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TRANSFER FUNCTION AND ARIMA MODEL FOR FORECASTING BI RATE IN INDONESIA

Khusnia Nurul Khikmah[1*], Kusman Sadik[2], Indahwati[3]

[1,2,3]*Department of Statistics, Faculty of Mathematics and Natural Sciences, IPB University*
Dramaga Street, Bogor, West Java, 16680, Indonesia

Corresponding author's e-mail: * **khusniank@gmail.com**

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ABSTRACT

Fluctuating gold prices can have an impact on various sectors of the economy. Some of the impacts of rising and falling gold prices are inflation, currency exchange rates, and the value of the Bank Indonesia benchmark interest rate (BI Rate). The data was taken from the Indonesian Central Statistics Agency's official website (BPS) for the Bank Indonesia benchmark interest rate (BI Rate) value. Therefore, research on the value of the Bank Indonesia benchmark interest rate (BI Rate) is essential with the gold price as a control. The purpose of this study is to forecast the value of the Bank Indonesia reference interest rate (BI Rate) with a transfer function model where the input variable used is the price of gold and forecast the value of the Bank Indonesia benchmark interest rate (BI Rate) with the ARIMA model. The analysis results show that the best model for forecasting the Bank Indonesia reference interest rate (BI Rate) is a transfer function model with a value of $b = 1$, $s = 0$, $r = 2$, *and a noise series model* $(p_n, q_n) = (1,0)$ *with the MAPE value is* 17.7723%.

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1. INTRODUCTION

Gold is one of the precious metals used as an investment destination because its value tends to increase from time to time [1]. Although this value tends to increase, the price of gold is very volatile, impacting several economic factors, one of which is the Bank Indonesia benchmark interest rate (BI Rate) [2]. The BI Rate itself is a policy interest rate whose fluctuating changes affect economic activity and create uncertainty, hence the need for further analysis. The price of gold and the BI Rate are cases whose value depends on time, so this is a time series case. The analysis that can be used as consideration for investors and Bank Indonesia to forecast the benchmark interest rate based on the price of gold is the autoregressive moving average and the transfer function.

Time-series data analysis is a statistical analysis method whose data considers time. This data is often found in data related to the economy, such as gold prices and BI rates. The autoregressive integrated moving average is a time-series data analysis method suitable for forecasting large amounts of data [3], [4]. Previous research on autoregressive integrated moving averages is in Sofro et al., who used the model on COVID-19 data in Indonesia [5]. Other research related to autoregressive integrated moving averages is also used to compare the results of the model's goodness against forecasting against the long short-term memory model see [6].

Furthermore, the value of the Bank Indonesia benchmark interest rate (BI Rate) can be forecasted using the transfer function [7], [8]. Research related to transfer functions is in [9], which examines the accuracy of the transfer function and vector autoregressive models in currency exchange rates. The transfer function forecasts the Bank Indonesia benchmark interest rate (BI Rate) if the value of gold prices fluctuates. This study chose these two methods to consider the need for more research on the transfer function model used in forecasting time series data for the economic field, especially the Bank Indonesia benchmark interest rate (BI Rate).

Gold price fluctuations impact several economic sectors, such as the global index and macroeconomic [10]. In addition, the volatile gold price has also resulted in stock market volatility, inflation, and the exchange rate in Indonesia [11]. On the other hand, the Bank Indonesia benchmark interest rate (BI Rate) affects the poverty rate due to its inflationary effect. Therefore, these two issues were selected in this study. Based on these problems, an advanced statistical analysis needs to be carried out to determine how the two influence each other and how the forecasting results are carried out. Based on previous research, this study has two statistical models (the autoregressive moving average and transfer function) with reasonable accuracy.

This research chooses the autoregressive moving average model because, based on previous research, this model has good forecasting accuracy, see in [12]–[14]. On the other hand, further research related to the influence of other factors on a variable is significant in the field, especially economics. One model approach in statistics that can provide this is the transfer function. This model can determine the influence of variables and predict well according to previous research on [15], [16]. Therefore, this study chose to compare the accuracy of the autoregressive moving average and the transfer function model was chosen to estimate the value of the Bank Indonesia benchmark interest rate (BI Rate) to assist the government in making decisions on its monetary policy.

