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DETERMINING THE VALUE OF DOUBLE BARRIER OPTION USING STANDARD MONTE CARLO, ANTITHETIC VARIATE, AND CONTROL VARIATE METHODS

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ABSTRACT

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In this paper, we applied the standard Monte Carlo, antithetic variate, and control variates methods to value the double barrier knock-in option price. The underlying asset used in the calculation of the double barrier knock-in option is the share of ANTM from April 1, 2019, until March 1, 2022. The value of the double barrier knock-in option is simulated using standard Monte Carlo, antithetic variate, and control variates methods. The results showed that all the methods converge to the exact solution, with the control variate method being the fastest. The Standard Monte Carlo method has the least computational time, followed by the control variate and antithetic variate methods. Compared to the other methods, control variate is the most effective and efficient in determining the value of the double barrier knock-in option based on the option value, relative error, and computational time. The antithetic variate method converges faster to the exact solution compared to standard Monte Carlo. However, it has the longest computation time compared to the other methods.

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1. INTRODUCTION

The development of financial assets in Indonesia today is increasing rapidly. Referring to the data from Kustodian Sentral Efek Indonesia (KSEI), by December 2021, stock market investors have increased to 92,99% in 2020 to 7,4 million investors [1]. The most popular investment product in stock is because it gives a higher yield. The risk in stock investments is also high, so it is necessary to hedge. As an alternative to hedge the risk, investors can use derivative products, one of which is an option.

According to Hull, exotic options are much more profitable than plain vanilla options [2]. One of the widely used exotic options is the barrier option because it has a barrier that gives additional protection for the investors and allows them to participate in determining the barrier. On the barrier option, payoff and option value depend on the achievement of a barrier from the underlying assets price during a certain period of time [3].

In this paper, we will discuss determining the option value of the double barrier. A double barrier is a barrier option that has two barriers (lower and upper barrier). Double barrier option that has been researched in financial mathematic literature were mostly option of double barrier knock-out as written by [4], [5], [6], [7], [8] and [9]. In addition to the literatures, this paper is focuses in the determination of double barrier knock-in option price. The benefit of double barrier knock-in option compared to the other barrier option is that this option gives protection to the investor when the underlying asset price is highly fluctuated.

Option valuation in the stock market is an important matter used to count the fair value of the options and to avoid the possibility of arbitrage. Black Scholes Model to determine stock price movement becomes fundamental in the calculation of the analytic value of the options. However, the analytic solution of exotic options in general is hard to determine, so numeric methods are needed to value this kind of option.

Various approaches of numeric methods to stipulate the barrier option value have been developed, such as the Monte Carlo method and the binomial lattice method. Noury and Abbasi researched the determination of the option value of double barrier knock-out using a modification of the Monte Carlo method [10]. The idea of this modified method is to use uniformly distributed random numbers and an exit probability of performing a robust estimation of the first time the stock price hits the barrier, resulting in the numeric solution being closer to the analytical value and smaller error when underlying asset touched the barrier for the first time.

In their research, Wang and Wang stated that Monte Carlo simulation is proven to be effective and simple in approximating the option value of barrier down-and-out option [11]. However, Monte Carlo's convergence rate is low, and it needs many simulations to gain a closer result to the analytic solution. The efficiency and accuracy of option value estimation of the barrier option can be increased by using the variance reduction technique. The Monte Carlo method with variance reduction consists of several types of techniques, such as control variates, antithetic variates, stratified sampling, Latin hypercube sampling, matching underlying assets, and importance sampling [12].

Monte Carlo Method of antithetic variates on research by Putri in determining the option value of barrier down-and-out, yielding a smaller error value and also shorter confidence interval compared to those using standard Monte Carlo simulation [13]. This finding is strengthened by Alzubaidi, which in his research, stated that the antithetic variates technique on the option calculation of barrier down-and-out option accelerated the Monte Carlo simulation and gave twice the accuracy than the standard Monte Carlo simulation [14]. Maulida, in her study of using Monte Carlo control variates, gave better calculation results on option value double barrier knock-out and standard deviation, also a smaller error than that of the standard Monte Carlo method [15].

To determine the best method in approximating the value of double barrier knock-in option, several methods were compared in this paper. The methods that were being compared are standard Monte Carlo, antithetic variate, and control variate methods. The values that were being compared are the option values, the relative errors, and the computation time of each method.

