

APPLICATION OF THE COPULA METHOD TO ANALYZE THE RELATIONSHIPS OF MACROECONOMIC FACTORS AFFECTING THE CSPI

Sri Endang Saleh^{1*}, Debyyansa Pakaya², Isran K. Hasan³, Ismail Djakaria⁴

¹Faculty of Economics, State University of Gorontalo

^{2,3,4}Department of Mathematics, Faculty of Mathematics and Natural Sciences, State University of Gorontalo
Prof. Dr. ing. BJ Habibie Street, Tilongkabila, Bone Bolango Regency 96554, Gorontalo, Indonesia

Corresponding author's e-mail: * sriendang_s@ung.ac.id

ABSTRACT

Article History:

Received: 27th December 2022

Revised: 18th April 2023

Accepted: 21st April 2023

Keywords:

Archimedean Copula; Ellipse Copula;

Composite Stock Price Index;

Inflation;

Interest Rate;

Exchange Rate.

The Composite Stock Price Index (CSPI) is a valuable number in assessing the performance of the stocks listed on the stock exchange; by looking at the Composite Stock Price Index, investors can determine their investment strategy. However, the rise and fall of the Composite Stock Price Index depend on a country's macroeconomic conditions; if the economy weakens, the company's performance will also undermine investors' confidence, and confidence decreases. Analyzing the relationship between the Composite Stock Price Index with macroeconomic factors can show how much the influence of these factors on the increase or decrease in the Composite Stock Price Index; the macroeconomic factors in question are inflation, interest rates, and the rupiah exchange rate. In this study, dependency analysis was carried out with the Copula approach method involving the Tau Kendal method for parameter estimation and the Maximum Likelihood Estimation (MLE) method to choose the best Copula model to explain the relationship between the Composite Stock Price Index and these macroeconomic factors. Research results in it are found that the best Copula that can explain the dependency structure between the Composite Stock Price, The index with inflation and interest rates is the Gumbel Copula with parameters $\theta \hat{=} 1.264$ and $\theta \hat{=} 1.174$, While the Copula model is the best that can explain the structure of the dependency between Composite Stock Price Index and the exchange rate is Copula Student-t with parameter $\theta \hat{=} -0.6037$.



This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution-ShareAlike 4.0 International License.

How to cite this article:

S. E. Saleh, D. Pakaya, I. K. Hasan and I. Djakaria, "APPLICATION OF THE COPULA METHOD TO ANALYZE THE RELATIONSHIPS OF MACROECONOMIC FACTORS AFFECTING THE CSPI," *BAREKENG: J. Math. & App.*, vol. 17, iss. 2, pp. 0903-0912, June, 2023.

Copyright © 2023 Author(s)

Journal homepage: <https://ojs3.unpatti.ac.id/index.php/barekeng/>

Journal e-mail: barekeng.math@yahoo.com; barekeng_journal@mail.unpatti.ac.id

Research Article • Open Access

1. INTRODUCTION

The Composite Stock Price Index (CSPI) is used to measure the performance of stocks combined in the capital market, allowing investors to look at market conditions and determine the investment strategy [1]. For the national economy, the rise and fall of the CSPI can describe macroeconomic conditions, which, if economic conditions weaken, can impact company performance in general so that business also cuts, eventually impacting company profits which decline. This also; this investor confidence to decrease.

The factors that cause the rise and fall of the CSPI are macroeconomic factors in the form of exchange or exchange rates, interest rates, inflation rates, and the number of regulations or economic deregulation. In addition, [2] describe several factors that affect the CSPI of the Indonesia Stock Exchange. Inflation is a wrong signal for investors, indicating that company earnings are declining due to weak purchasing power. The same thing can also happen if there is a change in interest rates, where when the economic situation worsens, the Central Bank will reduce interest rates to boost the economy because if there is a decline in the economy, this can weaken investor confidence so that the CSPI will also experience a fall. The same will also be experienced if the rupiah exchange rate continues to change; this will affect the company's balance sheet and impact price adjustments in the capital market. The exchange rate decline could reflect weaknesses, especially in exports and international finance [3].

