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COMPARISON OF WEIGHTED MARKOV CHAIN AND FUZZY TIME SERIES-MARKOV CHAIN METHODS IN AIR TEMPERATURE PREDICTION IN BANDA ACEH CITY

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ABSTRACT

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Air temperature prediction is needed for various needs, such as helping plan daily activities, agricultural planning, and disaster prevention. In this research, the Weighted Markov Chain (WMC) method and the Fuzzy Time Series-Markov Chain (FTS-MC) method are applied to predict the weekly air temperature in Banda Aceh City. The purpose of this study is to find out the results of the application and comparison of the accuracy of the WMC method and the FTS-MC method on weekly air temperature prediction in Banda Aceh City. The prediction result of air temperature in Banda Aceh city using the WMC method for the next three weeks obtained an air temperature of 26.5℃. The prediction results of air temperature in Banda Aceh city using the FTS-MC method for the next three weeks obtained predicted values of 26.66℃ for the 105th week, 26.79℃ for the 106th week, and 2.,83℃ for the 107th week. The MAPE accuracy level of the WMC method is 1.5% and the FTS-MC method is 1.7%. This shows that the MAPE of the WMC method is smaller than the FTS-MC method so it can be concluded that air temperature prediction using the WMC method is better than the FTS-MC method.

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1. INTRODUCTION

Changes in climate and weather recently are very extreme and difficult to predict. In an area or city, there may be continuous rain accompanied by strong winds and flooding. Meanwhile, in other areas, there was a prolonged drought, which dried up rice fields and water sources. Climate change occurs due to changes in climate variables, such as air temperature and rainfall, that occur continuously over a long period of time.

Banda Aceh, the capital of Aceh province, has undergone rapid urban development and a significant increase in its population. Over the past three decades, the residents of Banda Aceh have consistently experienced thermal discomfort, a condition most commonly felt due to elevated temperatures. This discomfort is primarily attributed to the rising air temperatures, particularly during daylight hours. These temperature fluctuations are closely tied to atmospheric energy exchange processes. During the day, greenhouse gases (GHGs) in the atmosphere absorb a portion of the sun's radiation, leading to an increase in air temperature, a phenomenon often referred to as global warming.

Global warming poses various risks to human health, primarily through increased air pollution. Additionally, heightened air pollution levels can negatively impact sunlight intensity, which, in turn, can reduce crop yields. This has significant implications for a country's economic well-being. Hence, there is a pressing need to predict air temperatures for a wide range of purposes, including determining optimal planting times and suitable crop choices, facilitating daily activity planning, and enhancing disaster preparedness efforts.

One of the methods that can be used to make predictions is the Weighted Markov Chain (WMC). This method has been applied by several researchers, including in **[1]**–**[3]**. This method uses historical data for the construction of probability models in predicting future trends. In 2015, Zhou estimated the stock price of China's sports industry using the WMC method. Then, he compared the stock price forecasting results with the Markov Chain method [3]. The results show that the prediction of the stock price of China's sports industry using the WMC method is closer to the true value, and the prediction using the Markov Chain method has a higher error than the WMC method. Other applications for the WMC method can also be seen in **[4]**–**[7]**.

Another method that can be used to predict the daily average of temperature is the Fuzzy Time Series (FTS). **[8]** have proposed the FTS method with two-factor time variance for temperature forecasting. This method has the advantage of obtaining good forecasting results. Other forecasting using the FTS method can be seen in **[9]**–**[11]**. Furthermore, in 2012, **[12]** in his research combined the Fuzzy Time Series technique with the Markov Chain to increase accuracy in forecasting the Taiwan exchange rate against the US dollar.

In research [13], the accuracy has been compared between the Fuzzy Time Series-Markov Chain (FTS-MC) and the WMC method for predicting stock price. The results show that the FTS-MC method has better accuracy. In another study, the FTS-MC method was compared to the Markov chain method for predicting temperature in Kendari City [14]. The results also show that the FTS-MC method has a smaller prediction error. Some applications of the FTS-MC method can also be seen in **[15]**–**[18]**.