2. RESEARCH METHODS

2.1 Autoregressive Moving Average

The autoregressive moving average is a univariate time series forecasting method that combines the autoregressive and moving average models for non-stationary data [5], [17]. The autoregressive moving average method can also overcome the problems with multivariate models. In general, the autoregressive moving average method can be written as $ARIMA(p, d, q)$, where p is the order of the autoregressive model, q is the order of the moving average model, and d is the number of differentiators so, that it becomes stationary data. In general, the autoregressive moving average equation with the backshift method can be written as follows:

$$\phi_p(B)(1-B)^d Y_t = \theta_q(B)e_t \tag{1}$$

Where ϕ_p is an autoregressive process with $\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)$, $\theta_q(B)$ is a moving average process with $\theta_q(B) = (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q)$, and e_t is the residual [18].

2.2 Transfer Function

The transfer function is a time series forecasting method that aims to predict future time series data (output time series data or Y_t) based on past values (input time series data or X_t) [19]. In general, the transfer function equation can be written as follows:

$$Y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} + \frac{\theta_q(B)}{\phi_p(B)} a_t \tag{2}$$

Where $\omega_s(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_s B^s$ and $\delta_r(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r$ [20], [21]. In general, modelling using a transfer function with input X_t and output Y_t has several stages as follows [8], [22]:

- Preprocessing data

  Prepare the data by specifying the input variable X_t and the output variable Y_t, checking the data's stationery, and performing the process for the stationary of the data if the data is not stationary.

- Pre-whitening input series

  Eliminates the X_t input series pattern so that only white noise is produced. For example, the pre-whitening result on the input series X_t is α_t. The equation α_t can be written as follows:

$$\alpha_t = \frac{\phi_x(B)}{\theta_x(B)} x_t \tag{3}$$

- Pre-whitening the output series

  The output series pre-whitening used is the result of the input series pre-whitening. Pre-whitening the output array does not necessarily produce in a white noise output series. Suppose the pre-whitening result in the Y_t output series is β_t. The equation β_t can be written as follows:

$$\beta_t = \frac{\phi_x(B)}{\theta_x(B)} y_t \tag{4}$$

  Calculating cross-correlation function

  The cross-correlation function detects the strong relationship between the results of pre-whitening the input series α_t and the pre-whitening output series β_t. The cross-correlation equation of the two series can be written as follows:

$$r_{xy}(k) = \hat{\rho}_{XY} = \frac{C_{XY}(k)}{\sqrt{C_{XX}(0)C_{YY}(0)}} = \frac{C_{XY}(k)}{S_X S_Y} \tag{5}$$

  Where k is the time lag, where $k = 0, \pm 1, 2, \ldots$ and $C_{xy}(k)$ is the cross-covariance between two variables and the value of $C_{xy}(k)$ can be calculated using the following equation:

$$C_{xy}(k) = \frac{1}{n}\sum_{t=1}^{n-k}(X_t - \underline{X})(Y_{t+k} - \underline{Y}) \tag{6}$$

- Direct assessment of impulse response weight

  Direct estimation of the impulse response weight or v_k with cross-correlation between the input and output series that have been pre-whitened $(r_{\alpha\beta})$ can be calculated using the following equation:

$$v_k = \frac{r_{\alpha\beta}(k) S_\beta}{S_\alpha} \tag{7}$$

- Determination of b, r, s, for the transfer function model

  b: delay logged on x_{t-b}

  s: how long the output series is affected by the input series

  r: how long the t-time output series is affected by the $k - t$-time output series

$$y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} + \frac{\theta_q(B)}{\phi_p(B)} a_t \tag{8}$$

- Preliminary testing of noise series

$$n_t = y_t - v(B)x_t = y_t - v_0x_t - v_1x_{t-1} - \cdots - v_gx_{t-g} \quad (9)$$

- Determination of the value (p_n, q_n) for the $ARIMA(p_n, 0, q_n)$ model of the residual

$$\phi_n(B)N_t = \theta_n(B)a_t \quad (10)$$

The $ARIMA(p_n, 0, q_n)$ model can be seen by identifying the ACF and PACF plots and the EACF value of the residuals.