Based on this, it is necessary to examine how much the above factors can affect the rise and fall of the CSPI. Bearing in mind that the CSPI can describe the economic condition of a country. In statistics, correlation analysis can be used to analyze the influence or relationship between variables. Correlation analysis is one of the methods used to detect relationships between associations that assume that the data is assumedly distributed [4] and [5]. However, sometimes this assumption cannot be fulfilled properly. Therefore is needed to overcome this so that the Copula can be an alternative.

The Copula is a method with flexibility because it does not have a data normality assumption requirement, so this method is considered appropriate for use in cases in the financial sector where in general, it is often found that the data has an abnormal distribution. Besides that, the Copula can also combine several marginal distributions into a joint distribution [6] and can also be collaborated with the empirical Copula [7]. Copula has several classes or families. The most popular Copula families are the Archimedean Copula family and the Elliptical Copula family. Copula Archimedean has three Copula classes: Clayton Copula, Gumbel Copula, and Frank Copula. Whereas Elliptical Copula has two classes, namely Copula Normal and Copula Student- t [8].

The Copula technique has been widely used for financial and actuarial applications. Cases in the financial sector are generally found to have data that is not normally distributed. Research that uses Copula in the financial industry includes [9], [10] which analyses the relationship between macroeconomic factors and the world gold price level. This study used the Copula Archimedean approach, in which the parameters were estimated using Tau Kendall. Then research by [10] has also researched to estimate claims reserves for several lines of business using the Archimedean Copula and the Generalized Linear Model. Apart from that, another study using the Ellips Copula Family is researched by [12] in calculating the value of risk in a stock portfolio.

The difference between this research and the research above is that this study uses two Copula families, namely Archimedean Copula and Elliptical Copula, in its analysis. So the purpose of this study is to apply the Copula approach in identifying the relationship of macroeconomic factors in the form of inflation, interest rates, and exchange rates to the Stock Price Index (CSPI) and to choose the best Copula model using the Maximum Likelihood Estimation (MLE) method by previously estimating parameters with Tau Kendall as part of the procedure.

2. RESEARCH METHODS

2.1 Research Variables

The variables used in this research are independent variables and dependent variables. The dependent variable is CSPI (Y) and the independent variables are macroeconomic factors in the form of inflation (X_1), interest rates (X_2) and exchange rates (X_3).

2.2 Data Sources

This study uses secondary data obtained from the official website of the Indonesia Stock Exchange for CSPI data, while inflation and interest rate data are obtained from the Central Statistics Agency (BPS) website via www.bps.go.id, and data on the Rupiah exchange rate against the USD obtained through the website satudata.kemendag.co.id. The data used is monthly data from 2018 to 2021.

2.3 Stages of Data Analysis

The stages in analyzing the data are as follows.

- 1) Describe the data to obtain an overview based on the descriptive statistics of the research data.
- 2) Testing the normality of the data to see whether the data distribution is normal. If any of the var are not normal, then the analysis is continued to Copula analysis with all normal variables still used.
- 3) Calculate the Tau Kendall correlation coefficient used in the Copula parameter estimation.
- 4) Identify the relationship between variables with Copula.
 - Transform random variables into a uniform distribution [0,1] using the Van der Waerden equation.
 - Make a *Scatterplot* of the transformation results.
 - Estimating Copula Archimedean and Copula Ellips parameters using *Tau* Kendall.
 - Selecting the best Copula using the *Maximum Likelihood Estimation* (MLE) method based on the largest *log-likelihood value*.
 - Make the selected Copula Scatterplot by first generating $n=5000$ data based on the selected Copula parameters so that the shape and tail of the copula can be seen clearly.
- 5) Create a Selected Copula Model based on the MLE fitting result Parameters.

2.4 Copula Family

Copula has two families, namely the archimedean Copula family and the elliptical Copula family. The Archimedean Copula family has a distribution with tail dependencies which indicates the possibility of a relationship between the two variables in a range of extreme regions. Archimedean Copula is divided into three classes, namely Copula Frank, Copula Clayton, and Copula Gumbel, each of which has a different tail. Copula Frank, who has no tail, Copula Clayton, who has a tail at the bottom, and Copula Gumbel, who has a tail at the top.

