BAREKENG: Journal of Mathematics and Its Applications
June 2023 Volume 17 Issue 2 Page 1007–1016
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol17iss2pp1007-1016

PRICING EUROPEAN BASKET OPTION USING THE STANDARD MONTE CARLO AND ANTITHETIC VARIATES

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ABSTRACT

Article History:

Received: 21st January 2023
Revised: 28th April 2023
Accepted: 30th April 2023

Keywords:

Antithetic Variates;
Basket Option;
Monte Carlo

Using the standard Monte Carlo and antithetic variates method in determining the price of the European basket option produces a good solution because they give small relative errors. Based on the relative error value, the antithetic variates method is faster in approaching the exact solution than the standard Monte Carlo method. The two methods show that the more simulations performed, the smaller the resulting relative error value and the closer to the exact solution. This paper also shows the effect of parameter values to the value of the basket option price using the antithetic variates method. The parameters used are strike price (K), maturity date (T), and volatility (σ). The simulation results from the antithetic variates method showed that the longer the option's maturity date, the higher the price of the call option and put option. Furthermore, the higher the strike prices are, the lower the call option prices are. Conversely, the higher the strike prices are, the higher the put option prices are. In addition, the greater the volatility values of the option are, the higher the prices of call options and put options will be.

How to cite this article:

S. B. Sitepu, D. C. Lesmana and R. Budiarti., "PRICING EUROPEAN BASKET OPTION USING THE STANDARD MONTE CARLO AND ANTITHETIC VARIATES," *BAREKENG: J. Math. & App.*, vol. 17, iss. 2, pp. 1007-1016, June, 2023.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article • Open Access

1. INTRODUCTION

Indonesian citizens are increasingly aware of the importance of investment. Investments can be made in real assets and financial assets. Real assets consist of property and gold, while financial assets include bonds, mutual funds, stocks, etc. This can be seen from the number of investors in the Indonesian capital market which continues to increase, especially during the Covid-19 pandemic since 2020. 2020 is also referred to as the year of the rise of domestic retail investors. Based on data from the Indonesia Stock Exchange, from 2019 to the end of 2020, capital market investors increased by 56% to 3.88 million [1]. Meanwhile, PT Kustodian Sentral Efek Indonesia (KSEI) noted that until the end of the first quarter of 2022, the number of Indonesian capital market investors reached 8.3 million, an increase of 12.13% from the position at the end of 2021 [2]. One of the most popular financial market instruments is stocks.

Stock investment is an investment with high risk due to the uncertainty of market prices on every trading day. To minimize the risk from this uncertainty and at the same time maximize profits, one of the efforts that investors can take is to hedge stocks. One of the hedging instruments that can be done by investors is to buy options.

Options are part of derivative products. In the world of capital markets, an option is a right, but not an obligation, based on an agreement (contract) to buy or sell an underlying asset. An option to sell (put option) is a contract that gives the option holder the right to sell the underlying asset at a certain time (expiration date or maturity date) and an agreed price (strike price). An option to buy (call option) is a contract that gives the option holder the right to buy the underlying asset at a certain time and an agreed price. Based on the execution time, options are divided into two: American and European. American options can be exercised at any time until the maturity date, while European options can only be exercised at the maturity date [3].

A single stock purchased will require one option to hedge. However, investors generally invest in several stocks at once, resulting in higher costs incurred for purchasing options. In this case, buying a basket option can be an optimal alternative because a basket option is an option whose payoff value depends on a group of (n) assets [4]. Basket options are one of the most popular exotic options in the commodity and equity markets [5] and are typically traded over the counter [6]. The advantage of the basket option is its ability to efficiently hedge risk on multiple assets at the same time [7]. Investors can manage risk for a basket or portfolio, in a single transaction, rather than protecting each asset [8]. When investors choose basket options, these options will be cheaper than choosing single stock options [9].

