BAREKENG: Journal of Mathematics and Its Applications
September 2023 Volume 17 Issue 3 Page 1393–1400
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol17iss3pp1393-1400

THE POWER GRAPH REPRESENTATION FOR INTEGER MODULO GROUP WITH POWER PRIME ORDER

Lalu Riski Wirendra Putra [1], Zata Yumni Awanis[2], Salwa[3], Qurratul Aini[4], I Gede Adhitya Wisnu Wardhana[5]*

[1,2,3,4,5]*Department of Mathematics, Faculty of Mathematics and Natural Science, University of Mataram*
Majapahit Street, No. 62, Mataram, 8311, Indonesia

*Corresponding author's e-mail: * **adhitya.wardhana@unram.ac.id***

Article History:

Received: 31st January 2023
Revised: 13th July 2023
Accepted: 24th July 2023

Keywords:

Power graph;
Group;
Integer modulo;
Power prime.

ABSTRACT

There are many applications of graphs in various fields. Starting from chemical problems, such as the molecular shape of a compound, to internet network problems, we can also use graphs to depict the abstract concept of a mathematical structure.. Groups in Algebra can be represented as a graph. This is interesting because Groups are abstract objects in mathematics. The graph of a group shows the physical form of the group by looking at the relationship between its elements. So, we can know the distance of the elements. In 2013, Abawajy et al. conducted studies related to power graphs. Power graph representation of groups of integers modulo with the order of prime numbers has been carried out in 2022 by Syechah et al. In this article, the author provides the properties of a power graph on a group of integers modulo with the order of powers of prime numbers.

How to cite this article:

L. R. W. Putra, Z. Y. Awanis, Salwa, Q. Aini and I. G. A. W. Wardhana., "THE POWER GRAPH REPRESENTATION FOR INTEGER MODULO GROUP WITH POWER PRIME ORDER,"*BAREKENG: J. Math. &App.*, vol. 17, iss. 3, pp. 1393-1400, September, 2023.

Research Article •**Open Access**

1. INTRODUCTION

There are many applications of graphs in various fields. Starting from chemical problems, namely the molecular shape of a compound to internet network problems, we can also use graphs to depict the abstract concept of a mathematical structure. Groups in algebra can be represented as a graph. This is interesting because groups are abstract objects in mathematics. The graph of a group shows the physical form of the group by looking at the relationship between its elements. So, we can know the distance of the elements.

There are many graph representations of a group and different representations give a new perspective. The coprime graph is a graph representation of group $\boldsymbol{G}$ in which its vertices consist of all the elements of groups, and two vertices $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{G}$ are adjacent whenever their order is prime relative or $\mathbf{gcd}(|\boldsymbol{a}|, |\boldsymbol{b}|) = \mathbf{1}$ **[1]–[5]**. And there is a dual representation of a coprime graph called a non-coprime graph with similar vertices, but two vertices $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{G}$ are adjacent whenever their order is not prime relative or $\mathbf{gcd}(|\boldsymbol{a}|, |\boldsymbol{b}|) \neq \mathbf{1}$ **[5]–[8]**. There is also a graph representation that its vertices consist of all the subgroups and two vertices are adjacent if only if its intersection is not trivial. This graph representation is called the intersection graph **[9]–[11]**. We can also define a representation graph on a ring, such us prime graph **[12]**.

In 2013, Abawajy, et al. carry out studies related to the power graphs of a group whose vertices consist of all of the elements of the group, and two different vertices $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{G}$ are adjacent if one is the power of the other in other words $\boldsymbol{a} = \boldsymbol{b}^{\boldsymbol{x}}$ or $\boldsymbol{b} = \boldsymbol{a}^{\boldsymbol{y}}$ for some $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{N}$ **[13]**. Later, the power graph is developed for dihedral groups **[14]–[16]** and groups of integers modulo($\mathbb{Z}_{\boldsymbol{n}}$) with the order of prime numbers in 2022**[15]**. In this article, the author provides the properties of a power graph in a group of integers modulo with an order of powers of prime numbers as well as some numerical invariants.

The type of research in this article is a literature review, which is a study conducted by examining various references with titles chosen by the author. The research is carried out by reading, analyzing, and comprehending references that come from books as well as other reading sources such as journals, papers that cover topics or materials related to groups, dihedral groups, coprime graphs, topological indices, and several topics needed to support this research.