The Copula archimedean family generator can be presented in **Table 1** below [13].

Table 1. Generators of the Archimedean Copula Family

Archimedean Copula	Generator
Frank Copula	$\hat{\tau} = 1 - \frac{4((1 - D_1(\theta_F))}{\theta_F}$
Clayton Copula	$\hat{\theta}_c = \frac{2\tau}{1 - \tau}$
Gumbel Copula	$\hat{\theta}_G = \frac{1}{1 - \tau}$

The Archimedean Copula generator is obtained from **Equation (6)**. Meanwhile for the Elliptical Copula Generator it can be done using the following Equation [10].

$$\rho = \sin \frac{\pi\tau}{2} \quad (1)$$

2.5 Uniform Distribution [0.1]

In the Copula analysis the initial stage is to change the random variable to form a uniform [0.1], If the marginal distribution of the random variable x is unknown, then the equation becomes:

$$F_X(x) = \frac{1}{n+1} \sum_{j=1}^n 1(X^{(j)} \leq x); \quad x \in \mathbb{R} \quad (2)$$

In the process of transforming these random variables, it is necessary to rank each random variable, namely $R_1^{(j)}, R_2^{(j)}, \dots, R_p^{(j)}$ which is the rank of X_1, X_2, \dots, X_p where previously formed matrices each divided by $n + 1$, for example as follows.

$$\left(\left(\frac{R_1^{(j)}}{n+1} \right) \right), \left(\left(\frac{R_2^{(j)}}{n+1} \right) \right), \dots, \left(\left(\frac{R_n^{(j)}}{n+1} \right) \right) \quad (3)$$

With this transformation, the Copula equation is obtained as follows:

$$C(\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_p) = \frac{1}{n} \sum_{j=1}^n I \left(\frac{R_1^{(j)}}{n+1} \leq \mathbf{u}_1, \frac{R_2^{(j)}}{n+1} \leq \mathbf{u}_2, \dots, \frac{R_n^{(j)}}{n+1} \leq \mathbf{u}_p \right) \quad (4)$$

with I = indicator function if each $X^j \leq x$ dan $\frac{R_i^{(j)}}{n+1} \leq u_i ; i = 1, 2, \dots, p$ [14]. Variable transformations are performed using the Van der Waerden transformation with the following equation.

$$T = \frac{r}{n+1} \quad (5)$$

with r is the rank and n is the amount of data [15].

2.6 Copula Parameter Estimation and Model Selection

The Tau Kendall correlation value can be used to estimate the parameters of Copula. The parameter estimation procedure uses the following equation [14].

$$\hat{\tau} = 1 + 4 \int_0^1 \frac{\phi(u)}{\phi'(u)} du \quad (6)$$

Furthermore, the Copula parameter estimator as well as the selection of the best copula model can use MLE. By describing the parameters given by Copula and the marginal distribution, the MLE will be obtained by maximizing the log function of the possible f density of d dimensions f with univariate marginal F_1, F_2, \dots, F_p and univariate density which f_1, f_2, \dots, f_p can be written as follows:

$$f(\mathbf{x}_1, \dots, \mathbf{x}_n) = c(F_1(\mathbf{x}_1), \dots, F_n(\mathbf{x}_n)) \prod_{i=1}^n f_i(\mathbf{x}_i) \quad (7)$$

with $c(\mathbf{u}_1, \dots, \mathbf{u}_n) = \frac{\partial C(\mathbf{u}_1, \dots, \mathbf{u}_n)}{\partial u_1 \dots \partial u_n}$ is the density of the copula dimensions n of $C = (u_1, \dots, u_p, \theta)$ [16].

3. RESULTS AND DISCUSSION

3.1 Description of Variable

Obtain an overview of all the variables in the study. Description of variables from the research data are presented in the following table.