2. RESEARCH METHODS

Steps used in this study are as follow:

a. To retrieve secondary data of several stock prices from www.finance.yahoo.com.
b. To calculate the return of the retrieved stock price then examined the normality of each return stock. Next, choosing the normal distributed stock, the study used the stock price of PT Aneka Tambang Tbk (ANTM) from April 1, 2019 until March 1, 2022.
c. To approximate the parameter value needed in simulating the option prices, which is the risk-free rate (r), volatility (σ), initial stock price (S_0), the strike price (K), the number of simulations (M), maturity date (T), time-span (Δt), upper barrier (U), lower barrier (L) and the number of monitoring points (N).

2.1 The Algorithm of Determining the Value of Double Barrier Option using Standard Monte Carlo Method

Steps of determining the value of double barrier option using Standard Monte Carlo method are as follow:

a. To generate the random variable of $E_{M\times N} = (\varepsilon_{ij}) \sim \phi(0,1)$ with $i = 1, 2, \cdots, M$ and $j = 1, 2, \cdots, N$.
b. To calculate stock price estimation on every monitoring point using the random variable that has been generated using **Equation (1)**.

$$S_j^i = S_{j-1}^i \, exp\left[\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon_{ij}\sqrt{\Delta t}\right] \tag{1}$$

c. To approximate the payoff of option call using **Equation (2)** and put using **Equation (3)**.

$$f(C^i) = \begin{cases} \max\left(S_N^i - K, 0\right); S_j^i \leq L \; or \; S_j^i \geq U \\ 0\,; L < S_j^i < U \end{cases} \tag{2}$$

$$f(P^i) = \begin{cases} \max\left(K - S_N^i, 0\right); S_j^i \leq L \; or \; S_j^i \geq U \\ 0\,; L < S_j^i < U \end{cases} \tag{3}$$

d. To calculate call option value using **Equation (4)** and put using **Equation (5)**.

$$C^i = e^{-rT}\left[f(C^i)\right] \tag{4}$$

$$P^i = e^{-rT}\left[f(P^i)\right] \tag{5}$$

e. To determine the average of call option value by **Equation (6)** and put using **Equation (7)** so it obtained the double barrier option value estimator using standard Monte Carlo method.

$$V_C = \frac{1}{M}\sum_{i=1}^{M} C^i \tag{6}$$

$$V_P = \frac{1}{M}\sum_{i=1}^{M} P^i \tag{7}$$

2.2 The Algorithm of Determining Double Barrier Option Value using Antithetic Variate Method

Steps of determining the double barrier option value using Monte Carlo antithetic variate method are as follow:

a. To generate the random variable of $E_{M\times N} = (\varepsilon_{ij}) \sim \phi(0,1)$ and $-E_{M\times N} = (-\varepsilon_{ij}) \sim \phi(0,1)$.
b. To calculate stock price estimation on every monitoring point using the **Equation (8)** and **Equation (9)**.

$$^{+}S_j^i = {}^{+}S_{j-1}^i \exp\left[\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon_{ij}\sqrt{\Delta t}\right] \tag{8}$$

$$^{-}S_j^i = {}^{-}S_{j-1}^i \exp\left[\left(r - \frac{\sigma^2}{2}\right)\Delta t - \sigma\varepsilon_{ij}\sqrt{\Delta t}\right] \tag{9}$$

c. To determine payoff of option call of two stock price estimations using **Equation (2)** and calculating the average of payoff by **Equation (10)**.

$$\bar{f}(C^i) = \frac{f({}^{+}C^i) + f({}^{-}C^i)}{2} \tag{10}$$

d. To determine the payoff of option put of two stock price estimations by **Equation (3)** and calculating the average of payoff using **Equation (11)**.

$$\bar{f}(P^i) = \frac{f({}^{+}P^i) + f({}^{-}P^i)}{2} \tag{11}$$

e. To determine the average of call and put options values using **Equation (12)** and **Equation (13)** obtaining the double barrier option value estimator using Monte Carlo Antithetic variate method.

$$V_{C\_AV} = e^{-rT}\left[\frac{1}{M}\sum_{i=1}^{M}\bar{f}(C^i)\right] \tag{12}$$

$$V_{P\_AV} = e^{-rT}\left[\frac{1}{M}\sum_{i=1}^{M}\bar{f}(P^i)\right] \tag{13}$$