2. RESEARCH METHODS

The type of research in this article is a basic research, which is a study conducted by examining various references with titles chosen by the author. The research is carried out by reading, analyzing, and comprehending references that come from articles as well as other reading sources such as journals, books, papers that cover topics or materials related to groups, dihedral groups, coprime graphs, topological indices, and several topics needed to support this research. The research begins by formulating the problem, then through a case study based on the order of the group, a conjecture will be developed, and with deductive reasoning, the conjecture is analyzed. If proven to be true, the conjecture is stated as a theorem, and if not yet proven, another case study will be sought.

3. RESULTS AND DISCUSSION

We give some properties of the power graph of a group from its form representation to its numerical invariances such as the graph metrics and the topological indices.

3.1 Power Graph Representation

The power graph of a group is defined as a graph whose vertices consist of all of the elements of the group, and two different vertices are adjacent if one is the power of the other.

Definition 1.[17] Suppose $\boldsymbol{G} = (\boldsymbol{G}, .)$ is a group. The set of vertices $\boldsymbol{V}(\boldsymbol{G})$ is equal to $\boldsymbol{G}$ and two distinct vertices are neighbors if and only if one of the vertices is the power of the other vertex. In other words, $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{G}$ are neighbors if and only if $\boldsymbol{a} = \boldsymbol{b}^{\boldsymbol{x}}$ or $\boldsymbol{a}^{\boldsymbol{y}} = \boldsymbol{b}$, for some $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{N}$

The group of integers modulo $(\mathbb{Z}_n)$of order,n is the set of equivalence classes of integers modulo n equipped with the addition operation modulo n $\left(+_{mod(n)}\right)$ which fulfills associative properties, has the identity, and has an inverse.

Definition 2. [18] Group G is said to be called a group of order n modulo integers, which is a group consisting of the set of all non-negative integers equipped with the addition operation modulo n. Or denoted by the set $\mathbb{Z}_n = \{0, 1, 2 \ \dots, n - 1\}$ with the $+_{mod(n)}$ operation.

Here are some examples of power graph representations on groups of integers modulo with the order of powers of primes.

Example 1. Given group $\mathbb{Z}_4 = \{0, 1, 2, 3\}$.

Table 1. The Neighborhood Table of $\mathbb{Z}_4$

Elements of $\mathbb{Z}_4$	0	1	2	3
0	−	$0 = 1^4$	$0 = 2^2$	$0 = 3^4$
1	$0 = 1^4$	−	$2 = 1^2$	$3 = 1^3$
2	$0 = 2^2$	$2 = 1^2$	−	$2 = 3^2$
3	$0 = 3^4$	$3 = 1^3$	$2 = 3^2$	−

Figure 1 represents the power graph of the group $\mathbb{Z}_4$ based on the neighborhood table.

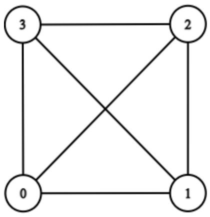

Figure 1. The power graph of $\mathbb{Z}_4$

Another example is the representation of a power graph in a group $\mathbb{Z}_8$ which is shown in Figure 2 below based on **Table 2**.

Example 2. Given group $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

Table 2. The Neighborhood Table of $\mathbb{Z}_8$

Elements of $\mathbb{Z}_8$	0	1	2	3	4	5	6	7
0	−	$0 = 1^8$	$0 = 2^4$	$0 = 3^8$	$0 = 4^2$	$0 = 5^8$	$0 = 6^4$	$0 = 7^8$
1	$0 = 1^8$	−	$2 = 1^2$	$3 = 1^3$	$4 = 1^4$	$5 = 1^5$	$6 = 1^6$	$7 = 1^7$
2	$0 = 2^4$	$2 = 1^2$	−	$2 = 3^6$	$4 = 2^2$	$2 = 5^2$	$2 = 6^3$	$2 = 7^6$
3	$0 = 3^8$	$3 = 1^3$	$2 = 3^6$	−	$4 = 3^4$	$3 = 5^7$	$6 = 3^2$	$3 = 7^5$
4	$0 = 4^2$	$4 = 1^4$	$4 = 2^2$	$4 = 3^4$	−	$4 = 5^4$	$4 = 6^2$	$4 = 7^4$
5	$0 = 5^8$	$5 = 1^5$	$2 = 5^2$	$3 = 5^7$	$4 = 5^4$	−	$6 = 5^6$	$5 = 7^3$
6	$0 = 6^4$	$6 = 1^6$	$2 = 6^3$	$6 = 3^2$	$4 = 6^2$	$6 = 5^6$	−	$6 = 7^2$
7	$0 = 7^8$	$7 = 1^7$	$2 = 7^6$	$3 = 7^5$	$4 = 7^4$	$5 = 7^3$	$6 = 7^2$	−

Figure 2 represents the power graph of the group $\mathbb{Z}_8$ based on the neighborhood table.