Table 2. Description of Variable

Variable	N	Minimum	Maximum	Average	Deviation Standard
CSPI (Y)	48	4538	6605	5946	537.7304
Inflation (X_1)	48	-0.27	0.68	0.1948	0.2250578
Interest Rate (X_2)	48	3.5	6.0	4,625	0.9311398
Exchange Rate (X_3)	48	13413	16367	14340	470.1319

Based on **Table 2**. The descriptive statistics of each variable used in the study show $N = 48$, which means that the research data for each variable is 48. The value in the variable Y indicates the level of the CSPI, while the value in the variable shows X_1 the X_2 percentage from inflation and interest rates, and the value in the variable X_3 shows the price of the Rupiah exchange rate against the USD.

3.2 Normality Testing

Data normality testing was carried out to test the distribution of research data meets the normal distribution or not. For the case of the Copula approach method, it does not require normally distributed data. If all variables or some variables are not normally distributed, then Copula is the right method to use. Below is presented the p-value of normality test results with the Shapiro-Wilk test.

Table 3. Shapiro-Wilks test

Variable	<i>p</i> - values
<i>Y</i>	0.0000
<i>X</i> ₁	0.3891
<i>X</i> ₂	0.0001
<i>X</i> ₃	0.0001

Table 3 shows that the variable *X*₁ with a significant level *p*-value (0.3981) > 0.05 fulfills the normal distribution. While the other variables do not meet the normal distribution. The above causes the correlation cannot be used properly because most of the variables do not follow a normal distribution. So that the analysis of the dependency of the CSPI on macroeconomic factors in the form of inflation, interest rates, and exchange rates can be carried out using the Copula approach.

3.3 Tau Kendall Correlation Coefficient

The value of the correlation coefficient is useful for estimating Copula parameters. The largest positive correlation coefficient is 1, and the largest negative correlation coefficient is -1, while the smallest is 0. The relationship between variables is said to be perfect if the correlation coefficient is 1 or -1 because the smaller the correlation coefficient, the greater the error. The results of the analysis of the coefficients are as follows.

Table 4. Results of the Tau Kendall Correlation Coefficient

Variable	Tau - Kendall	
	Coefficient Correlation	<i>p</i> - values
<i>Y</i> and <i>X</i> ₁	0.1904 4	0.0581
<i>Y</i> and <i>X</i> ₂	0.14341	0.1691
<i>Y</i> and <i>X</i> ₃	-0.375 9	0.0001

Table 4 shows the *p*-value of the three correlation coefficients. It can be seen that the CSPI variable and the exchange rate have a significant (*p*-value < 0.05), meaning that the variables have a close relationship. While the CSPI variable with inflation and interest rates is not significant (*p*-value > 0.05), meaning that the two variables have a weak close relationship.

The Tau Kendall correlation coefficient obtained in **Table 4** is used to estimate the Archimedean Copula and Elliptical Copula parameters.

3.4 Copula Parameter Estimation

1. Transformation of Random Variables to *Uniform Distribution* [0,1]

In the analysis with Copula, the variable is transformed to a uniform marginal distribution [0,1]. The following shows the scatter plot of the transformed data in the form of a plot of the relationship pattern of variable *Y* (CSPI) with each variable *X* (inflation, interest rates, and exchange rates).

Based on **Figure 1**, a scatterplot of Transformation Results to Uniform Distribution [0.1], it can be seen that the spread of data between *Y* and *X*₁ dots has a spread but is one pattern in the form of a straight line. This means that the relationship between the two variables is quite low. Meanwhile, for the distribution of data between *Y* and *X*₂, there are more patterns in the form of lines. This means that there is a relationship between variables. Meanwhile, the distribution of data between *Y* and *X*₃ has a negative relationship because when the variable *X*₃ increases, it is followed by the variable *Y* decreasing. To identify the relationship further, the Copula method is used.