In this paper, the European basket option price will be determined using the standard Monte Carlo and Antithetic Variates method with the assumption that the asset follows a correlated geometric Brownian motion process. The Monte Carlo method is used to determine the estimated value of the option by using a numerical simulation. The advantage of the Monte Carlo method is that the more simulations carried out, the closer the estimate will be to the analytical solution [10]. The antithetic variates technique is one of the variance reduction techniques which is an improvement from the Monte Carlo method. The variance reduction method aims to achieve a more precise estimate [11]. [9] and [12] in their scientific papers state that the determination of European basket options using the Monte Carlo method can be quite good because each case's relative error is quite small. The Monte Carlo approach has proven to be an efficient and flexible computational tool in modern finance [13]. The antithetic variates technique is used to reduce variance by replacing random variables with other negatively correlated random variables without increasing the sample size in simulating option prices [14]. [15] and [16] in their scientific work, stated that the Variance Reduction method can significantly increase the accuracy of estimates and reduce the number of simulations required.

Based on this background, the purpose of this study is to determine the price of European basket options using the standard Monte Carlo and the antithetic variates method.

2. RESEARCH METHODS

2.1. Research data

The data used in this paper was taken from https://finance.yahoo.com/ in the form of stock price data with normally distributed returns so that it fulfills the model assumptions. The stock price data selected in this case are ASII, UNVR, ANTM, TLKM, and BBCA stocks with a monthly period of 3 years from March 2019 to March 2022.

2.2. Research Flow

This research was conducted using the following steps:

1. To retrieve stock price data from https://finance.yahoo.com/
2. To calculate the log return of each stock

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right), t = 1,2,3,\dots,n \tag{1}$$

3. To test the normality of each logarithm of return
4. To determine the volatility of the i-th the stock σ_i

$$\hat{\sigma_i} = \frac{s}{\sqrt{\tau}} \tag{2}$$

5. To calculate the correlation between stocks, i.e. ρ_{ij} with $i,j = 1,2,\dots d$
6. To arrange ρ_{ij} into a correlation matrix with size $d \times d$

$$R = \begin{pmatrix} \rho_{11} & \cdots & \rho_{1d} \\ \vdots & \ddots & \vdots \\ \rho_{d1} & \cdots & \rho_{dd} \end{pmatrix} \tag{3}$$

7. To calculate matrix L with Cholesky factorization

$$R = L^T L \tag{4}$$

2.3. Determining the price of a basket option using the standard Monte Carlo method

In determining the price of a basket option using the standard Monte Carlo method, we use the following steps:

1. To input the number of simulations M, the prevailing interest rate r, the volatility of each stock σ_i, the initial stock price $S_i(0), i = 1,2,\dots d$ where d is the number of stocks, and the strike price K
2. To generate a random variable that is normally distributed i.i.d $\varepsilon_i^k \sim N(0,1)$
3. To simulate the stock price at the time of T

$$S_i^k(T) = S_i(0)exp\left[\left(r - \frac{\sigma_i^2}{2}\right)T + \sigma_i\sqrt{T}\sum_{j=1}^{i} L_{ij}\,\varepsilon_j^k\right] \tag{5}$$

4. To determine the option price:
   i. To determine the price of call options
      a. To determine the value of Y_C^k

$$Y_C^k = e^{-rT}\left(\sum_{i=1}^{d} w_i\, S_i^k(T) - K\right)^+ \tag{6}$$

      b. To determine the average of Y_C^k which is an estimator of the call option price

$$\hat{Y}_C = \frac{\sum_{k=1}^{M} Y_C^k}{M} \tag{7}$$

   ii. To determine the price of put options
      a. To determine the value of Y_P^k

$$Y_P^k = e^{-rT}\left(\sum_{i=1}^{d} K - w_i\, S_i^k(T)\right)^+ \tag{8}$$

b. To determine the average of Y_P^k which is an estimator of the put option price

$$\hat{Y}_P = \frac{\sum_{k=1}^{M} Y_P^k}{M} \quad (9)$$

2.4. Determining the price of a basket option using the antithetic variates method

In determining the price of a basket option using the antithetic variates method, use the following steps:

1. To input the number of simulations M, the prevailing interest rate r, the volatility of each stock σ_i, the initial stock price $S_i(0)$, and the strike price K
2. To generate a random variable that is normally distributed i.i.d $\varepsilon_i \sim N(0,1)$ and get $-\varepsilon_i^k \sim N(0,1)$
3. To simulate the stock price at the time of T using **Equation (5)**
4. To determine the option price

   i. To determine the price of call options

   a. To determine the value of Y_C^k using **Equation (6)**

   b. To determine the average of Y_C^k which is an estimator of the call option price using **Equation (7)**

   ii. To determine the price of put options

   a. To determine the value of Y_P^k using **Equation (8)**

   b. To determine the average of Y_P^k which is an estimator of the put option price using **Equation (9)**

3. RESULTS AND DISCUSSION

3.1. Data Description

In determining the price of the European basket option, historical data on stock prices are used to calculate the logarithmic return which is normally distributed as assumed by the Black-Scholes model. The historical data is also needed to determine the volatility of the data for each stock used. The data used in this study is monthly historical data from the closing prices of ASII, UNVR, ANTM, TLKM, and BBCA stocks for three years in the period March 19, 2019 – March 19, 2022. In graphical form, monthly data on stock prices will be shown in **Figure 1** below. Stock price movements from March 19, 2019, to March 19, 2022.

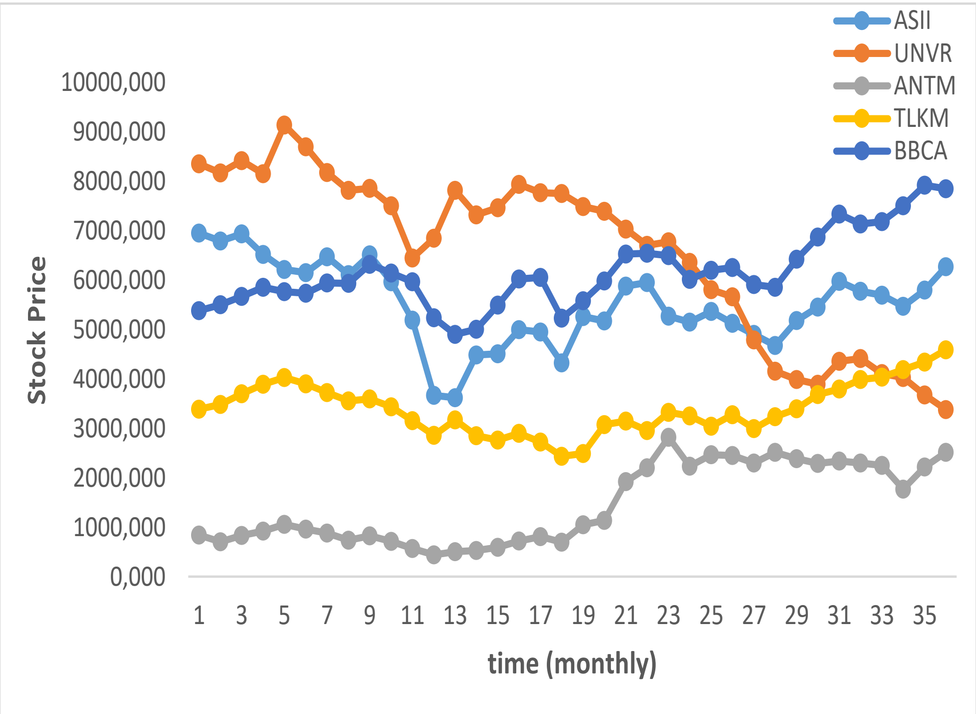

Figure 1. The chart of stock price movements for three years

3.2. Determining the logarithm of return from stock data

The logarithmic return is determined by using Equation (1) and presented in Figure 2.