2.3 The Algorithm of Determining the Double Barrier option value using Control Variate Method

Steps in determining the double barrier option value using Monte Carlo control variate method are as follow **[16]**:

a. To calculate the European call and put options value of Black-Scholes Model using **Equation (14)** and **Equation (15)**.

$$c = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2) \tag{14}$$

$$p = Ke^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1) \tag{15}$$

with $\mathbf{d_1} = \frac{\ln\left(\frac{S_0}{K}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ and $\mathbf{d_2} = \frac{\ln\left(\frac{S_0}{K}\right)+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \mathbf{d_1} - \sigma\sqrt{\mathbf{T}}$.

b. To generate the random variable of $E_{M\times N} = (\varepsilon_{ij}) \sim \phi(0,1)$.

c. To calculate the stock price estimation on every monitoring point using the generated random variable by **Equation (1)**.

d. To determine the payoff of option call and put of double barrier using **Equation (2)** and **Equation (3)**. Then determining the European payoff of option call and put by the same stock price movement using **Equation (16)** and **Equation (17)**.

$$f(C_E^i) = max\,(S_N^i - K, 0) \tag{16}$$

$$f(P_E^i) = max\,(K - S_N^i, 0) \tag{17}$$

e. To determine the call and put options values of double barrier by **Equation (4)** and **Equation (5)**. Then determining the European call and put options values by **Equation (18)** and **Equation (19)**.

$$C_E^i = e^{-rT}\left[f(C_E^i)\right] \tag{18}$$

$$P_E^i = e^{-rT}\left[f(P_E^i)\right] \tag{19}$$

f. To calculate the correlation of double barrier option value with the European option values. The European option with the highest correlation with the double barrier call option is denoted V_C^E. The European option with the highest correlation with the double barrier put option is denoted V_P^E.

g. To calculate the value of $\theta_C = \frac{cov\left(C^i, V_{Ci}^E\right)}{var\left(V_{Ci}^E\right)}$ and $\theta_P = \frac{cov\left(P^i, V_{Pi}^E\right)}{var\left(V_{Pi}^E\right)}$.

h. To calculate call and put double barrier control variate options value using **Equation (20)** and **Equation (21)**.

$$\tilde{C}^i = C^i + \theta_C\left(c - V^E_{C^i}\right) \tag{20}$$

$$\tilde{P}^i = P^i + \theta_P\left(p - V^E_{P^i}\right) \tag{21}$$

i. To determine the average of call and put options values using **Equation (22)** and **Equation (23)**, so that it would obtained the double barrier option value estimator using Monte Carlo control variate method.

$$V_{C\_cv} = \frac{1}{M}\sum_{i=1}^{M}\tilde{C}^i \tag{22}$$

$$V_{P\_cv} = \frac{1}{M}\sum_{i=1}^{M}\tilde{P}^i \tag{23}$$

3. RESULTS AND DISCUSSION

Underlying assets used in this paper are the monthly non-dividend stock prices of PT Aneka Tambang Tbk (ANTM) from April 1, 2019 until March 1, 2022. The stock price of ANTM was chosen because the return of stock price is normally distributed so that it fulfilled the model assumption. ANTM stock price movement on the April 1, 2019 to March 1, 2022 shown by **Figure 1**.