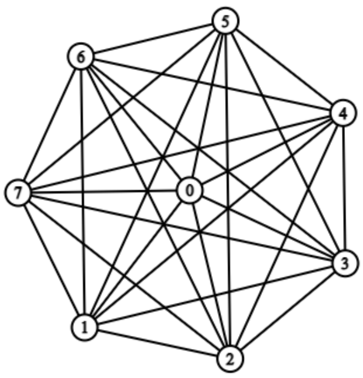

Figure 2. The power graph of $\mathbb{Z}_8$ group

Theorem 1. *Given a group of integers modulo $n(\mathbb{Z}_n)$with the operation $+_{mod(n)}$. If $n = p^k$ for some $k \in \mathbb{N}$ and p is a prime number. Then the power graph of the group $\mathbb{Z}_n(\Gamma_{\mathbb{Z}_n})$is a complete graph of K_n.*

Proof. Let $\mathrm{n} = \mathrm{p}^{\mathrm{k}}$ for some $\mathrm{k} \in \mathbb{N}$ and p is a prime number. Take any $\bar{\mathrm{a}}, \bar{\mathrm{b}} \in \mathbb{Z}_{\mathrm{n}}$

1) If $(\mathrm{a}, \mathrm{n}) \neq 1$ and $(\mathrm{b}, \mathrm{n}) \neq 1$, then $\mathrm{a}|\mathrm{n}$ and $\mathrm{b}|\mathrm{n}$. Consequently $\mathrm{a} = \mathrm{p}^{\mathrm{l}}$ and $\mathrm{b} = \mathrm{p}^{\mathrm{m}}$ for an $\mathrm{l}, \mathrm{m} \in \mathbb{Z}$ with $\mathrm{l}, \mathrm{m} \leq \mathrm{k}$. Without reducing the generalization, let $\mathrm{l} < \mathrm{m}$, then $\exists \mathrm{q} \in \mathbb{N} \ni \mathrm{m} = \mathrm{l} + \mathrm{q}$. Consequently, $\mathrm{b} = \mathrm{p}^{\mathrm{m}} = \mathrm{p}^{\mathrm{l}+\mathrm{q}} = \mathrm{ap}^{\mathrm{q}}$. Then $\exists \mathrm{r} = \mathrm{p}^{\mathrm{q}} \in \mathbb{N}$ so that $\mathrm{b} = \mathrm{a}.\mathrm{r} = \underbrace{\mathrm{a} + \mathrm{a} + \ldots + \mathrm{a}}_{\mathrm{r}}$. In other words, $\bar{\mathrm{b}} = \bar{\mathrm{a}}^{\mathrm{r}}$. As a result, $\bar{\mathrm{a}}$ and $\bar{\mathrm{b}}$ are neighbors.
2) Without reducing generalization, if $(\mathrm{a}, \mathrm{n}) = 1$ and$(\mathrm{b}, \mathrm{n}) \neq 1$. Then a is relatively prime with n, so that $\exists \alpha, \beta \in \mathbb{Z}$ so that $\alpha \mathrm{a} + \beta \mathrm{n} = 1$. Because the applicable operation is the addition modulo operation, then $1 = \alpha \mathrm{a} = \underbrace{\mathrm{a} + \mathrm{a} + \ldots + \mathrm{a}}_{\alpha}$, in other words, $\bar{\mathrm{a}}^{\alpha} = \bar{1}$. That is, $\bar{\mathrm{a}}^{\alpha}$ is a generator of $\mathbb{Z}_{\mathrm{n}}$. So there is $\gamma \in \mathbb{N} \ni \bar{\mathrm{b}} = \bar{\mathrm{a}}^{\gamma}$. That is, $\bar{\mathrm{a}}$ and $\bar{\mathrm{b}}$ are neighbors.
3) If $(\mathrm{a}, \mathrm{n}) \neq 1, (\mathrm{b}, \mathrm{n}) \neq 1$, then a, b are relatively prime with n. So there are$\alpha_1, \alpha_2, \beta_1$, and $\beta_2 \in \mathbb{Z}$ so that $\alpha_1 \mathrm{a} + \beta_1 \mathrm{n} = \alpha_2 \mathrm{b} + \beta_2 \mathrm{n} = 1$. That is, $1 = \mathrm{a}\alpha_1 = \mathrm{b}\alpha_2$, with a same techique like before we have $\bar{\mathrm{a}}^{\alpha_1} = \bar{\mathrm{b}}^{\alpha_2} = \bar{1}$. Consequently, $\bar{\mathrm{a}}^{\alpha_1}$and $\bar{\mathrm{b}}^{\alpha_2}$ are the generators of $\mathbb{Z}_{\mathrm{n}}$. So there are $\gamma \in \mathbb{N}$ so that $\bar{\mathrm{a}} = \bar{\mathrm{b}}^{\gamma}$. That is, $\bar{\mathrm{a}}$ and $\bar{\mathrm{b}}$ are neighbors.