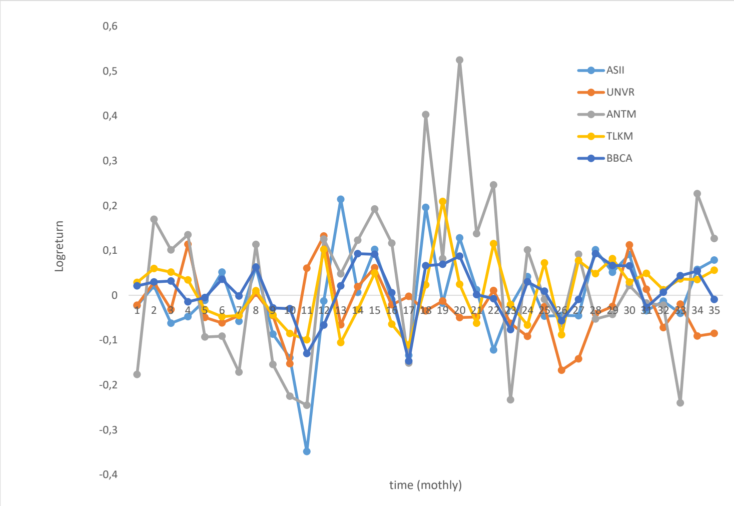

Figure 2. The graph of the logarithmic return

3.3. Determining the Volatility

To calculate the volatility from historical stock price data, we use Equation (2). The results are as follows:

The volatility of ASII	:	$\sigma_1 = 35\%$ per year
The volatility of UNVR	:	$\sigma_2 = 24\%$ per year
The volatility of ANTM	:	$\sigma_3 = 62\%$ per year
The volatility of TLKM	:	$\sigma_4 = 25\%$ per year
The volatility of BBCA	:	$\sigma_5 = 20\%$ per year

3.4. Determination of the Correlation Matrix

The correlations between stocks' returns are given in the following table:

Table 1. The correlation between stocks

Number	Stocks	Weight w_i	Volatility σ_i	Correlation (ρ_{ij}) ASII	UNVR	ANTM	TLKM	BBCA
1	ASII	0.2	35%	1.0000	0.1568	0.0507	0.6007	0.3182
2	UNVR	0.2	24%	0.1568	1.0000	-0.783	-0.374	-0.750
3	ANTM	0.2	62%	0.0507	-0.783	1.0000	0.3269	0.6939
4	TLKM	0.2	25%	0.6007	-0.374	0.3269	1.0000	0.6725
5	BBCA	0.2	20%	0.3182	-0.750	0.6939	0.6725	1.0000

From this correlations, a square matrix R of order 5×5 is formed as follows:

$$R = \begin{vmatrix} 1.0000 & 0.1568 & 0.0507 & 0.6007 & 0.3182 \\ 0.1568 & 1.0000 & -0.7837 & -0.3743 & -0.7504 \\ 0.0507 & -0.7837 & 1.0000 & 0.3269 & 0.6939 \\ 0.6007 & -0.3743 & 0.3269 & 1.0000 & 0.6725 \\ 0.3182 & -0.7504 & 0.6939 & 0.6725 & 1.0000 \end{vmatrix}$$

Using the Cholesky factorization in **Equation** (**4**), the lower triangular matrix L is obtained as follows:

$$L = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.156773 & 0.987635 & 0 & 0 & 0 \\ 0.050669 & -0.80156 & 0.595761 & 0 & 0 \\ 0.60066 & -0.47433 & -0.14053 & 0.62807 & 0 \\ 0.318239 & -0.81027 & 0.047453 & 0.165074 & 0.46117 \end{vmatrix}$$

3.5. Numerical Result of Option Price

The previously estimated parameter values are used to approximate the stock price at maturity date. This stock prices are then used to find the option price using the Monte Carlo and antithetic variates method. To use the Monte Carlo method, we use the number of simulations (M), time interval $(\tau) = \frac{1}{12}$ year, maturity date of one year, the risk-free interest rate of 3.5% per year, and the initial stock price (S_0) of each stock is as follows:

Initial stock price ASII : $S_{01} = 5273$

Initial stock price UNVR : $S_{02} = 6778$

Initial stock price ANTM : $S_{03} = 2820$

Initial stock price TLKM : $S_{04} = 3323$

Initial stock price BBCA : $S_{05} = 6496$

Table 2 presents the simulation results using the standard Monte Carlo and antithetic variates method for call option prices with various specified values, along with the relative error values of each simulation. The relative error value of each simulation can be calculated from the percentage difference in the price of the option to the exact solution.