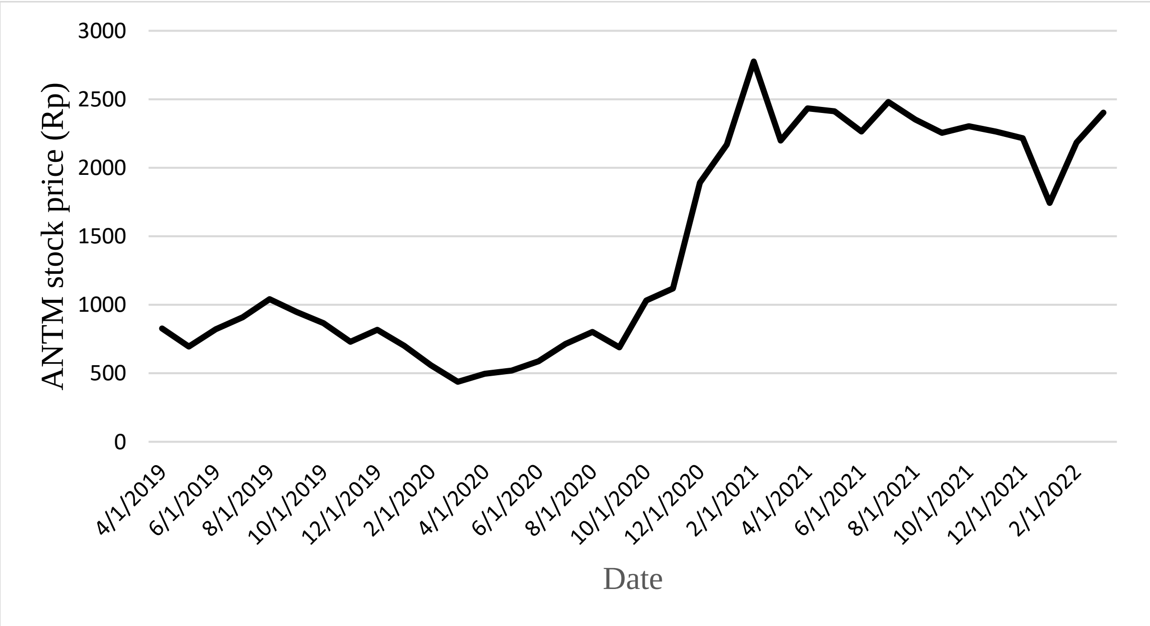

Figure 1. ANTM's monthly stock price movement april 1, 2019 to march 1, 2022 period

In **Figure 1**, it can be seen that the stock prices were highly fluctuated, resulting in the investors coming up with strategies it would have high returns with low risks. One of the ways to hedge is using a double barrier knock-in option. The parameter used in computing the option value of double barrier knock-in is as follows.

1. The initial stock price that is the ANTM stock price on March 1, 2021, (S_0 = 2199.15381).
2. Volatility calculated using ANTM return stock price on April 1, 2019, until March 1, 2022, so it would obtain σ = 0,61 per annum.
3. The risk-free rate used of r = 0.035 per annum is the interest rate issued by Bank of Indonesia on March 2022.
4. Maturity date T = 1 year with the monthly monitoring point (N = 12) and $\Delta t = \frac{T}{N}$.
5. The strike price amounted of K = 2400, and upper barrier U = 2600, as well as lower barrier L = 1800.

Based on the above parameter, the option value estimator of double barrier knock-in would then be simulated using Monte Carlo standard, antithetic variate, and control variate methods using Julia Software 1.7.3 with an increasing number of simulations.

3.1 The Numeric Result of Call Double Barrier Knock-in Option Value

n the calculation of the call double barrier option value using the Monte Carlo control variate method, it was first determined the analytic value from the European call option, which is its control variate. The analytic value of the European call option is the European call option of the Black-Scholes Model, which its value obtained with the help of software RMFI v1.00a amounted of $c = 483.94982$. The numerical result of call double barrier knock-in option value using standard Monte Carlo, antithetic variate, and control variate methods are presented in **Table 1**.

Table 1. The Simulation Result of Call Double Barrier Knock-in Option

M	MC_S	MC_AV	MC_CV	Error MC_S	Error MC_AV	Error MC_CV
10	174.2950	345.9906	483.9498	0.640184	0.282532	0.000152
100	499.6162	452.8337	483.9498	0.031413	0.060975	0.000152
1000	492.5617	475.4432	483.9498	0.016850	0.014090	0.000152
10000	490.8383	478.3275	483.9194	0.013291	0.000725	8.928E-05
100000	486.2505	479.2192	483.8793	0.003820	0.006260	6.407E-06
1000000	485.6455	481.7056	483.8783	0.002571	0.001104	4.340E-06
10000000	483.3909	482.3315	483.8779	0.002083	0.000194	3.513E-06

It was shown in **Table 1** the call double barrier knock-in option value and the relative error value by standard Monte Carlo, antithetic variate and control variate methods. MC_S denotes the standard Monte Carlo method, MC_AC is the Monte Carlo antithetic variate while MC_CV is the Monte Carlo control variate. The relative error value is calculated using this equation

$$\text{relative error} = \frac{|\text{numerical value} - \text{exact value}|}{\text{exact value}}.$$