The application of these two methods in the stochastic problem of air temperature is yet to be found. Therefore, in this study, the Weighted Markov Chain and Fuzzy Time Series-Markov Chain methods were applied to predict the weekly air temperature in Banda Aceh city. Then, the accuracy of the two methods will be compared to find out the best method for predicting the weekly average of air temperature data.

2. RESEARCH METHODS

The data used in this study is the weekly average of air temperature data from 04 November 2019 to 04 November 2021 in Banda Aceh city. An overview of the data is given in **Figure 1**.

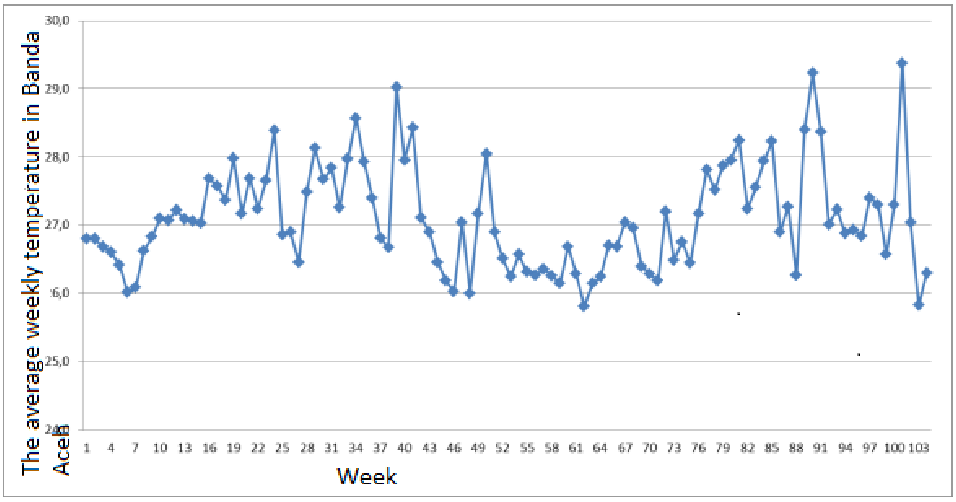

Figure 1. The weekly average temperature in Banda Aceh City

Furthermore, the weekly air temperature will be predicted for the next 3 weeks using the Weighted Markov Chain and Fuzzy Time Series-Markov Chain methods. The Markov chain method can only be applied in a process if the process has Markov properties. So, before applying the Markov chain to data, the weekly air temperature data must be tested. Details on how to test Markov properties can be seen in [19]. Our test results show that the data satisfy Markov properties.

2.1 Weighted Markov Chain (WMC)

The following are the steps for predicting air temperature using the WMC method.

a) Calculating the mean and standard deviation of 104 weeks of air temperature data using **Equation (1)** and **Equation (2)**:

$$\bar{X} = \frac{\sum_{t=1}^{n} x_t}{n}, \tag{1}$$

$$S = \sqrt{\frac{\sum_{t=1}^{n}(x_t - \bar{X})^2}{n-1}}, \tag{2}$$

with x_t represents the average air temperature in week t, $t = 1,2,\ldots,104$, and $n = 104$.

b) Forming state and grouping air temperatures into the appropriate state groups. The selection of the number of states is based on historical data trends and the calculation of the mean and standard deviation.

c) Forming a transition frequency matrix obtained by calculating the number of stochastic processes that move from air temperature state i to air temperature state j [20]. Then based on the transition frequency matrix, the transition probability between states can be determined using **Equation (3)**:

$$P_{ij} = \frac{f_{ij}}{\sum_{j=1}^{m} f_{ij}}. \tag{3}$$

d) In predicting weekly air temperature, it will involve as many as n steps in the previous time. It is necessary to calculate the weight of each k for every $k = 1,2,\ldots,n$. The weight calculation uses **Equation (4)**:

$$w_k = \frac{|r_k|}{\sum_{\ell=1}^{K}|r_\ell|}, \qquad k = 1, 2, \ldots, K \tag{4}$$

where r_k denotes the autocorrelation coefficient. Calculation of the autocorrelation coefficient is obtained using **Equation (5)**:

