameters that will be linked to proxy data and utilized to determine the mortality model at a specific period (in this case, GDP)
3. Plot the model parameters for mortality and GDP for the specified period and nation.
4. Design a function that most accurately captures the connection between the two.
5. Use RMSE to assess the best relationship's function.

Based on the above steps of the algorithm, we first need to determine the mortality model and associated time parameters. The model used is the Lee-Carter model [7], with the model given by **Equation (1)**, and the parameter considered as the time parameter is k_t. **Figure 3** shows examples of how k_t is estimated for Austria, Canada, Australia, and Chile. Although the four countries show a consistent trend, we can see that the k_t charts of the four countries differ in detail. Canada has experienced a stronger rate of decline since the 2000s, while Austria and Australia started earlier, around the 1970s.

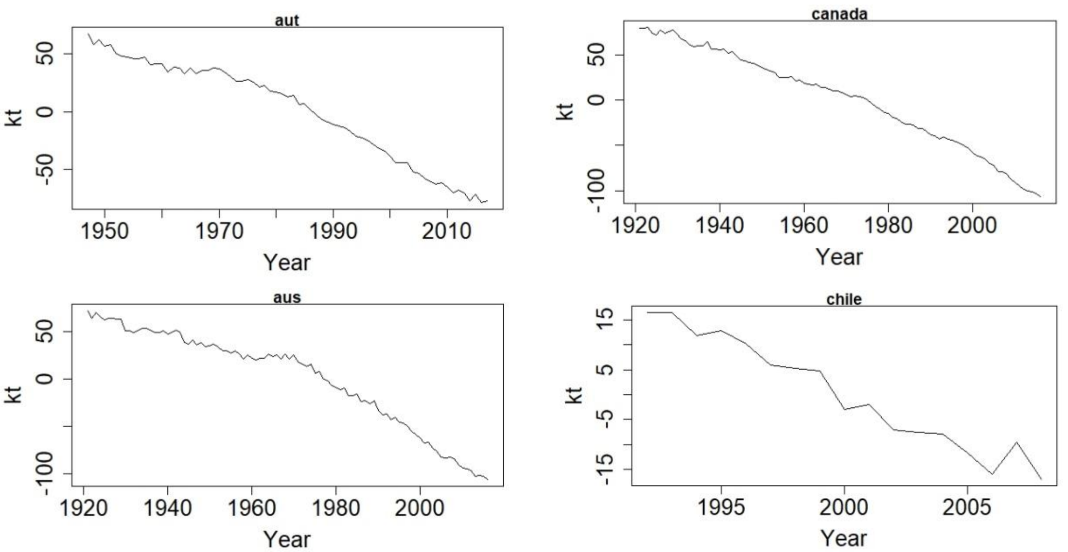

Figure 3. **Parameter estimates of k_t of Lee-Carter model for Austria, Australia, Canada, and Chile**

The Lee-Carter model's parameter k_t, in our opinion, is what controls how mortality varies with time and location. With the aid of data that is simpler to collect, in this case, statistics on gross domestic product, the goal of this study is to identify trends that lead to death rates. Almost all nations have access to GDP data, which is significantly easier to find than mortality rates. Not every country has a mortality rate, particularly not over the long term. Since we are all familiar with statistical methods, the third step in the strategy we propose is to plot and look for the relationship between the gross domestic product. **Figure 5** provides an overview of the relationship between the two in Austria, Chile, and Australia. The associations for all of the nations included in this study are gathered in the lower right corner plot of **Figure 4**.