The exact value of double barrier knock-in option is the maximal simulation iteration that could be processed by the available computer, i. e. with M = 15000000 for the calculation of the exact solution. On that simulation it generated the call option value of the standard Monte Carlo (MC_S) = 484,3999, the call option value of the antithetic variate (MC_AV) = 482,2381 and the call option value of the control variate technique (MC_CV) = 483,8762. Based on the relative error value in **Table 1**, it could be seen that as the simulation number increases, then the relative error value decreases. The relative error value showed that the control variates method is the fastest method in approaching the exact solution compared to the other methods, while using the standard Monte Carlo method took a long time to converge to the exact solution.

3.2 The Numeric Result of Put Double Barrier Knock-in Option Value

In the calculation of the put double barrier option value using the Monte Carlo control variate method, the analytic value was determined first from the European put option, which is its control variate. The European put analytic option value is the European put option of the Black-Scholes Model that its value earned with the help of software RMFI v1.00a amounted of $p = 602{,}24901$. he numerical result of putting double barrier knock-in option value using the standard Monte Carlo, antithetic variate, and control variate methods were presented in **Table 2**.

Table 2. The Numeric Result of Put Double Barrier Knock-in Option

M	MC_S	MC_AV	MC_CV	Error MC_S	Error MC_AV	Error MC_CV
10	809.9871	591.7230	602.249	0.347497	0.010860	0.001783
100	504.8186	596.7321	602.249	0.160182	0.002486	0.001783
1000	622.7478	595.4963	601.5289	0.036006	0.004552	0.000585
10000	594.8424	597.4031	601.2763	0.010418	0.002282	0.00017
100000	603.8574	597.6448	601.2426	0.004579	0.000960	0.000109
1000000	600.6933	598.0731	601.1702	0.000685	0.000244	1.148E-05
10000000	601.3540	598.2668	601.1785	0.000415	7.939E-05	2.329E-06

The exact value of the double barrier knock-in option is the maximal simulation iteration that could be processed by the available computer, i. e. with M = 15000000 for the calculation of the exact solution. In that simulation, it generated the put option value of the standard Monte Carlo (MC_S) = 601,10478, the call option value of the antithetic variate (MC_AV) = 598,2193, and the call option value of the control variate (MC_CV) = 601,1771. Based on the relative error value in **Table 2**, it could be seen that as the simulation number increased, then the relative error value decreased. The relative error value showed the control variate method is the fastest in approaching the exact solution compared to the other methods.

In **Table 2**, it can be seen that the obtained put option value is getting smaller with more simulations done. Different from the call option value, that got bigger with the increasing number of simulations done (**Table 1**). The relative error value generated in the standard Monte Carlo and antithetic variate in the calculation of the put option is smaller compared with the calculation of the call option.

3.3 Computation Time of the Double Barrier Knock-in Option Value Simulation

The efficiency of the standard Monte Carlo, antithetic variate, and control variate methods can be seen from the time needed to compute the double barrier knock-in option value. The computation time in the simulation of double barrier knock-in option value using the standard Monte Carlo, antithetic variate, and control variate methods were presented in **Table 3**.

Table 3. Computation Time of the Double Barrier Knock-in Option Value Simulation

M	MC_S (s)	MC_AV (s)	MC_CV (s)
10	0.000193	0.000284	0.003964
100	0.001014	0.00679	0.005109
1000	0.008969	0.019883	0.017336
10000	0.113359	0.173064	0.133457
100000	1.050989	2.544738	1.379683
1000000	9.123926	18.621362	13.509016
10000000	113.02977	164.35614	149.294751

Based on **Table 3**, it could be seen that the more simulations being done, then the time it took to compute the double barrier knock-in option value is also longer. The Standard Monte Carlo method has the shortest computation time compared to any other method. The control variate method has a relatively short computation time compared to the antithetic variate method. The computation time needed for the antithetic variate method is the longest of the other methods.

3.4 The Effect of Parameter Change to Double Barrier Knock-in Option Value

Next to be simulated is the random variable estimator of the double barrier knock-in option value of standard Monte Carlo, antithetic variate, and control variate methods with the change of value parameter of the strike price (K), initial stock price (S_0), maturity date (T), and volatility (σ) by simulation M = 100000. Followed by showing the effects of each parameter to the double barrier knock-in option value.

a. The effect of the strike price (K) to option value

The effect of strike price changes to the double barrier knock-in option value estimator using the standard Monte Carlo, antithetic variate and control variate methods presented in **Figure 2**.