$$r_k = \frac{\sum_{t=1}^{n-k}(x_t - \bar{X})(x_{t+k} - \bar{X})}{\sum_{t=1}^{n}(x_t - \bar{X})^2}. \tag{5}$$

e) Predicting future air temperature is done by predicting the initial temperature up to n times to come. The weekly temperature prediction is obtained using **Equation (6)**:

$$\hat{P}_{ij} = \sum_{k=1}^{K} w_k P_{ij}^{(k)} \tag{6}$$

where $P_{ij}^{(k)}$ represents the probability of transition from state i to state j in k steps. The prediction result is a state j which is obtained from the maximum $\hat{P}_{ij}$ value.

2.2 Fuzzy Time Series-Markov Chain (FTS-MC)

The following are the steps for predicting air temperature using the FTS-MC method.

a) Determine the universal set U using **Equation (7)**:

$$U = [D_{min} - D_1 , D_{max} + D_2] \tag{7}$$

where D_{min} and D_{max} represent the smallest data value and the largest data value. According to **[12]**, the value D_1 and D_2 are arbitrary positive real numbers determined by the researcher so that all historical data are within the interval of the universal set U.

b) Partition the universal set U into n equal intervals using **Equation (8)**:

$$n = 1 + 3.322 \log N. \tag{8}$$

Then determine the value of the interval length (l) using **Equation (9)**:

$$l = \frac{[(D_{max} + D_2) - (D_{min} - D_1)]}{n} \tag{9}$$

so, for each interval is obtained $u_1, u_2, \dots, u_n$ that are the elements of the fuzzy set.

c) Determine the fuzzy sets of the universal set U using **Equation (10)**:

$$\mu_{ij} = \begin{cases} 1 & ; i = j \\ 0.5 & ; j = i - 1 \text{ or } j = i + 1 \\ 0 & ; \text{others} \end{cases} \tag{10}$$

[21]. Furthermore, the formed Fuzzy set is

$$\begin{aligned} \tilde{A}_1 &= \left\{\frac{1}{u_1}, \frac{0.5}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \dots, \frac{0}{u_n}\right\}, \\ \tilde{A}_2 &= \left\{\frac{0.5}{u_1}, \frac{1}{u_2}, \frac{0.5}{u_3}, \frac{0}{u_4}, \dots, \frac{0}{u_n}\right\}, \\ &\vdots \\ \tilde{A}_n &= \left\{\frac{0}{u_1}, \frac{0}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \dots, \frac{0.5}{u_{n-1}}, \frac{1}{u_n}\right\}. \end{aligned} \tag{11}$$

The fuzzy set $\tilde{A}_i$ that is formed corresponds to the number of u_i for $i = 1, 2, \dots n$.

d) Performing fuzzification of historical data. If a time series data is in the interval u_i, then the data is fuzzified into $\tilde{A}_i$. For example, the air temperature data for the 1st week is in the interval u_3 then it is fuzzified into $\tilde{A}_3$.

e) Determine Fuzzy Logical Relationship (FLR). FLR is the relationship between each sequence of air temperature data to the next air temperature data in the form of a fuzzy set. For example, the 1st data is $\tilde{A}_1$ and the 2nd data is $\tilde{A}_2$. Then the FLR formed is $\tilde{A}_1 \to \tilde{A}_2$.

f) Form a Fuzzy Logical Relationship Group (FLRG). If the FLR *state* $\tilde{A}_2$ (example $\tilde{A}_2 \to \tilde{A}_2, \tilde{A}_2 \to \tilde{A}_2, \tilde{A}_2 \to \tilde{A}_3$), then the FLR is grouped into FLRG and written as $\tilde{A}_2 \to \tilde{A}_1, \tilde{A}_2, \tilde{A}_3$.