2. Estimation of Archimedean Copula and Elliptical Copula Parameters Using Kendall 's *Tau*

Based on **Table 5**, if the parameter estimate is positive, it means that there is an indication of a positive relationship between the CSPI variables and macroeconomic factors, whereas if the parameter estimate is negative, there is an indication of a negative relationship between the variables. The positive relationship

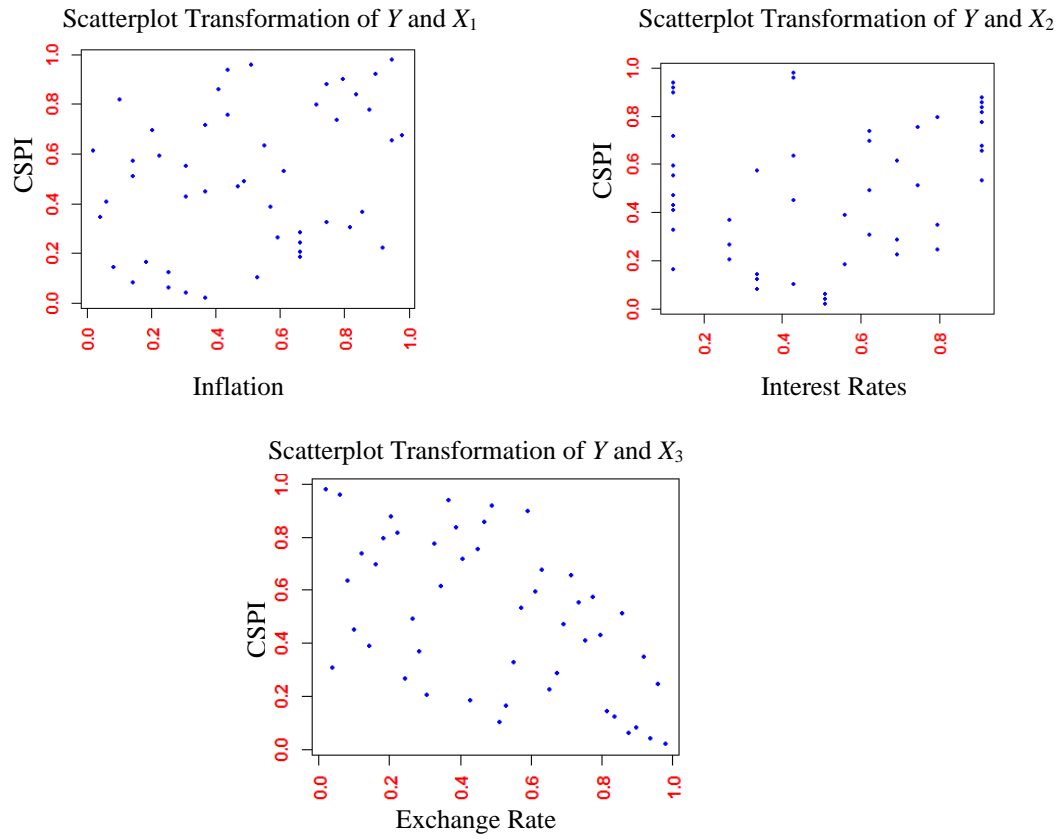


Figure 1. Scatterplot of Transformation Results to *Unifrom Distribution* [0.1]

in question is that variable X is only related to variable Y when variable X is very high. Furthermore, to obtain the best Copula model that can explain the dependency structure between CSPI variables and macroeconomic factors.

Table 5. Estimation of Copula Parameters with the Tau Kendall Approach

Variable	Copula type	Estimation Parameter
Y and X₁	Gumbel	1235
	Clayton	0.4705
	Franks	1,766
	Normal	0.2947
	t	0.2947
Y and X₂	Gumbel	1.167
	Clayton	0.3348
	Franks	1,313
	Normal	0.2234
	t	0.2234
Y and X₃	Gumbel	1
	Clayton	-0.5464
	Franks	-3,838
	Normal	-0.5567
	t	-0.5567

3. Selection of the best Copula using the *Maximum Likelihood Estimation* (MLE) Method

The best copula that can explain the best dependency structure is chosen based on the largest log-likelihood value using the maximum likelihood ratio (MLE) method. Calculations for each variable with Copula types are presented in the following table.