Because it applies to all $\bar{\mathrm{a}}, \bar{\mathrm{b}} \in \mathbb{Z}_{\mathrm{n}}$, and all the possibilities that exist, then the power graph of the group $\mathbb{Z}_{\mathrm{n}}$ with $\mathrm{n} = \mathrm{p}^{\mathrm{k}}$for a $\mathrm{k} \in \mathbb{N}$ and p prime numbers is a complete graph K_{n}. ■

3.2 Numerical Invariance

Apart from representing graphs in groups, another interesting thing to discuss is the numerical invariance of a graph. The nature of the graph will make it easier to find the numerical invariance of the graph in that group. Following are some definitions of the numerical invariance of a graph.

3.2.1 Clique and Chromatic Number

Definition 3.[19]. Suppose $\boldsymbol{G}$ is a graph where $\boldsymbol{V}(\boldsymbol{G})$ is a set of vertices, and $\boldsymbol{E}(\boldsymbol{G})$ is a set of edges.

1) Suppose $\boldsymbol{x} \in \boldsymbol{V}(\boldsymbol{G})$, the eccentricity of $\boldsymbol{x}$ is the maximum distance $\boldsymbol{x}$ to another vertex, denoted by $\boldsymbol{ec}(\boldsymbol{x}) = \boldsymbol{max}\,\{\boldsymbol{d}(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{y} \in \boldsymbol{V}(\boldsymbol{G})\}$.
2) The radius of a graph G is the minimum eccentricity of all points in $\boldsymbol{V}(\boldsymbol{G})$, denoted $\boldsymbol{rad}(\boldsymbol{G}) = \boldsymbol{min}\,\{\boldsymbol{ec}(\boldsymbol{x}) | \boldsymbol{x} \in \boldsymbol{V}(\boldsymbol{G})\}$.
3) The diameter of a graph $\boldsymbol{G}$ is the maximum eccentricity of all points in $\boldsymbol{V}(\boldsymbol{G})$, denoted $\boldsymbol{diam}(\boldsymbol{G}) = \boldsymbol{max}\,\{\boldsymbol{ec}(\boldsymbol{x}) | \boldsymbol{x} \in \boldsymbol{V}(\boldsymbol{G})\}$.
4) The chromatic number of a graph is the minimum number of colors to color graph $\boldsymbol{G}$, denoted by $\boldsymbol{\chi}(\boldsymbol{G})$.
5) The clique number of a graph G is the maximum number of vertices of a graph $\boldsymbol{G}$ that form a complete subgraph, denoted by $\boldsymbol{\omega}(\boldsymbol{G})$.

Theorem 2. Suppose that given a group of integers modulo $(\mathbb{Z}_{\boldsymbol{n}})$. If n is the power of prime numbers, then we get $\boldsymbol{rad}(\boldsymbol{\Gamma}_{\mathbb{Z}_n}) = \boldsymbol{diam}(\boldsymbol{\Gamma}_{\mathbb{Z}_n}) = \mathbf{1}$.