g) Calculate the initial predicted value using previous historical data. First, determine the value of the probability matrix using **Equation (12)**:

$$P_{ij} = \frac{M_{ij}}{M_i};\ i,j = 1,2,3,\dots,n \tag{12}$$

to obtain the following **Equation (13)**:

$$R = \begin{pmatrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ \vdots & \vdots & \ddots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{pmatrix} \tag{13}$$

which is a square matrix with elements containing the possibility of changing from state $\tilde{A}_i$ to state $\tilde{A}_j$, based on the FLRG table in the previous stage. Then based on the value of the probability matrix, the initial predicted value can be calculated using the rules contained in **Equation (14)** and **Equation (15)**:

$$F(t) = m_k P_{ik} = m_k \tag{14}$$

$$F(t) = m_1 P_{i1} + m_2 P_{i2} + \cdots + m_{i-1} P_{i(i-1)} + Y_{(t-1)} P_{ii} + m_{i+1} P_{i(i+1)} + \cdots + m_n P_{in}. \tag{15}$$

h) Adjusts the trend of the predicted value. This step aims to reduce the amount of deviation in the prediction results. To adjust the predicted value, the following **Equation (16)** and **Equation (17)** are used:

$$D_{t1} = \left(\frac{l}{2}\right) s\ ;\ (1 \le s \le n - i) \tag{16}$$

$$D_{t2} = -\left(\frac{l}{2}\right) v\ ;\ \ (1 \le v \le i) \tag{17}$$

where D_{t1}, D_{t2} represent the adjustment value of the air temperature prediction, s represents the forward transition displacement, and v represents the backward transition displacement.

i) Determine the final predicted value using **Equation (18)**:

$$F'(t) = F(t) \pm D_{t1} \pm D_{t2} \tag{18}$$

to predict the upcoming air temperature for the next k weeks.

2.3 Calculating the Accuracy of Predicted Results

To calculate the average value of the absolute error predicted from the actual value in percentage form, we used the Mean Absolute Percentage Error (MAPE) contained in **Equation (19)**:

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y(t) - F(t)}{Y(t)}\right| \times 100\% \tag{19}$$

where $Y(t)$ represents the actual air temperature at time t, $F(t)$ represents the predicted air temperature at time t, and n represents the amount data of predicted air temperature.

3. RESULTS AND DISCUSSION

3.1 Air Temperature Prediction Using the Weighted Markov Chain Method

The average value and standard deviation of the weekly air temperature data for Banda Aceh city from 1 November 2019 to 4 November 2021 for 104 weeks using **Equation (1)** and **Equation (2)** were obtained $\bar{X} = 27.1$℃ and $S = 0.754$℃. Furthermore, the air temperature data is grouped into the appropriate state group. The selection of the number of states is based on historical data trends and the calculation of the average and standard deviation. Based on the trend of air temperature plot results in **Figure 2**, the air temperature is grouped into 6 states so that a table of air temperature groupings was formed as shown in **Table 1.**

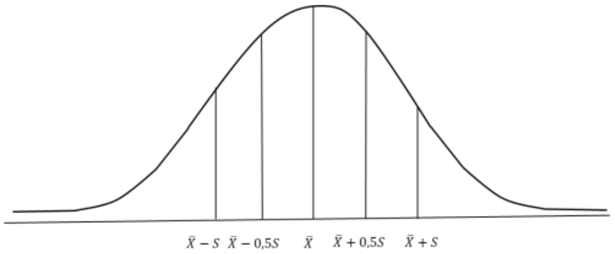

Figure 2. Distribution of state groupings

Table 1. Air Temperature Grouping

State	Grouping Intervals	State Limit
1	$X < \bar{X} - S$	$X < 26.3$
2	$\bar{X} - S \le X < \bar{X} - 0.5S$	$26.3 \le X < 26.7$
3	$\bar{X} - 0.5S \le X < \bar{X}$	$26.7 \le X < 27.1$
4	$\bar{X} \le X < \bar{X} + 0.5S$	$27.1 \le X < 27.5$
5	$\bar{X} + 0.5S \le X < \bar{X} + S$	$27.5 \le X < 27.9$
6	$X \ge \bar{X} + S$	$X \ge 27.9$

Based on Table 1, the weekly air temperature of Banda Aceh city can be grouped as shown in Table 2.