2.3 Accuracy

1. Akaike Information Criterion (AIC)

The Akaike information criterion measures the model's goodness, where the best model chosen is the model with the smallest AIC value. The AIC value can be calculated using the following equation:

$$AIC = 2k - 2\ ln\ (L) \quad (11)$$

Where k is the model's parameter and L is the maximum value of the likelihood function used to estimate the model [23], [24].

2. Mean Absolute Percentage Error (MAPE)

The mean absolute percentage error is the percentage of error forecasting results to the actual value in a certain period. The percentage value of this error can be calculated using the equation [25], [26]:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Y_t-\hat{Y}_t}{Y_t}\right| \times 100\% \quad (12)$$

2.4 Data Description

The data used in this study is secondary data, namely data taken from the official website of the Indonesian Central Statistics Agency (BPS) for the value of the Bank Indonesia benchmark interest rate (BI Rate) and the price of gold in Indonesia from price-gold.org per month. The data taken is 100 observations or data taken **from April 2012 to July 2020**. This research also uses R to model the data.

This research is divided into two methods, namely ARIMA and transfer function. For the ARIMA model, the variable used for modeling is the Bank Indonesia benchmark interest rate (BI Rate) value. As for the transfer function method, the variable used as the input series X_t is the price of gold, while the Y_t variable used as the output series is the value of the Bank Indonesia benchmark interest rate (BI Rate).

3. RESULTS AND DISCUSSION

The case study used in this research is data on gold prices and the value of the Bank Indonesia benchmark interest rate (BI Rate). This data is time-series data, where visually, this time-series data pattern is shown in Figure 1 and Figure 2.

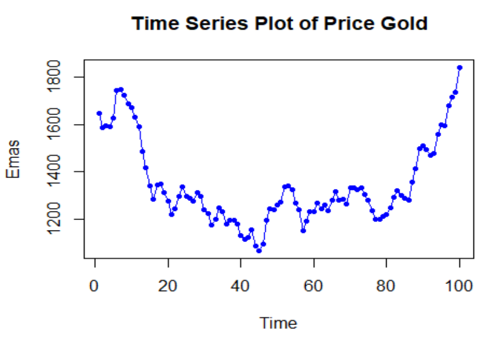

Figure 1. Time-series plot of gold prices

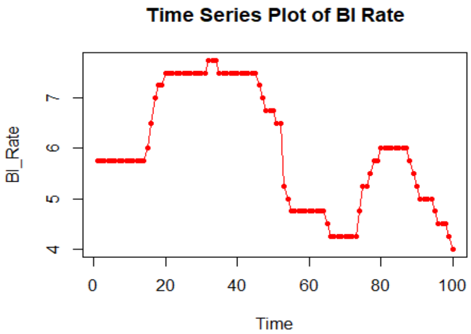

Figure 2. Time-series plot of BI rate

The input series, namely the price of gold (X_t), is first modelled using ARIMA. Because the data used is data that contains trend patterns, the differencing process is carried out once. The results of the one-time differencing show that the data is stationary, which can be continued to determine the candidate model where the ACF and PACF plots of differencing results are shown in **Figure 3** and **Figure 4**.

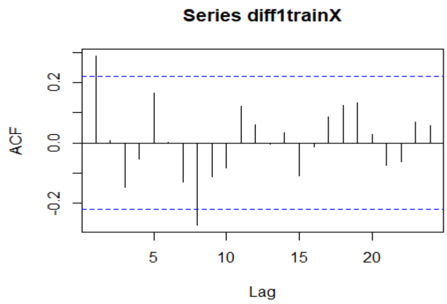

Figure 3. ACF plot of the first difference of gold prices training dataset

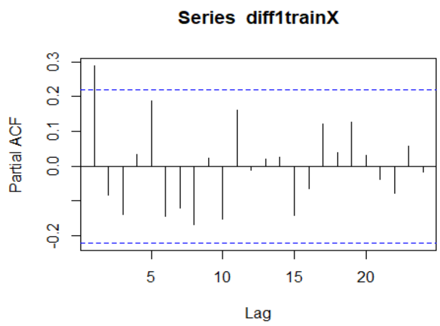

Figure 4. PACF plot of the first difference of gold prices training dataset

ARIMA model candidates from the gold prices or the X_t input series are presented in **Table 1**.