Proof. According to **Theorem 1**, graph $\boldsymbol{\Gamma}_{(\mathbb{Z}_n)}$ is a complete graph. This means that the distance of all elements of $\boldsymbol{V}(\boldsymbol{\Gamma}_{\mathbb{Z}_n})$ to each other is 1. By definition, the eccentricity of all elements of $\boldsymbol{V}(\boldsymbol{\Gamma}_{\mathbb{Z}_n})$ is 1. As a result, we get $\boldsymbol{rad}(\boldsymbol{\Gamma}_{\mathbb{Z}_n}) = \boldsymbol{diam}(\boldsymbol{\Gamma}_{\mathbb{Z}_n}) = \mathbf{1}$. ■

Theorem 3. Suppose that given a group of integers modulo $\mathbb{Z}_{\boldsymbol{n}}$. If n is the power of prime numbers then we get $\boldsymbol{\omega}(\boldsymbol{\Gamma}_{\mathbb{Z}_n}) = \boldsymbol{\chi}(\boldsymbol{\Gamma}_{\mathbb{Z}_n}) \;= \boldsymbol{n}$.

Proof. Based on **Theorem 1**, it is obtained that $\boldsymbol{\Gamma}_{\mathbb{Z}_\mathbf{n}}$is a complete graph of $\boldsymbol{K}_{\boldsymbol{n}}$, so that every two vertices are adjacent to each other. Because every two points are neighbors, each point must have a different color, so the chromatic number of a power graph $\boldsymbol{\Gamma}_{\mathbb{Z}_\mathbf{n}}$is the number $|\boldsymbol{V}(\boldsymbol{\Gamma}_{\mathbb{Z}_\mathbf{n}})| = \boldsymbol{n}$. Then because graph $\boldsymbol{\Gamma}_{\mathbb{Z}_\mathbf{n}}$is a complete graph, it is clear that the maximum number of complete subgraphs is $\boldsymbol{\Gamma}_{\mathbb{Z}_\mathbf{n}}$itself, so the clique number of a power graph $\boldsymbol{\Gamma}_{\mathbb{Z}_\mathbf{n}}$is a number $|\boldsymbol{V}(\boldsymbol{\Gamma}_{\mathbb{Z}_n})| = \boldsymbol{n}$. ■

3.2.2 Zagreb, Harmonic, Randic, Gutman, and Wiener Index

In this section, we will give some topological indices, such as the Zagreb index, the Harmonic index, the Gutman index, and the Wiener index.

Definition 4. [20] Suppose $\boldsymbol{G}$ is a graph where $\boldsymbol{V(G)}$ is a set of vertices. Degree of $\boldsymbol{x} \in \boldsymbol{V(G)}$ is the number of sides adjacent to $\boldsymbol{x}$, denoted by $\boldsymbol{deg\ (x)}$.

Definition 5.[21] Let $\boldsymbol{G}$ be a connected graph without loops and multiple edges where $\boldsymbol{V(G)}$ is a set of vertices, and $\boldsymbol{E(G)}$ is a set of edges. The first Zagreb index $\boldsymbol{M_1}$and the second Zagreb index $\boldsymbol{M_2}$ of $\boldsymbol{G}$ are defined as follows.

$$M_1 = M_1(G) = \sum_{u\in V(G)} (\deg(u))^2 \tag{1}$$

$$M_2 = M_2(G) = \sum_{\{u,v\}\in E(G)} (\deg(u)\deg(v)) \tag{2}$$

Theorem 4. Suppose that given a group of integers modulo $(\mathbb{Z}_n)$. If $\boldsymbol{n}$ is the power of prime numbers, then $\boldsymbol{M_1(\Gamma_{\mathbb{Z}_n}) = n(n-1)^2}$and $\boldsymbol{M_1(\Gamma_{\mathbb{Z}_n}) = \frac{n}{2}(n-1)^3}$.