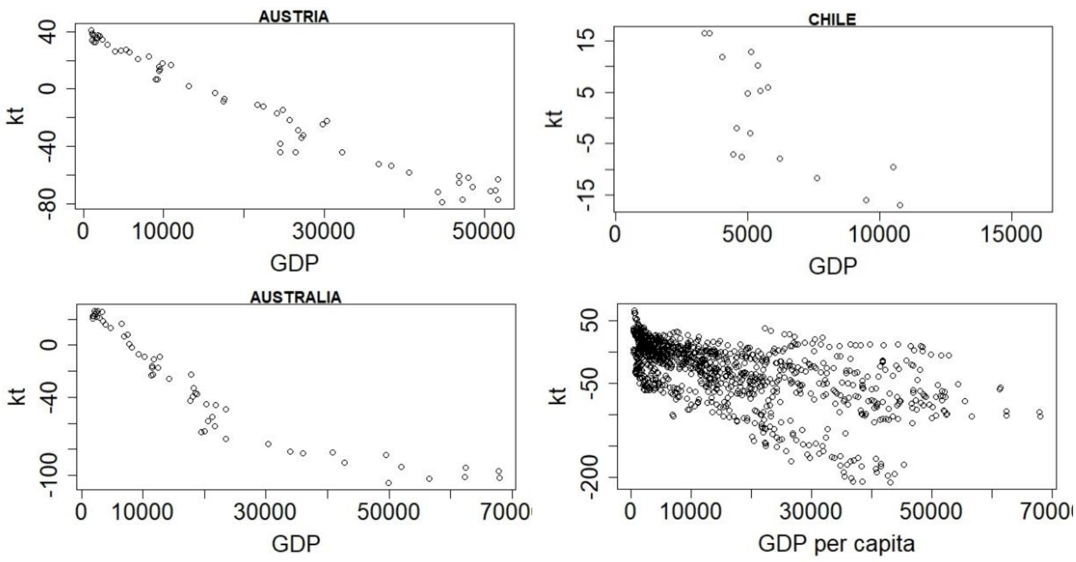

Figure 4. **Plots of PDB vs year's indices k_t of Lee-Carter Model**

The next step is to identify the appropriate pattern based on the data we obtain after graphing GDP versus k_t. We shall attempt the easiest method to complete this problem in this study, employing kernel regression, local linear models, and linear models. The simplest model, the linear one, is used first, followed by a somewhat more sophisticated model, the linear local regression and Kernel regression, which may handle non-linear interactions. Of course, there are numerous possibilities in this regard, but the researcher purposefully chooses a straightforward model because the main point of this work is the concept of using proxy data for longevity analysis rather than elaborating on the link between the two variables. **Figure 5** and **Figure 6** show the plots and their growth curves based on the proposed model. The three suggest that there is much space for improvement in future studies.

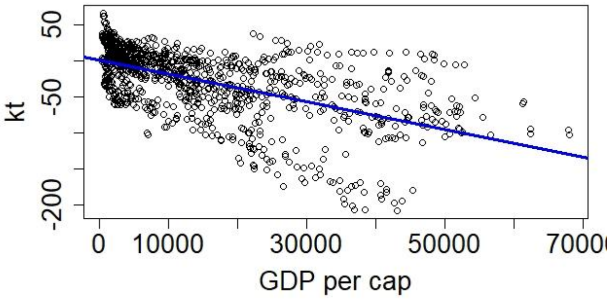

Figure 5. Visual representation of linear model

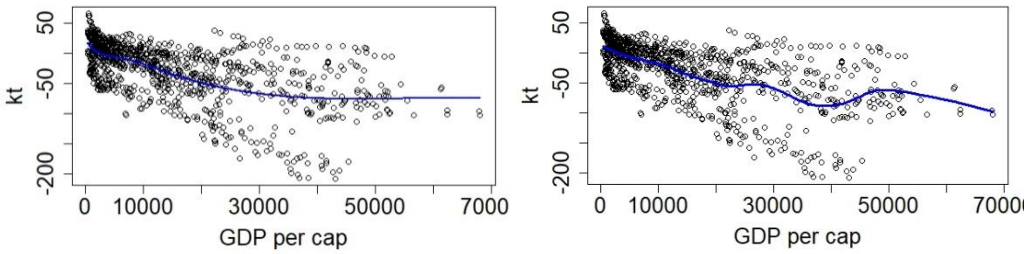

Figure 6. Visual representation of local linear (left) and kernel regressions (right)

The best model is determined by calculating the RMSE of the built model compared to the parameter estimates for the Indonesian data. Lee-Carter parameter estimates for the Indonesian data are from Safitri **[12]**. The model used is the one that gives the smallest RMSE. **Table 1** summarizes the RMSE values for each year. The table shows that the lowest RMSE values are obtained using linear regression. Therefore, in this study, a linear model was used to confirm the relationship between annual parameters of k_t and GDP. This model can then be used in conjunction with forward projections of GDP to project mortality models and determine future pension plan's liabilities.

Tabel 1. Summary of the Root Mean Squared Error (RMSE's) of the model used

	1970	1975	1980	1985	1990	1995	2000	2005	2010	2015	RMSE
Safitri [12]	0.0032	0.0114	-0.0697	0.0104	0.0139	0.0057	-0.029	0.0407	0.0407	0.0639	
Linear	1.5331	1.2347	0.73192	0.6824	0.5498	-0.308	0.1702	-0.765	-4.370	-4.8002	2.19234
Cubic spline	22.372	20.039	16.4335	16.100	15.224	10.174	12.863	7.897	-2.421	-2.994	14.17262
Spline	13.276	12.500	11.1929	11.064	10.719	8.5099	9.7368	7.363	-0.614	-1.418	9.583479

4. CONCLUSIONS

Longevity risk analysis in a country like Indonesia with very limited mortality data is very challenging. Establishing a method using proxy data based on the relationship between GDP data and longevity risk parameters is an alternative to overcome the problem of limited data. The Preston curve can be adapted for this purpose. Using the Lee-Carter model to model mortality and gross domestic product is one way to get there.

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