Table 1. Candidates model of ARIMA model

candidate model	coefficient of parameter	AIC
$ARIMA(0,1,1)$	$\theta=-0.2888$ $\mu=-5.6114$	811.64
$ARIMA(1,1,0)$	$\phi=\ 0.2930$ $\mu=-5.6475$	811.68
$ARIMA(1,1,1)$	$\phi=\ 0.1590$ $\theta=-0.1510$ $\mu=-5.6464$	813.37

***Data source**: the output of estimation parameter from the R program*

Based on **Table 1**, the best ARIMA model obtained is $ARIMA(0,1,1)$ with the smallest AIC value of 811.64. This best model is then performed residual diagnostics and fulfils the assumption of residual white noise. Then pre-whitening the X_t input series by changing the X_t series to an α_t series where the value of α_t is as follows:

$$\alpha_t = e_t - X_t - X_{t-1} + 0.2888e_{t-1}$$

Pre-whitening the input series X_t is also applied to pre-whitening the output series Y_t, using the input series X_t model $ARIMA(0,1,1)$. The results of pre-whitening the output series Y_t, namely β_t, are as follows:

$$\beta_t = e_t - Y_t - Y_{t-1} + 0.2888e_{t-1}$$

The results of pre-whitening the input series α_t and the output series β_t are then calculated for the cross-correlation. The results of the cross-correlation between the two are presented in **Figure 5**.

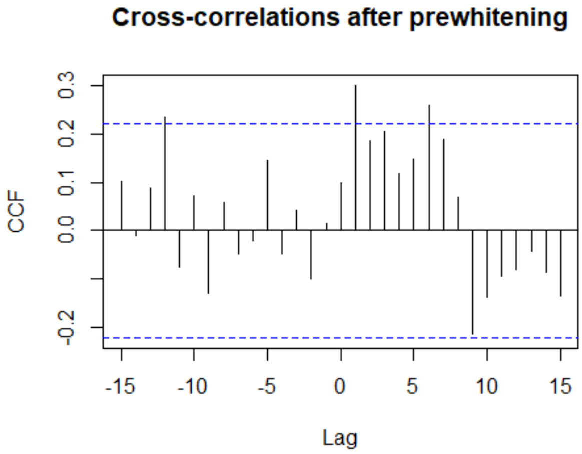

Figure 5. Cross-correlation plot between α_t and β_t

Based on the cross-correlation plot above, it can be obtained that it is significant for the first time in the first lag, thus indicating $b = 1$, $s = 0$ and $r = 2$. The value of the order r is obtained by matching the cross-correlation plot with the impulse function. In addition, the value of $b = 1$ can be interpreted as follows. Namely, a one-month delay before the gold prices (input series X_t) affects the value of the BI rate (output series Y_t).

The transfer function model obtained based on the b, s, and r orders is then checked for residuals using the ACF, PACF, and EACF values to determine the order (p_n, q_n). The best transfer function model is determined based on the smallest AIC value and significant model parameters. The best transfer function model is the transfer function model with $(b, r, s) = (1,2,0)$ and the noise series model $(p_n, q_n) = (1,0)$. The model formed is as follows:

$$Y_t = \frac{\omega_s(B)}{\delta_r(B)} x_{t-b} + \frac{\theta_q(B)}{\phi_p(B)} a_t$$

$$Y_t = \frac{\omega_0(B)}{\delta_2(B)} x_{t-1} + \frac{\theta_0(B)}{\phi_1(B)} a_t$$

$$Y_t = \frac{1}{1 + \delta_1 B + \delta_2 B^2} x_{t-1} + \frac{1}{(1 - \phi_1 B)} a_t$$

$$Y_t = \frac{1}{1 + 0.39027B + 0.43043B^2} x_{t-1} + \frac{1}{(1 - 0.99891B)} a_t$$

The transfer function model $(b, r, s) = (1,2,0)$ and the noise series model $(p_n, q_n) = (1,0)$ were then carried out diagnostic tests for residuals and fulfilled the assumption of residual white noise. This transfer model is then compared with the standard ARIMA model to predict the value of the Bank Indonesia benchmark interest rate (BI Rate), where the results of the forecasting accuracy of the two models are presented in **Table 2**.