Table 6 . Parameter estimation results and Log likelihood values with MLE

Variable	Copula type	Parameter($\hat{\theta}$)	Logs likelihood
Y and X ₁	Gumbel	1,264	2,509
	Clayton	-	-
	Franks	1,749	1941
	Normal	0.3 5	2,399
	Q	0.35	2,399
Y and X ₂	Gumbel	1.175	0.8959
	Clayton	-	-
	Franks	1.14	0.8106
	Normal	0.1916	0.5574
	Q	0.1916	0.5573
Y and X ₃	Gumbel	1	-4.724e-07
	Clayton	-	-
	Franks	-3.73	7,573
	Normal	-0.6276	10.21
	t	-0.6037	10.25

Based on **Table 6**, the dependency model between the CSPI and Inflation variables follows the Gumbel Copula because it has the largest log-likelihood value compared to other Copulas with a parameter of $\hat{\theta}=1.264$. Likewise, the dependency model between the CSPI variable and interest rates follows the Gumbel Copula with parameter $\hat{\theta}=1.175$. While the CSPI and exchange rate variables follow Copula Student-t with parameters $\hat{\theta}=-0.6037$. Next, make a plot to see the pattern of relationships between variables. In this case, because the amount of data is only $n=48$, it is generated as many as $n=5000$ points so that the pattern of relationships between variables with Copula types can be seen clearly. The dependency plots and selected Copula types based on parameters between variables can be shown in **Figure 2**.

Based on **Figure 2**, it can be seen the pattern of each selected Copula. It is known that the pattern formed by Copula Gumbel has a tail at the top, while Copula Student-t forms an ellipse with a tail above and below.

The Gumbel Copula is part of the Archimedean Copula, which has a dependency tail at the top, which means that when the explanatory variable is very high, it will have a relationship with the response variable. When the explanatory variable is low, the relationship between the two is low or has no relationship. So it can be concluded that inflation only has a dependency or relationship with the Composite Stock Price Index (CSPI) when inflation is very high, but when inflation is low or very low, then the relationship with the CSPI is very low or has no relationship.

The same thing also happens if the interest rate set by Bank Indonesia is very high, then the interest rate will have a relationship with the Composite Stock Price Index (CSPI), otherwise, if the interest rate set by Bank Indonesia is low, then the interest rate has no relationship with the Index-Joint stock price. While the relationship between the CSPI and the exchange rate follows the Copula student-t, where this Copula is part of the Ellips Copula, which has a tail at the top and bottom, this means that the exchange rate can have a close relationship with the Composite Stock Price Index (CSPI) when the exchange rate conditions very high or very low.

3.5 Selected Copula Models

- a. Gumbel Copula model between CSPI and Inflation variables with parameters $\hat{\theta}=1.264$

$$C_{\theta}^{Gu}(u_1, u_2) = \exp\left(-\left[\left(-\ln u_1\right)^{\theta} + \left(-\ln u_2\right)^{\theta}\right]^{\frac{1}{\theta}}\right)$$

$$= \exp\left(-\left[\left(-\ln u_1\right)^{1.264} + \left(-\ln u_2\right)^{1.264}\right]^{\frac{1}{1.264}}\right)$$