Table 2. Classification of Air Temperature Data

Week	Temperature	State	State Transition
1	26.8	3	
2	26.8	3	3_3
⋮	⋮	⋮	⋮
102	27.0	3	6_3
103	25.8	1	3_1
104	26.3	2	1_2

Furthermore, the transition probability between states is determined using Equation (3) with the transition probability matrix P:

$$P = \begin{bmatrix} 0.333 & 0.25 & 0.25 & 0.16666 & 0 & 0 \\ 0.333 & 0.333 & 0.111 & 0.11111 & 0.05555 & 0.05556 \\ 0.083 & 0.291 & 0.375 & 0.16666 & 0.04166 & 0.04166 \\ 0 & 0.15 & 0.2 & 0.25 & 0.2 & 0.2 \\ 0 & 0 & 0 & 0.3 & 0.3 & 0.4 \\ 0 & 0 & 0.2631 & 0.21052 & 0.05263 & 0.47368 \end{bmatrix}.$$

Calculation of the autocorrelation coefficient values is carried out using Equation (4) and then the calculation of the Markov chain weights is carried out using Equation (5), where the variable r_k represents the autocorrelation coefficient. Following are the autocorrelation coefficient values and the Markov chain weights which can be seen in Table 3 and Table 4.

Table 3. Autocorrelation Coefficient Values

$\boldsymbol{k}$	1	2	3
$\boldsymbol{r_k}$	0.5352	0.2717	0.171

Table 4. Markov Chain Weights for Each $\boldsymbol{k}$

$\boldsymbol{k}$	$\boldsymbol{w_k}$		
	$\boldsymbol{K=1}$	$\boldsymbol{K=2}$	$\boldsymbol{K=3}$
1	1	0.6633	0.5473
2		0.3367	0.2778
3			0.1749

The final step in the WMC method is the prediction process. The resulting prediction is in the form of a state that represents the range of the weekly air temperature for Banda Aceh city. To determine the predicted value of the weekly air temperature, **Equation (6)** is used. The prediction state can be identified by looking at the maximum probability value. By using $K = 3$, the calculation of air temperature prediction in the 105th week will involve the state in the previous three weeks (104, 103, and 102) which can be seen in **Table 5** below.

Table 5. Prediction of Air Temperature in the 105th Week with $K = 3$

Week	State (i)	k	State Weight (w_k)	$P_{ij}^{(k)}$					
				$j = 1$					$j = 1$
104	2	1	0.54729	0.1824	104	2	1	0.54729	0.1824
103	1	2	0.277835	0.0598	103	1	2	0.277835	0.0598
102	3	3	0.174875	0.0273	102	3	3	0.174875	0.0273
$\hat{P}_{ij}$	0.2695	0.2941	0.1650	0.1389	$\hat{P}_{ij}$	0.2695	0.2941	0.1650	0.1389

Table 5 shows that the air temperature in the 105th week will be in state 2 with a probability of 29.41%. Using the same steps, the air temperature prediction for the next three weeks can be calculated. Based on the prediction results for the next three weeks, it can be seen that the air temperature of Banda Aceh City is in the same state (state 2). The prediction results show that the weekly air temperature is in state 2 with the median of the weekly air temperature range of 26.5°C. So, it can be obtained that the prediction result for the weekly air temperature of Banda Aceh city is 26.5 °C.

3.2 Air Temperature Prediction Using the Fuzzy Time Series-Markov Chain Method

Prediction of the weekly air temperature using the Fuzzy Time Series-Markov Chain (FTS-MC) method is carried out in the following steps.

Step 1: Determine the universal set U.

Based on the air temperature data obtained $D_{min} = 25.8$, $D_{max} = 29.4$, and the values used are $D_1 = 0.1$ and $D_2 = 0.2$ then from **Equation (7)** we get the universal set $U = [25.7\ ;\ 29.6]$.

Step 2: Determine the number of fuzzy intervals.

The universal set U will be partitioned into n equal intervals using the Sturges formula in **Equation (8)** so that the number of intervals $n = 7.7 \approx 8$ is obtained with the interval length $(l) = 0.48$. The limit value for each interval is obtained from the lowest interval value added to the length of the interval to obtain 8 intervals in the universal set U and the median value can be seen in **Table 6**.