Table 2. Accuracy of transfer function model and ARIMA model

Accuracy	ARIMA	Transfer Function
RMSE	5.271958	1.074888
MASE	49.69294	7.846724
MAE	5.230835	0.825970

Accuracy	ARIMA	Transfer Function
MAPE	99.6623	17.7723

Data source: *the output of accuracy from the R program*

Based on the accuracy of the forecast results of the two models, it can be obtained that the accuracy of the data model using the transfer function has greater accuracy than that modelled using the ARIMA model. This accuracy can be seen from the accuracy of the RMSE, MASE, MAE, and MAPE transfer function smaller than ARIMA.

4. CONCLUSIONS

Based on the analysis that has been done, the best model obtained is the transfer function model with a value of $b = 1$, $s = 0$, $r = 2$, and the noise series model $(p_n, q_n) = (1,0)$. In addition, the transfer function model produces a better forecasting value than the ARIMA model, with a smaller accuracy value of 0.1777233 based on the MAPE value. The results of this analysis can be used as a reference in the application of the transfer function method and its accuracy results compared to the ARIMA model. For further research, it is expected to be able to use time-series data containing seasonal patterns.

REFERENCES

[1] D. G. Baur and L. A. Smales, "Hedging geopolitical risk with precious metals," *J Bank Financ*, vol. 117, p. 105823, 2020.

[2] N. M. Nugraha, D. Herlambang, D. N. S. Nugraha, and S. Amalia, "The Influence of Macroeconomic Factors on the Volatility of Composite Price Stock Index: A Study on the Indonesia Stock Exchange," *International Journal of Psychosocial Rehabilitation*, vol. 24, no. 1, pp. 2507–2513, 2020.

[3] H. Tandon, P. Ranjan, T. Chakraborty, and V. Suhag, "Coronavirus (COVID-19): ARIMA based time-series analysis to forecast near future," *arXiv preprint arXiv:2004.07859*, 2020.

[4] H. Tandon, P. Ranjan, T. Chakraborty, and V. Suhag, "Coronavirus (COVID-19): ARIMA-based Time-series Analysis to Forecast near Future and the Effect of School Reopening in India," *J Health Manag*, p. 09720634221109087, 2022.

[5] K. N. Khikmah, "COMPARISON OF ARIMA AND GARMA'S PERFORMANCE ON DATA ON POSITIVE COVID-19 CASES IN INDONESIA," *BAREKENG: Jurnal Ilmu Matematika dan Terapan*, vol. 16, no. 3, pp. 919–926, 2022.

[6] S. Siami-Namini, N. Tavakoli, and A. S. Namin, "A comparison of ARIMA and LSTM in forecasting time series," in *2018 17th IEEE international conference on machine learning and applications (ICMLA)*, IEEE, 2018, pp. 1394–1401.

[7] G. S. Osho, "A General Framework for Time Series Forecasting Model Using Autoregressive Integrated Moving Average-ARIMA and Transfer Functions," *Int J Stat Probab*, vol. 8, no. 6, pp. 1–23, 2019.

[8] S. A. Mulema and A. Carrión García, "Quality and productivity in aquaculture: Prediction of oreochromis mossambicus growth using a transfer function arima model," *International Journal for Quality Research*, vol. 12, no. 4, pp. 823–834, 2018.

[9] T. Purwa, U. Nafngiyana, and S. Suhartono, "Comparison of arima, transfer function and var models for forecasting cpi, stock prices, and indonesian exchange rate: Accuracy vs. explainability," *Statistics (Ber)*, 2020.

[10] D. A. Kusumawati and N. Asandimitra, "Impact of Global Index, Gold Price and Macro Economic Variable for Indonesia Composite Index," *Research Journal of Finance and Accounting*, vol. 8, no. 2, pp. 53–62, 2017.