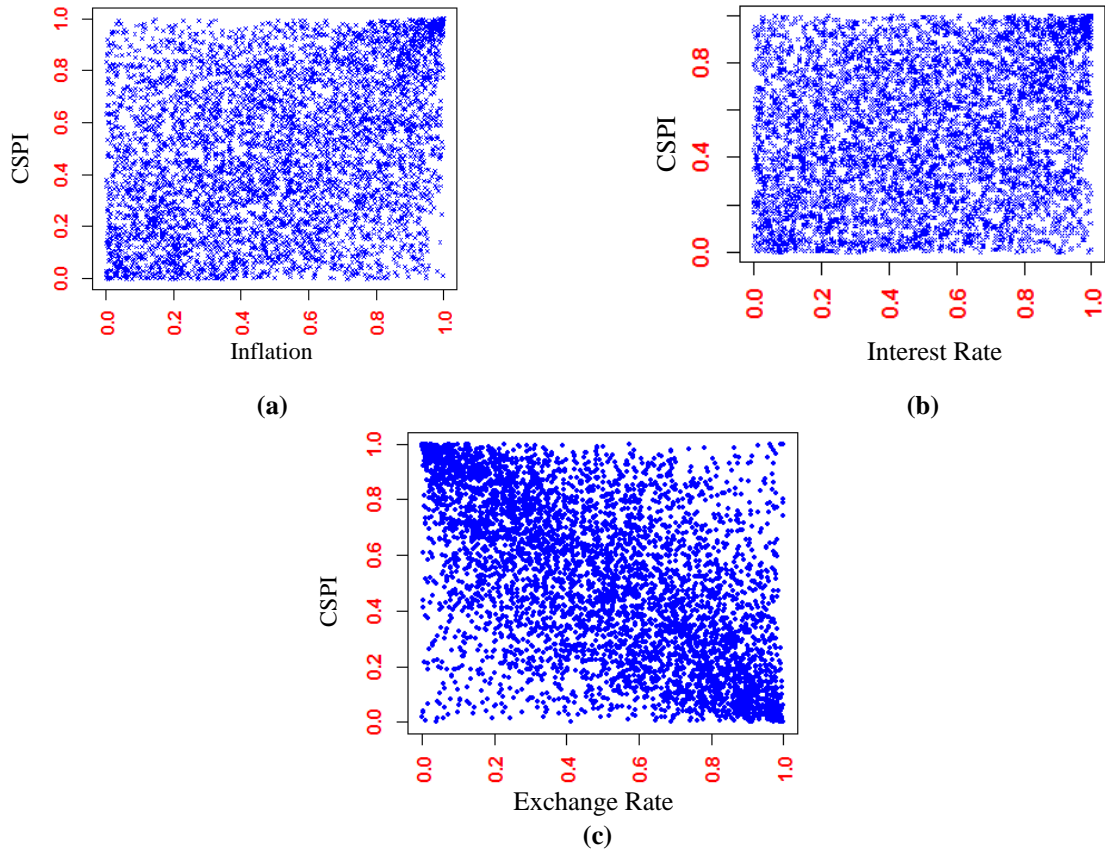


Figure 2. Scatterplot of the best Copula model with $N = 5000$ generation data with selected parameters: (a) Copula Gumbel; (b) Copul Gumbel; and (c) Copula Student- t

- b. Gumbel Copula model between CSPI and interest rates flower with parameter $\hat{\theta} = 1.175$.

$$C_{\theta}^{Gu}(u_1, u_2) = \exp\left(-\left[\left(-\ln u_1\right)^{\theta} + \left(-\ln u_2\right)^{\theta}\right]^{\frac{1}{\theta}}\right)$$

$$= \exp\left(-\left[\left(-\ln u_1\right)^{1.175} + \left(-\ln u_2\right)^{1.175}\right]^{\frac{1}{1.175}}\right)$$

- c. Copula *Student-t* model Among CSPI variables and values swap with parameters $\hat{\theta} = -0.6037$ and degrees of freedom $\nu = 7.0103$.

$$C_{\theta}(u_1, u_2, \dots, u_m) = t_{\nu, \Sigma}\left(t_{\nu}^{-1}(u_1), \dots, t_{\nu}^{-1}(u_m)\right)$$

$$= t_{7.0103, \Sigma}\left(t_{7.0103}^{-1}(u_1), \dots, t_{7.0103}^{-1}(u_m)\right)$$