Table 2. The prices and relative errors of simulated call options

Simulations (M)	Monte Carlo Call Option	Monte Carlo Relative Error	Antithetic Variates Put Option	Antithetic Vartiates Relative Error
10	330.988	0.051998	360.779	0.033113
100	379.636	0.087337	345.363	0.011031
1000	352.728	0.010268	349.178	0.009332
10000	348.228	0.002621	349.504	0.002529
100000	350.138	0.002850	349.505	0.002525
1000000	349.257	0.000327	349.341	0.000362

In determining the relative error value, the value of the exact solution used is the solution value from the maximal simulation. The maximal simulation iteration that could be processed according to the maximum memory capacity available on the computer, i.e M = 10000000. In that simulation the price of the call option using the standard Monte Carlo method is 349.143, and the price of the call option using the antithetic variates method is 349.215. The relative error of call options using the standard Monte Carlo method is 0.000326514 ≈ 0.0327%, and the relative error of call options using the antithetic variates method is 0.000361955 ≈ 0.0362%.

From **Table 2**, it can be seen that the relative error value is getting smaller when the more simulations are carried out. This shows that simulations are processed, the closer the results will be to the exact solution. Based on the relative error value, the antithetic variates method approaches the exact solution faster.

The following figures show graphs that compare the price of an estimate call option with an exact solution.

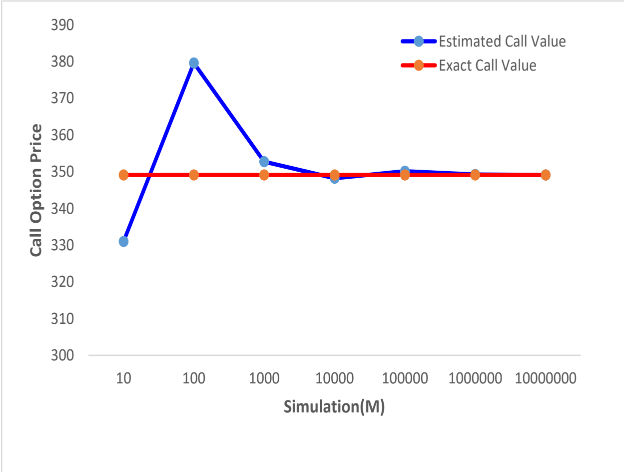

Figure 3. The comparison of estimated call option prices with exact solutions using standard Monte Carlo

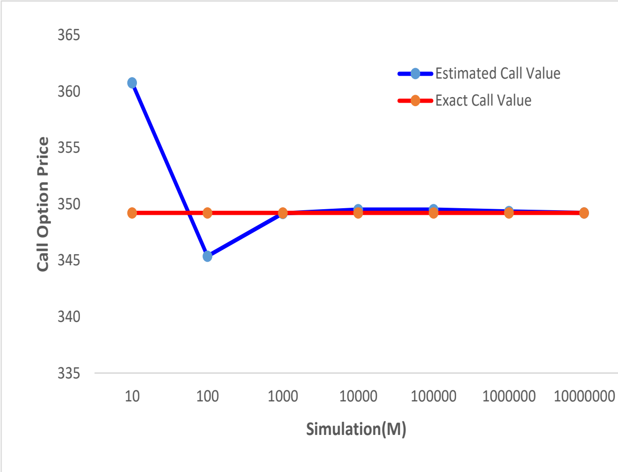

Figure 4. The comparison of estimated call option prices with exact solutions using using antithetic variates

Table 3 presents the simulation results using the standard Monte Carlo method and antithetic variates for call option prices with various selected values of M accompanied by the relative error values of each simulation.

Table 3. The prices and relative errors of simulated put options

Simulations (M)	Monte Carlo Put Option	Monte Carlo Relative Error	Antithetic Variates Put Option	Antithetic Variates Relative Error
10	260.668	0.08681	263.5202	0.099046
100	227.322	0.05222	228.5962	0.046609
1000	246.169	0.02636	244.809	0.021008
10000	236.272	0.01491	239.3915	0.001586
100000	238.386	0.00609	239.2461	0.002193
1000000	240.073	0.00094	239.5915	0.000752

Table 3 shows the relative error value obtained with many simulations M = 10000000. On that simulation, the price of put options using the Monte Carlo method is 239.848, and the price using the antithetic variates method is 239.7718. The relative error of the put option using the standard Monte Carlo method is 0.00094 ≈ 0.094%, and the relative error of the put option using the antithetic variates method is 0.000752 ≈ 0.0752%.