[11] E. Endri, Z. Abidin, T. P. Simanjuntak, and I. Nurhayati, "Indonesian stock market volatility: GARCH model," *Montenegrin Journal of Economics*, vol. 16, no. 2, pp. 7–17, 2020.

[12] F. Mahia, A. R. Dey, M. A. Masud, and M. S. Mahmud, "Forecasting electricity consumption using ARIMA model," in *2019 International Conference on Sustainable Technologies for Industry 4.0 (STI)*, IEEE, 2019, pp. 1–6.

[13] M. R. Abonazel and A. I. Abd-Elftah, "Forecasting Egyptian GDP using ARIMA models," *Reports on Economics and Finance*, vol. 5, no. 1, pp. 35–47, 2019.

[14] H. Alabdulrazzaq, M. N. Alenezi, Y. Rawajfih, B. A. Alghannam, A. A. Al-Hassan, and F. S. Al-Anzi, "On the accuracy of ARIMA based prediction of COVID-19 spread," *Results Phys*, vol. 27, p. 104509, 2021.

[15] A. L. Schaffer, T. A. Dobbins, and S.-A. Pearson, "Interrupted time series analysis using autoregressive integrated moving average (ARIMA) models: a guide for evaluating large-scale health interventions," *BMC Med Res Methodol*, vol. 21, no. 1, pp. 1–12, 2021.

[16] C. Karaman, O. Karaman, P.-L. Show, H. Karimi-Maleh, and N. Zare, "Congo red dye removal from aqueous environment by cationic surfactant modified-biomass derived carbon: equilibrium, kinetic, and thermodynamic modeling, and forecasting via artificial neural network approach," *Chemosphere*, vol. 290, p. 133346, 2022.

[17] S. Siami-Namini, N. Tavakoli, and A. S. Namin, "A comparison of ARIMA and LSTM in forecasting time series," in *2018 17th IEEE international conference on machine learning and applications (ICMLA)*, IEEE, 2018, pp. 1394–1401.

[18] L. L. Zhao, J. Mbachu, and Z. Liu, "Modelling Residential Building Costs in New Zealand: A Time-Series Transfer Function Approach," *Math Probl Eng*, vol. 2020, pp. 1–18, Jan. 2020, doi: 10.1155/2020/7028049.

[19] J.-Y. Lee, "Search trends preceding increases in suicide: A cross-correlation study of monthly Google search volume and suicide rate using transfer function models," *J Affect Disord*, vol. 262, pp. 155–164, 2020.

[20] W. Ahmad, S. A. Khan, M. M. M. Islam, and J.-M. Kim, "A reliable technique for remaining useful life estimation of rolling element bearings using dynamic regression models," *Reliab Eng Syst Saf*, vol. 184, pp. 67–76, 2019.

[21] E. Aivazidou and I. Politis, "Transfer function models for forecasting maritime passenger traffic in Greece under an economic crisis environment," *Transportation Letters*, vol. 13, no. 8, pp. 591–607, 2021.

[22] B. Aljoumani, J. A. Sànchez-Espigares, N. Cañameras, G. Wessolek, and R. Josa, "Transfer function and time series outlier analysis: modelling soil salinity in loamy sand soil by including the influences of irrigation management and soil temperature," *Irrigation and drainage*, vol. 67, no. 2, pp. 282–294, 2018.

[23] H. De and G. Acquah, "Comparison of Akaike information criterion (AIC) and Bayesian information criterion (BIC) in selection of an asymmetric price relationship," *J Dev Agric Econ*, vol. 2, pp. 1–6, Feb. 2010.

[24] S. Ray and B. Bhattacharyya, "Availability in different source of irrigation in India: a statistical approach," *Ecosystem*, p. 109, 2015.

[25] T. Ahmad and H. Chen, "Short and medium-term forecasting of cooling and heating load demand in building environment with data-mining based approaches," *Energy Build,* vol. 166, pp. 460–476, 2018.

[26] K. Wang, X. Qi, and H. Liu, "A comparison of day-ahead photovoltaic power forecasting models based on deep learning neural network," *Appl Energy*, vol. 251, p. 113315, 2019.