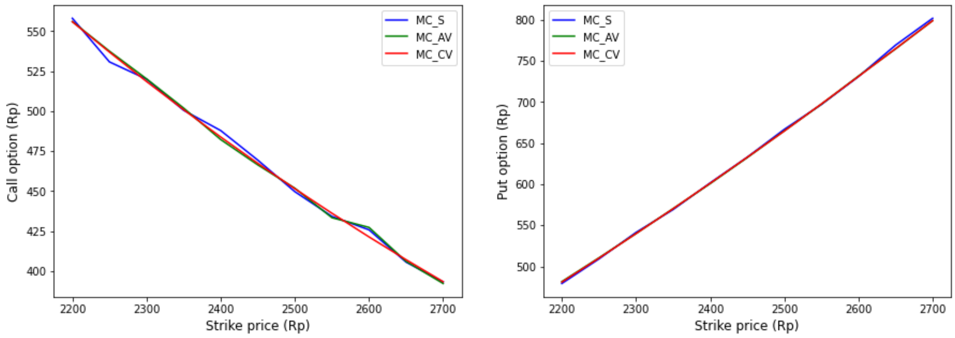

Figure 2. The effect of the strike price to the double barrier knock-in option value

In **Figure 2**, the call double barrier knock-in option value shown is getting lower as the strike price is gets higher. On the other hand, the put double barrier knock-in option value is getting higher as the strike price is getting higher too. The put double barrier knock-in option value resulted from standard Monte Carlo, antithetic variate and control variate methods are relatively the same, while in the calculation of call of standard, Monte Carlo earned a little different option value than the other methods.

b. The Effect of Initial Stock price (S_0) to the option value

The effect of the Initial stock price changes to the double barrier knock-in option value by standard Monte Carlo, antithetic variate, and control variate methods presented in **Figure 3**.

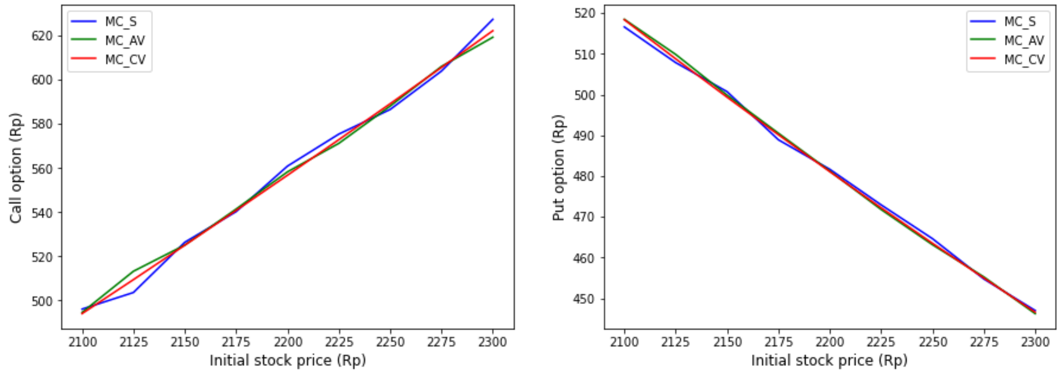

Figure 3. The effect of initial stock price to the double barrier knock-in option value

In **Figure 3**, the call double barrier knock-in option value is seen to gets bigger as the Initial stock price get bigger too. On the contrary, the put double barrier knock-in option value is getting lower as the Initial stock price gets higher. The option value generated from the antithetic variate and control variate methods has relatively the same values. However, the standard Monte Carlo method generated a little different option value compared to any other methods.

c. The Effect of Maturity Date (T) to Option Value

The effect of maturity date changes to the double barrier knock-in option value estimator using the standard Monte Carlo method, antithetic variate, and control variate are presented in **Figure 4**.