4. CONCLUSIONS

Based on the results of the analysis that has been done, it can be concluded that analysis of the relationship between the CSPI and macroeconomic factors in the form of inflation, interest rates, and the rupiah exchange rates using the Copula approach found that the CSPI and inflation and interest rates only have a relationship when inflation and interest rates are very high, while when both are low, they have no relationship with CSPI or can not affect the CSPI. While the exchange rate has a relationship with the CSPI both when the exchange rate is very high or very low. Best Copula models:

$$C_{\theta}^{Gu}(u_1, u_2) = \exp\left(-\left[\left(-\ln u_1\right)^{1.264} + \left(-\ln u_2\right)^{1.264}\right]^{\frac{1}{1.264}}\right)$$

$$C_{\theta}^{Gu}(u_1, u_2) = \exp\left(-\left[\left(-\ln u_1\right)^{1.175} + \left(-\ln u_2\right)^{1.175}\right]^{\frac{1}{1.175}}\right)$$

$$C_{\theta}(u_1, u_2, \dots, u_m) = t_{7.0103, \Sigma}\left(t_{7.0103}^{-1}(u_1), \dots, t_{7.0103}^{-1}(u_m)\right)$$

REFERENCES

- [1] Sunariyah, *Introduction to Capital Market Knowledge*, Third. Yogyakarta: UPP AMPYKPN, 2013.
- [2] P. Nano, and Putra, B. A, "The Factors Affecting the Composite Stock Price Index (CSPI) of Indonesia Stock Exchange", *Jurnal Ekonomi & Studi Pembangunan*, vol.21, no. 1, 2020, pp. 69-83
- [3] B. Setianto, *Fair Value Stock Investment (Value Investing)*. Jakarta: Bumisaka Kurnia, 2014.
- [4] P. Embrechts, "Copulas: A Persolan View," *J. Risk Insur.*, vol. 76, no. 3, pp. 639–650, 2009.
- [5] J. Segers, "Asymptotics of empirical copula processes under non-restrictive smoothness assumptions", *Bernoulli*, vol 18, no. 3, 2012, 764–782
- [6] OW Ningrum, Tarno, and DAI Maruddani, "Calculation of Value At Risk in Stock Portfolios Using Copula," *J. Gaussian*, vol. 6, no. 2, pp. 231–240, 2017, [Online]. Available: <https://ejournal3.undip.ac.id/index.php/gaussian/article/view/16952/16249>
- [7] J. Segers, M. Sibuya, & H. Tsukahara, "The empirical beta copula", *Journal of Multivariate Analysis*, vol. 155, 2017, pp. 35–51
- [8] J. Chvosta, DJ Erdman, and M. Little, "Modeling Financial Risk Factor Correlation with the COPULA," *Stat. Anal Data.*, pp. 1–14, 2011.
- [9] S. Wati, M. Hadijati, and N. Fitriyani, "Dependency Analysis of Macroeconomic Factors on the Gold Price Level," *Eig. Math. J.*, vol. 2, no. 2, 2019.
- [10] K. Valentovičová, "Claim Reserving With Copulae For Multiple Line of Business." Master Thesis. Charles University in Prague, 2015.
- [11] Y. Wilandari, SH Kartiko, and AR Effendie, "Estimation of Reserve Claims Using Generalized Linear Model (GLM) and Copula," *J. Gaussian*, vol. 9, no. 4, pp. 411–420, 2020.
- [12] QR Sartika, T. Widiharih, and MA Mukid, "Value at Risk in Stock Portfolio Using t-Copula: Case Study of PT. Indofood Makmur, Tbk. and Bank Mandiri (Persero), Tbk.," *J. Media Stat.*, vol. 12, no. 2, pp. 175–187, 2019.
- [13] Q. Zhang, V. Singh, J. Li, F. Jiang, and Y. Bai, "Spatio-temporal Variations of Precipitation Extremes in Xinjiang, China," *J. Hydrology*, vol. 434–435, pp. 7–18, 2011.
- [14] C. Genest and J. Neslehova, *Copulas: Introduction to the Theory and Implementation in R with Applications in Finance and Insurance*. Canada: University Laval and McGill University, 2010
- [15] Pevšta, M. dan Okhrin, O. 2014. Conditional Least Squares and Copulae in Claims Reserving for a Single Line of Business. *Journal of Insurance: Mathematics and Economics*. Volume 56, 2014, pp. 28–37.
- [16] C. Barbara, I. Rustam, and P. Elena, "Copula Estimation," in *In Copula Theory and Its Application*, Lecture Notes in Statistics, 2011, pp 93-109