From **Table 3**, it can be seen that the relative error value of the put option gets smaller the more simulations are carried out. This shows that the more simulations are processed, the closer the results will be to the exact solution. Based on the relative error value, the antithetic variates method approaches the exact solution faster.

The following figures show a comparison chart of approximate put option prices with exact solutions.

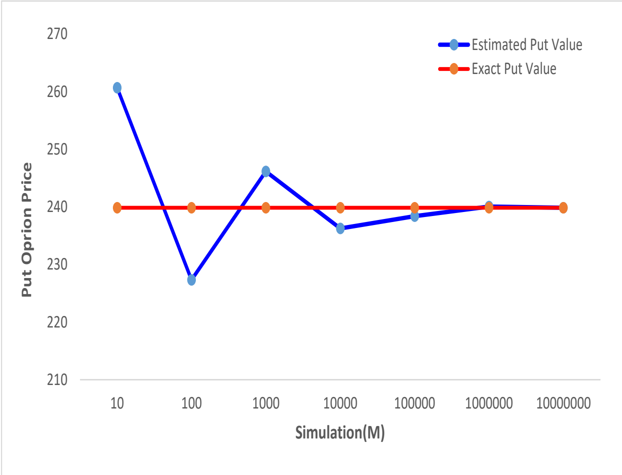

Figure 5. The comparison of estimation put option prices with exact solutions using the standard Monte Carlo

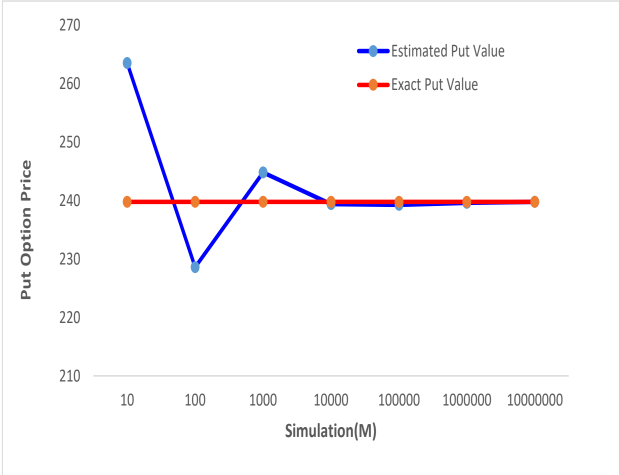

Figure 6. The comparison of the estimation put option price with an exact solution using antithetic variates

3.6. Changes in the price of European basket options against changes in parameter values

The following will show changes in the value of parameters that can affect the price of European basket options using the antithetic variates method. The parameters used are strike price (K), maturity date (T), and volatility (σ). The simulation is carried out using 100000 simulation and the risk-free interest rate (r) of 3.5% per year.

1. Changes in the price of European basket options to changes in maturity (T)

Six-time periods will be taken to see the change in the price of the call option against change in maturity, starting from 6 months to 3 years, and the strike price used is 5000.

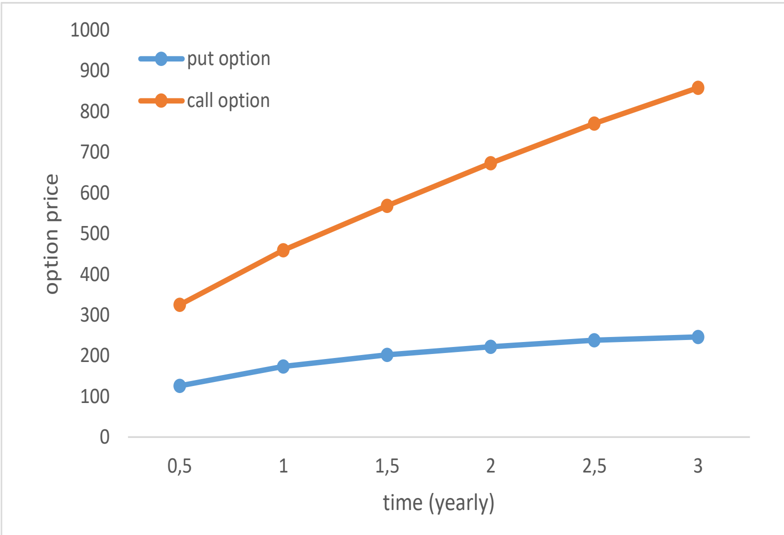

Figure 7. The graph of price changes in call options and put options against changes in maturity

Figure 7 shows that the longer the option expiration date, the higher the price of the call option and put option will be. This shows that the maturity date (T) positively affects call options and put options.