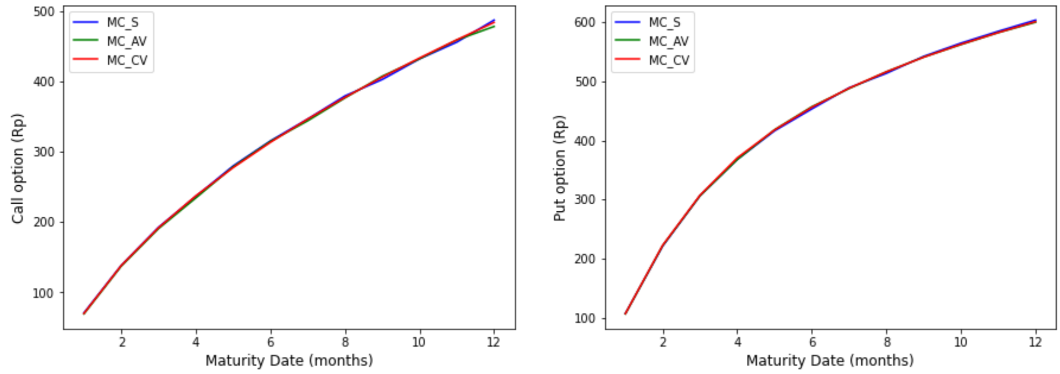

Figure 4. The effect of maturity date to the double barrier knock-in option value

In **Figure 4**, it can be seen that the double barrier knock-in option value is getting bigger if the maturity date is also getting longer. The double barrier knock-in option values resulting from the standard Monte Carlo, antithetic variate, and control variate methods are relatively similar. The maturity date has a positive correlation with the option value. If the maturity date is getting longer, the option value is also getting bigger.

d. The Effect of Volatility (σ) to Option Value

The effect of the volatility changes to the double barrier knock-in option value by standard Monte Carlo, antithetic variate, and control variate methods are presented in **Figure 5**.

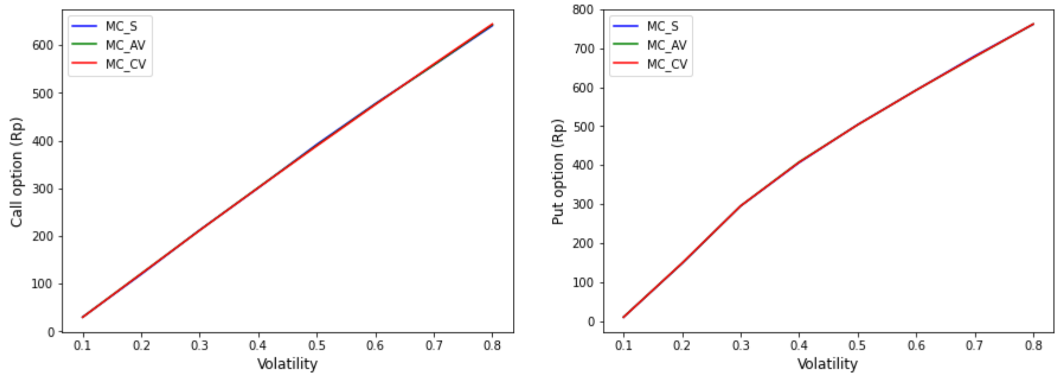

Figure 5. The effect of volatility to the double barrier knock-in option value

In Figure 5, it can be seen that the double barrier knock-in option value is getting bigger as the volatility is also getting bigger. The call option value is bigger than the put option value, with a volatility of 10%. However, the put option value is bigger than the call option value if the volatility is more than 10%. Volatility has a positive correlation with the option value. If the volatility is getting bigger, then the option value is getting bigger too.

4. CONCLUSIONS

In this paper, it is found that the Monte Carlo control variate method gave the value of the double barrier knock-in option faster than those of antithetic variate and standard Monte Carlo. The control variate method has the smallest relative error, followed by antithetic variate and standard Monte Carlo. The Standard Monte Carlo method has the shortest computation time, followed by control variate and antithetic variates. Compared to the other methods, the control variate methods are the most effective and efficient in determining the double barrier knock-in option value based on the option value, relative error, and computation time.

The increase in strike price makes the put double barrier knock-in option value to be higher and calls the double barrier knock-in option value to be lower. The increase in initial stock price makes the call double barrier knock-in option value to be higher and put double barrier knock-in option value to be lower. The increase in maturity date makes the double barrier knock-in option value to be higher. The increase in volatility makes the double barrier knock-in option to be higher.

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