Table 6. Interval (u) and Median (m) in the FTS-MC Method

No	Interval (u)	Median (m)
1	[25.7 ; 26.2]	26.0
2	[26.2 ; 26.7]	26.4
3	[26.7 ; 27.2]	26.9
4	[27.2 ; 27.6]	27.4
5	[27.6 ; 28.1]	27.9
6	[28.1 ; 28.6]	28.4
7	[28.6 ; 29.1]	28.8
8	[29.1 ; 29.6]	29.3

Step 3: Determine the fuzzy sets of the universal set U.

Based on the rules of membership degrees in **Equation (10**) and **Equation (11)**, the fuzzy sets formed are as follows:

$$\tilde{A}_1 = \left\{\frac{1}{u_1}, \frac{0.5}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0}{u_6}, \frac{0}{u_7}, \frac{0}{u_8}\right\},$$
$$\tilde{A}_2 = \left\{\frac{0.5}{u_1}, \frac{1}{u_2}, \frac{0.5}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0}{u_6}, \frac{0}{u_7}, \frac{0}{u_8}\right\},$$
$$\vdots$$
$$\tilde{A}_8 = \left\{\frac{0}{u_1}, \frac{0}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0}{u_6} +, \frac{0.5}{u_7}, \frac{1}{u_8}\right\}.$$

Step 4: Fuzzification of air temperature data.

Air temperature data can be fuzzified based on the fuzzy set that has been defined in the previous step. For example, for 1st week data ($t = 1$) 26.8°C is included in the u_3 interval so the data for 1st week is fuzzified on the fuzzy set $\tilde{A}_3$. The following is the fuzzification result of weekly air temperature data notated into linguistic values.

Table 7. Data Fuzzification in the FTS-MC Method

t	Actual Value	Linguistic Value
1	26.8	$\tilde{A}_3$
2	26.8	$\tilde{A}_3$
3	26.7	$\tilde{A}_3$
4	26.6	$\tilde{A}_2$
⋮	⋮	⋮
102	27.0	$\tilde{A}_3$
103	25.8	$\tilde{A}_1$
104	26.3	$\tilde{A}_2$

Step 5-6: Determine FLR and Form FLRG.

Based on the fuzzification results of the weekly air temperature data in the previous step, the FLR will be determined. For example, for the 1st data $\tilde{A}_3$ and the 2nd data $\tilde{A}_3$, the FLR formed is $\tilde{A}_3 \to \tilde{A}_3$. FLR can be seen in **Table 8**. Then based on **Table 8**, the FLRG is formed. FLRG results can be seen in **Table 9**.

Table 8. FLR in the FTS-MC Method

Data Order	FLR
$1 \to 2$	$\tilde{A}_3 \to \tilde{A}_3$
$2 \to 3$	$\tilde{A}_3 \to \tilde{A}_3$
$3 \to 4$	$\tilde{A}_3 \to \tilde{A}_2$
⋮	⋮
$101 \to 102$	$\tilde{A}_8 \to \tilde{A}_3$
$102 \to 103$	$\tilde{A}_3 \to \tilde{A}_1$
$103 \to 104$	$\tilde{A}_1 \to \tilde{A}_2$