Proof. Given a graf $\boldsymbol{\Gamma_{\mathbb{Z}_n}}$. Based on **Theorem 1**, we get $\deg(u) = \deg(v) = n-1, \forall u, v \in V(G)$. So that,

$$M_1(\Gamma_{\mathbb{Z}_n}) = \sum_{u\in V(\Gamma_{\mathbb{Z}_n})} (\deg(u))^2 = \underbrace{(n-1)^2 + (n-1)^2 + \cdots + (n-1)^2}_{n\ times} = n(n-1)^2$$

Then, we get

$$\begin{aligned} M_2(\Gamma_{\mathbb{Z}_n}) &= \sum_{\{u,v\}\in E(\Gamma_{\mathbb{Z}_n})} (\deg(u)\deg(v)) \\ &= \underbrace{(n-1)(n-1) + \cdots + (n-1)(n-1)}_{\binom{n}{2}\ times} \\ &= \binom{n}{2}(n-1)^2 \\ &= \frac{n(n-1)(n-2)!}{(n-2)!\,2!}(n-1)^2 \\ &= \frac{n}{2}(n-1)^3. \blacksquare \end{aligned}$$

Definition 6.[7]. Let $\boldsymbol{G}$ be a connected graph where $\boldsymbol{E(G)}$ is a set of edges. The first harmonic index $\boldsymbol{H(G)}$ and Randic index $\boldsymbol{R(G)}$ of $\boldsymbol{G}$ are defined as follows

$$\boldsymbol{H(G) = \sum_{\{u,v\}\in E(G)} \frac{2}{\deg(u)+\deg(v)}} \tag{3}$$

$$\boldsymbol{R(G) = \sum_{\{u,v\}\in E(G)} (\deg(u)\deg(v))^{-\frac{1}{2}}} \tag{4}$$

Theorem 5. Suppose that given a group of integers modulo $(\mathbb{Z}_n)$. If n is the power of prime numbers, then $\boldsymbol{H(\Gamma_{\mathbb{Z}_n}) = R(\Gamma_{\mathbb{Z}_n}) = \frac{n}{2}}$.

Proof. Given a graf $\boldsymbol{\Gamma_{\mathbb{Z}_n}}$. Based on **Theorem 1**, we get $\boldsymbol{\deg(u) = \deg(v) = n-1, \forall u, v \in V(G)}$. So that,

$$\begin{aligned} H(\Gamma_{\mathbb{Z}_n}) &= \sum_{\{u,v\}\in E(\Gamma_{\mathbb{Z}_n})} \frac{2}{\deg(u)+\deg(v)} \\ &= \underbrace{\frac{2}{(n-1)+(n-1)} + \cdots + \frac{2}{(n-1)+(n-1)}}_{\binom{n}{2}\ times} \\ &= \binom{n}{2}\frac{1}{n-1} \\ &= \frac{n(n-1)(n-2)!}{(n-2)!\,2!}\frac{1}{(n-1)} \end{aligned}$$

$$= \frac{n}{2}.$$

.Also, we get

$$R(\Gamma_{\mathbb{Z}_n}) = \sum_{\{u,v\}\in E(\Gamma_{\mathbb{Z}_n})} (\deg(u)\deg(v))^{-\frac{1}{2}}$$
$$= \underbrace{((n-1)(n-1))^{-\frac{1}{2}} + \cdots + ((n-1)(n-1))^{-\frac{1}{2}}}_{\binom{n}{2}\ times}$$
$$= \binom{n}{2}((n-1)^2)^{-\frac{1}{2}}$$
$$= \frac{n(n-1)(n-2)!}{(n-2)!\,2!}\frac{1}{(n-1)}$$
$$= \frac{n}{2}. \blacksquare$$

Definition 7. [22] Let $\boldsymbol{G}$ be a connected graph where $\boldsymbol{V(G)}$ is a set of vertices. The Gutman index is defined as

$$\boldsymbol{Gut(G) = \sum_{u,v\in V(G)} \deg(u)\deg(v)\, d(u,v)} \quad (5)$$

Theorem 6. Suppose that given a group of integers modulo ($\boldsymbol{\mathbb{Z}_n}$). If n is the power of prime numbers, then $Gut(\Gamma_{\mathbb{Z}_n}) = \frac{n}{2}(n-1)^3$.