2. Changes in the price of European Basket options to changes in strike price (K)

To see the change in the price of the call option against the change in the strike price, the maturity period of one year is taken and the strike price used is seven values starting from 4500 to 6000.

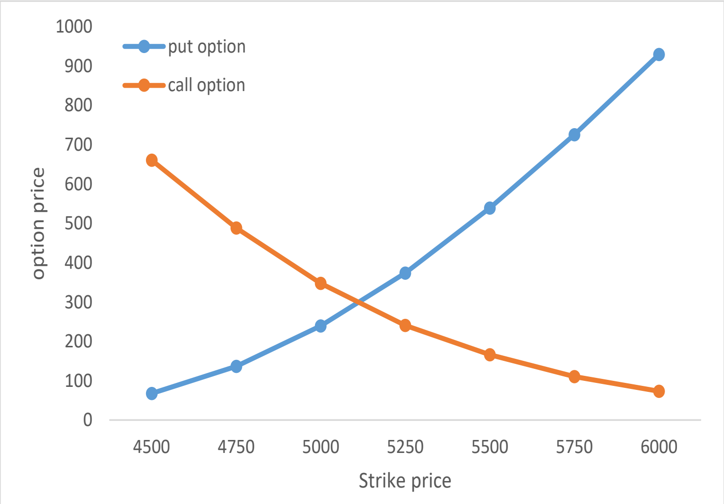

Figure 8. The graph of change in the price of options against changes in strike price (K)

Figure 8 shows that the higher the strike price, the lower the call option price. Conversely, the higher the strike price, the higher the put option price. This shows that the strike price has a negative effect on changes in the call option price but has a positive effect on changes in the put option price.

3. Changes in the price of European basket options against changes in volatility (σ)

To see changes in option prices against changes in volatility, a period of 1 year will be taken, and the strike price used is 5000, and the volatility used starts from 20% to 38% per year.

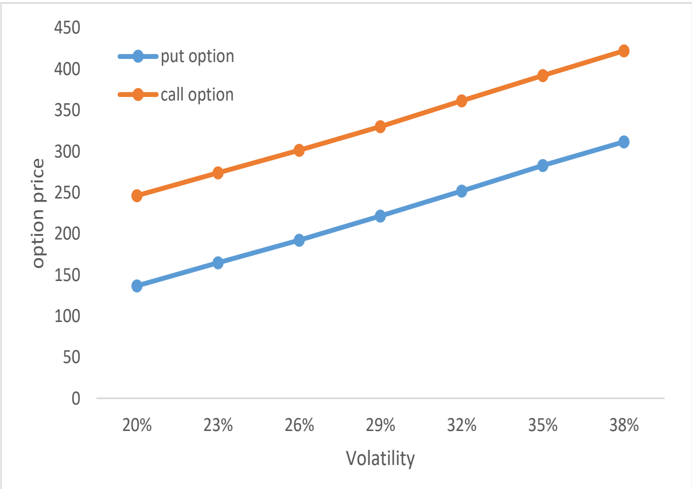

Figure 9. The graph of price changes for call and put options against changes in volatility (σ)

Figure 9 shows that the greater the volatility of the options, the higher the price of call options and put options. This shows that volatility positively affects call options and put options.

4. CONCLUSIONS

The conclusions of this research are as follows:

1. Using the standard Monte Carlo and antithetic variates method to determine the price of European basket options produce a good solution because they give relatively small errors.
2. Based on the relative error value, this study found that the antithetic variates method approached the exact solution faster than the standard Monte Carlo method.
3. The two methods show that the more simulations performed, the smaller the relative error value is and the closer the result is to the exact solution.
4. The simulation results from the standard Monte Carlo method confirmed that the longer the expiration time of the option, the higher the price of the call and put options will be.