Table 9. FLRG in the FTS-MC Method

No.	Current State	Next State
1	$\tilde{A}_1$	$2(\tilde{A}_1)$, $3(\tilde{A}_2)$, $2(\tilde{A}_3)$, $(\tilde{A}_4)$
2	$\tilde{A}_2$	$4(\tilde{A}_1)$, $8(\tilde{A}_2)$, $4(\tilde{A}_3)$, $4(\tilde{A}_4)$, $(\tilde{A}_6)$
3	$\tilde{A}_3$	$2(\tilde{A}_1)$, $8(\tilde{A}_2)$, $14(\tilde{A}_3)$, $4(\tilde{A}_4)$, $(\tilde{A}_5)$, $(\tilde{A}_7)$
4	$\tilde{A}_4$	$3(\tilde{A}_2)$, $3(\tilde{A}_3)$, $(\tilde{A}_4)$, $8(\tilde{A}_5)$, $(\tilde{A}_6)$, $(\tilde{A}_8)$
5	$\tilde{A}_5$	$(\tilde{A}_3)$, $6(\tilde{A}_4)$, $4(\tilde{A}_5)$, $4(\tilde{A}_6)$, $(\tilde{A}_7)$
6	$\tilde{A}_6$	$4(\tilde{A}_3)$, $(\tilde{A}_4)$, $(\tilde{A}_5)$, $(\tilde{A}_8)$
7	$\tilde{A}_7$	$2(\tilde{A}_5)$
8	$\tilde{A}_8$	$(\tilde{A}_3)$, $(\tilde{A}_6)$

Step 7: Determine the initial predicted value.

Based on **Equation (13)**, each element is a probability value obtained from **Equation (12)** where P_{ij} is the probability of transition from state i to state j. The following is obtained matrix R as follows:

$$R = \begin{bmatrix} \frac{2}{8} & \frac{3}{8} & \frac{2}{8} & \frac{1}{8} & 0 & 0 & 0 & 0 \\ \frac{4}{21} & \frac{8}{21} & \frac{4}{21} & \frac{4}{21} & 0 & \frac{1}{21} & 0 & 0 \\ \frac{2}{30} & \frac{8}{30} & \frac{14}{30} & \frac{4}{30} & \frac{1}{30} & 0 & \frac{1}{30} & 0 \\ 0 & \frac{3}{17} & \frac{3}{17} & \frac{1}{17} & \frac{8}{17} & \frac{1}{17} & \frac{1}{17} & 0 \\ 0 & 0 & \frac{1}{16} & \frac{6}{16} & \frac{4}{16} & \frac{4}{16} & \frac{1}{16} & 0 \\ 0 & 0 & \frac{4}{7} & \frac{1}{7} & \frac{1}{7} & 0 & \frac{1}{7} & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Furthermore, the initial predicted value is determined based on the rules contained in **Equation (14)** and **Equation (15)**. The results of the initial prediction can be seen in **Table 10**.

Table 10. The Results of the Initial Prediction in the FTS-MC Method

t	Actual Value	Initial Prediction $F(t)$
1	26.8	0.0
2	26.8	26.8
3	26.7	26.8
4	26.6	26.8
⋮	⋮	⋮
102	27.0	27.6
103	25.8	26.9
104	26.3	26.5

Step 8: Trend adjustment from the initial predicted value.

Trend adjustment of the predicted value is carried out for each relationship between the current state and the next state of the FLR using **Equation (16)** and **Equation (17)**. The results of the D_t adjustment values are presented in **Table 11**.

Table 11. The Adjustment Value D_t

Data Order	FLR	Adjustment Value
$1 \rightarrow 2$	$\tilde{A}_3 \rightarrow \tilde{A}_3$	0
$2 \rightarrow 3$	$\tilde{A}_3 \rightarrow \tilde{A}_3$	0
$3 \rightarrow 4$	$\tilde{A}_3 \rightarrow \tilde{A}_2$	-0.24
⋮	⋮	⋮
$101 \rightarrow 102$	$\tilde{A}_8 \rightarrow \tilde{A}_3$	-1.21
$102 \rightarrow 103$	$\tilde{A}_3 \rightarrow \tilde{A}_1$	-0.48
$103 \rightarrow 104$	$\tilde{A}_1 \rightarrow \tilde{A}_2$	0.24

Step 9: Determinate the final prediction results of the air temperature data.

For the calculation of the final predicted value follow the rules in **Equation (18)**. The final prediction results are presented in **Table 12**.