Proof. Given a graf $\boldsymbol{\Gamma_{\mathbb{Z}_n}}$. Based on **Theorem 1**, we get $\deg(u) = \deg(v) = n-1, \forall u,v \in V(G)$. So, we get

$$Gut(\Gamma_{\mathbb{Z}_n}) = \sum_{u,v\in V(G)} \deg(u)\deg(v)\, d(u,v)$$
$$= \underbrace{(n-1)(n-1)1 + \cdots + (n-1)(n-1)1}_{\binom{n}{2}\ times}$$
$$= \binom{n}{2}(n-1)^2$$
$$= \frac{n}{2}(n-1)^3. \blacksquare$$

Definition 8.[23]. Let $\boldsymbol{G}$ be a connected graph where $\boldsymbol{V(G)}$ is a set of vertices and $\boldsymbol{E(G)}$ is a set of edges. The Wiener index $\boldsymbol{W(G)}$ and Wiener edge index $\boldsymbol{W_e(G)}$ are defined as

$$W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d(u,v) \quad (6)$$

$$W_e(G) = \sum_{\{u,v\}\in E(G)} d(u,v) \quad (7)$$

Theorem 7. Suppose that given a group of integers modulo ($\boldsymbol{\mathbb{Z}_n}$). If n is the power of prime numbers, then $\boldsymbol{W(\Gamma_{\mathbb{Z}_n}) = \frac{n(n-1)}{4}}$ and $\boldsymbol{W_e(\Gamma_{\mathbb{Z}_n}) = \frac{n(n-1)}{2}}$.

Proof. Given a graph $\boldsymbol{\Gamma_{\mathbb{Z}_n}}$. By **Equation (6)** and **Equation (7)** we get,

$$W(\Gamma_{\mathbb{Z}_n}) = \frac{1}{2}\sum_{u,v\in V(\Gamma_{\mathbb{Z}_n})} d(u,v)$$
$$= \frac{1}{2}\binom{n}{2}\ 1$$
$$= \frac{\binom{n}{2}}{2}$$
$$= \frac{n(n-1)}{4}$$

and

$$
\begin{aligned}
W_e(\Gamma_{\mathbb{Z}_n}) &= \sum_{\{u,v\}\in E(\Gamma_{\mathbb{Z}_n})} d(u,v) \\
&= \binom{n}{2} 1 \\
&= \frac{n(n-1)}{2}. \blacksquare
\end{aligned}
$$

4. CONCLUSIONS

The power graph of the group $\mathbb{Z}_n(\Gamma_{\mathbb{Z}_n})$for n is a prime power is a complete graph of K_n, and its radius and its diameter are equal to one. And the other numerical invariant such as the clique number and the chromatic numberis equal to n. Last we found its first Zagreb, second Zagreb, Harmonic, Randic, and Gutman indexes are $n(n-1)^2$, $\frac{n}{2}(n-1)^3$, $\frac{n}{2}$, and $\frac{n}{2}(n-1)^3$.

REFERENCES

[1] A. G. Syarifudin, Nurhabibah, D. P. Malik, and I. G. A. W. dan Wardhana, “Some characterizatsion of coprime graph of dihedral group D2n,” *J Phys Conf Ser*, vol. 1722, no. 1, 2021, doi: 10.1088/1742-6596/1722/1/012051.

[2] X. Ma, H. Wei, and L. Yang, “The Coprime graph of a group,” *International Journal of Group Theory*, vol. 3, no. 3, pp. 13–23, 2014, doi: 10.22108/ijgt.2014.4363.

[3] N. Nurhabibah, A. G. Syarifudin, and I. G. A. W. Wardhana, “Some Results of The Coprime Graph of a Generalized Quaternion Group Q_4n,” *InPrime: Indonesian Journal of Pure and Applied Mathematics*, vol. 3, no. 1, pp. 29–33, 2021, doi: 10.15408/inprime.v3i1.19670.

[4] M. R. Gayatri, Q. Aini, Z. Y. Awanis, S. Salwa, and I. G. A. W. Wardhana, “The Clique Number and The Chromatics Number Of The Coprime Graph for The Generalized Quarternion Group,” *JTAM (Jurnal Teori dan Aplikasi Matematika)*, vol. 7, no. 2, pp. 409–416, 2023, doi: 10.31764/jtam.v7i2.13099.

[5] N. Nurhabibah, I. G. A. W. Wardhana, and N. W. Switrayni, “NUMERICAL INVARIANTS OF COPRIME GRAPH OF A GENERALIZED QUATERNION GROUP,” *J. Indones. Math. Soc*, vol. 29, no. 01, pp. 36–44, 2023.

[6] F. Mansoori, A. Erfanian, and B. Tolue, “Non-coprime graph of a finite group,” *AIP Conf Proc*, vol. 1750, no. June 2016, 2016, doi: 10.1063/1.4954605.