5. In the strike price parameter, the higher the strike prices are, the lower the call option prices are. The higher the strike prices are, the higher the put option prices are.
6. In the volatility parameter, the greater the volatility value of the option is, the higher the prices of call options and put options are.

REFERENCES

[1] S. Sidik, "Naik 56%, Jumlah Investor Pasar Modal RI Mencapai 3,88 juta," *cnbcindonesia.com*, 2021. https://www.cnbcindonesia.com/market/20210629153854-17-256818/naik-56-jumlah-investor-pasar-modal-ri-mencapai-388-juta (accessed Feb. 14, 2022).

[2] T. Purwanti, "Investor Pasar Modal RI Makin Banyak, Tembus 8,3 Juta!," *cnbcindonesia.com/*, 2022. https://www.cnbcindonesia.com/market/20220414130213-17-331715/investor-pasar-modal-ri-makin-banyak-tembus-83-juta (accessed May 06, 2022).

[3] J. C. Hull, "Options, Futures and Other Derivatives 7th John Hull.pdf." Pearson Education, Inc., 2009.

[4] C. Albanese and G. Campolieti, *Advanced Derivatives Pricing and Risk Management*. Elsevier Academic Press, 2006.

[5] K. Shiraya and A. Takahashi, "An approximation formula for basket option prices under local stochastic volatility with jumps: An application to commodity markets," *J. Comput. Appl. Math.*, vol. 292, no. 25380389, pp. 230–256, 2016, doi: 10.1016/j.cam.2015.06.027.

[6] A. Leccadito, T. Paletta, and R. Tunaru, "Pricing and hedging basket options with exact moment matching," *Insur. Math. Econ.*, vol. 69, pp. 59–69, 2016, doi: 10.1016/j.insmatheco.2016.03.013.

[7] G. Karipova and M. Magdziarz, "Pricing of basket options in subdiffusive fractional Black–Scholes model," *Chaos, Solitons and Fractals*, vol. 102, pp. 245–253, 2017, doi: 10.1016/j.chaos.2017.05.013.

[8] C. Mitchell, "Basket Option," 2021. https://www.investopedia.com/terms/b/basketoption.asp (accessed May 06, 2022).

[9] M. I. Maulana, "Penentuan Harga Opsi Beli Tipe Eropa Menggunakan Metode Monte Carlo," Bogor, 2018. [Online]. Available: http://etd.repository.ugm.ac.id/home/detail_pencarian/165111

[10] P. Lötstedt, J. Persson, L. von Sydow, and J. Tysk, "Space-time adaptive finite difference method for European multi-asset options," *Comput. Math. with Appl.*, vol. 53, no. 8, pp. 1159–1180, 2007, doi: 10.1016/j.camwa.2006.09.014.

[11] F. Mehrdoust, K. Fathi, and A. A. Rahimi, "Numerical simulation for multi-asset derivatives pricing under black-scholes model," *Chiang Mai J. Sci.*, vol. 40, no. 4, pp. 725–735, 2013.

[12] J. Zeng, "Pricing Asian Options and Basket Options by Monte Carlo Methods," Wuhan Technology University, Wuhan, China, 2017.

[13] P. Boyle, M. Broadie, and P. Glasserman, "Monte Carlo methods for security pricing," *J. Econ. Dyn. Control*, vol. 21, pp. 1267–1321, 1997, [Online]. Available: Elsevier

[14] B. Wang and L. Wang, "Pricing Barrier Options using Monte Carlo Methods," 2011.

[15] Y. Sun and C. Xu, "A hybrid Monte Carlo acceleration method of pricing basket options based on splitting," *J. Comput. Appl. Math.*, vol. 342, pp. 292–304, 2018, doi: 10.1016/j.cam.2018.03.045.

[16] L. H. T. W. Putri, K. Dharmawan, and S. I. Wayan, "Penentuan Harga Jual Opsi Barrier Tipe Eropa dengan Metode Antithetic Variate pada Simulasi Monte Carlo," vol. 7 (2), pp. 71–78, 2018, doi: https://doi.org/10.24843/MTK.2018.v07.i02.p187.