Figure 3 **Table 12. Prediction Results After Adjustment**

t	$Y(t)$	$F(t)$	$D(t)$	$F'(t)$
1	26.8	0.0	0	0.0
2	26.8	26.8	0	26.8
3	26.7	26.8	0	26.8
4	26.6	26.8	-0.24	26.5

t	$Y(t)$	$F(t)$	$D(t)$	$F'(t)$
⋮	⋮	⋮	⋮	⋮
102	27.0	27.6	-1.21	26.4
103	25.8	26.9	-0.48	26.5
104	26.3	26.5	0.24	26.8

where $Y(t)$ represents actual value, $D(t)$ represents adjustment value, and $F'(t)$ represents final value. presented a comparison graph between the actual and predicted values using the FTS-MC method at air temperature for 104 weeks.

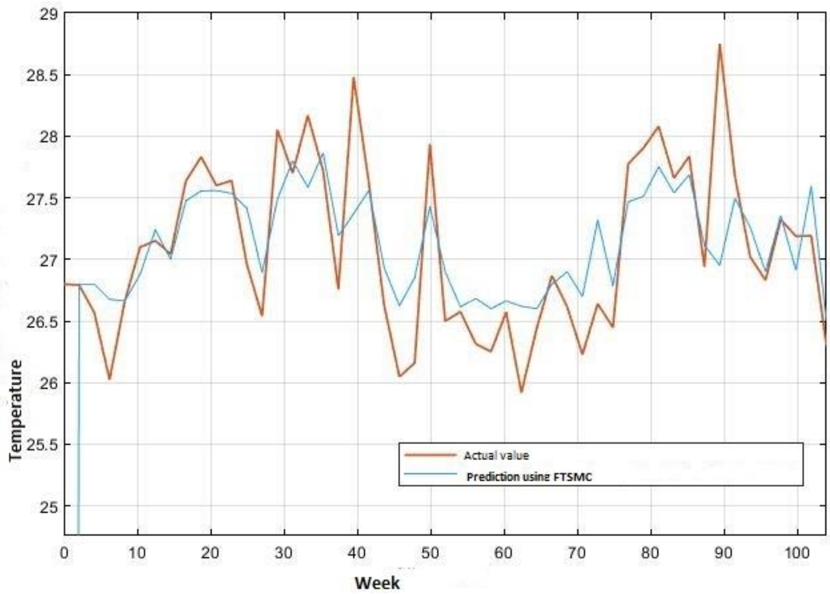

Figure 3. Comparison graph between the actual and predicted values using the FTS-MC method

Furthermore, the air temperature prediction of Banda Aceh city for the next 3 weeks is carried out using the previous steps (initial predictions, adjustment values, and final predictions) that can be seen in **Table 13**. The adjustment value will be assumed to be equal to 0 because the steady state of the air temperature at time $\boldsymbol{t+1}$ is unknown.

Table 13. Prediction of Air Temperature from 105th Week to 107th Week

t	$Y(t)$	$F(t)$	D_t	$F'(t)$
105	25.89	26.66	0	26.66
106	26.30	26.79	0	26.79
107	26.86	26.83	0	26.83

3.3 Comparison Between the Result of the WMC and FTS-MC Methods

From the results, it was obtained that the percentage error of the air temperature prediction of Banda Aceh City for the next 3 weeks using the WMC method was 1,5% and the FTS-MC method was 1,7%. A comparison of MAPE with WMC and FTS-MC methods can be seen in **Table 14**. Based on the results, the MAPE of the WMC method is smaller than the FTS-MC method, so it can be said that in this study, the air temperature prediction using the WMC method is better than the FTS-MC method. However, the prediction of air temperature with both methods is only suitable for short-term use and is not ideal for long-term predictions because the resulting error will be even greater.

Table 14. Comparison of MAPE with WMC and FTS-MC Methods

Method	MAPE Value	Accuracy Criteria
WMC	$1.5\% \leq 10\%$	Very good
FTS-MC	$1.7\% \leq 10\%$	Very good

4. CONCLUSIONS

Based on the discussion in the previous chapter, conclusions can be obtained as follows:

1. The prediction of the average temperature of the Banda Aceh City using the WMC method produces the same value for each subsequent time period. Meanwhile, by using the FTS-MC method the prediction results can be different for the next three weeks.
2. The accuracy level of the MAPE for the WMC method is smaller than the FTS-MC method. It can be concluded that the prediction of air temperature using the WMC method is better than the FTS-MC method.

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