[7] Nurhabibah, D. P. Malik, H. Syafitri, and I. G. A. W. Wardhana, “Some results of the non-coprime graph of a generalized quaternion group for some n,” *AIP Conf Proc*, vol. 2641, no. December 2022, p. 020001, 2022, doi: 10.1063/5.0114975.

[8] W. U. Misuki, G. A. W. Wardhana, and N. W. Switrayni, “Some Characteristics of Prime Cyclic Ideal On Gaussian Integer Ring Modulo,” *IOP Conf Ser Mater Sci Eng*, vol. 1115, no. 1, p. 012084, 2021, doi: 10.1088/1757-899X/1115/1/012084.

[9] D. S. Ramdani, I. G. A. W. Wardhana, and Z. Y. Awanis, “THE INTERSECTION GRAPH REPRESENTATION OF A DIHEDRAL GROUP WITH PRIME ORDER AND ITS NUMERICAL INVARIANTS,” *BAREKENG: Jurnal Ilmu Matematika dan Terapan*, vol. 16, no. 3, pp. 1013–1020, Sep. 2022, doi: 10.30598/barekengvol16iss3pp1013-1020.

[10] S. Akbari, F. Heydari, and M. Maghasedi, “The intersection graph of a group,” *J Algebra Appl*, vol. 14, no. 5, Jun. 2015, doi: 10.1142/S0219498815500656.

[11] N. Nurhabibah, A. G. Syarifudin, I. G. A. W. Wardhana, and Q. Aini, “The Intersection Graph of a Dihedral Group,” *Eigen Mathematics Journal*, vol. 4, no. 2, pp. 68–73, 2021, doi: 10.29303/emj.v4i2.119.

[12] J. Abawajy, A. Kelarev, and M. Chowdhury, “Power Graphs: A Survey,” 2013. [Online]. Available: www.ejgta.org

[13] E. Y. Asmarani, A. G. Syarifudin, I. G. A. W. Wardhana, and N. W. Switrayni, “The Power Graph of a Dihedral Group,” *Eigen Mathematics Journal*, vol. 4, no. 2, pp. 80–85, 2021, doi: 10.29303/emj.v4i2.117.

[14] B. N. Syechah, E. Y. Asmarani, A. G. Syarifudin, D. P. Anggraeni, and I. G. A. W. W. Wardhana, “Representasi Graf Pangkat Pada Grup Bilangan Bulat Modulo BerordeBilanganPrima,” *Evolusi: Journal of Mathematics and Sciences*, vol. 6, no. 2, pp. 99–104, 2022.

[15] T. Chelvam and M. Sattanathan, “Power graph of finite abelian groups,” *Algebra and Discrete Mathematics*, vol. 16, no. 1, pp. 33–41, 2013.

[16] J. H. Silverman, *Abstract Algebra An Integrated Approach*. 2022.

[17] A. G. Syarifudin, I. G. A. W. Wardhana, N. W. Switrayni, and Q. Aini, “The Clique Numbers and Chromatic Numbers of The Coprime Graph of a Dihedral Group,” *IOP Conf Ser Mater Sci Eng*, vol. 1115, no. 1, p. 012083, 2021, doi: 10.1088/1757-899x/1115/1/012083.

[18] R. G. Godsil C., *Algebraic graph theory*. Springer, 2001.

[19] M. N. Husni, H. Syafitri, A. M. Siboro, A. G. Syarifudin, Q. Aini, and I. G. A. W. Wardhana, “THE HARMONIC INDEX AND THE GUTMAN INDEX OF COPRIME GRAPH OF INTEGER GROUP MODULO WITH ORDER OF PRIME POWER,” *BAREKENG: Jurnal Ilmu Matematika dan Terapan*, vol. 16, no. 3, pp. 961–966, Sep. 2022, doi: 10.30598/barekengvol16iss3pp961-966.

[20] J. P. Mazorodze, S. Mukwembi, and T. Vetrík, “The Gutman index and the edge-Wiener index of graphs with given vertex-connectivity,” *DiscussionesMathematicae - Graph Theory*, vol. 36, no. 4, pp. 867–876, 2016, doi: 10.7151/dmgt.1900.

[21] N. I. Alimon, N. H. Sarmin, and A. Erfanian, “The Szeged and Wiener indices for coprime graph of dihedral groups,” in *AIP Conference Proceedings*, American Institute of Physics Inc., Oct. 2020. doi: 10.1063/5.0018270.