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A MULTI-ITEM INVENTORY MODEL WITH VARIOUS DEMAND FUNCTIONS CONSIDERING DETERIORATION AND PARTIAL BACKLOGGING

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ABSTRACT

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Inventory management is an important thing to be considered in order to run the activities of a company smoothly. By considering the deterioration factor, partial backlogging policy, and different types of demand functions, we develop a mathematical model for a multi-item inventory system. In this paper, three inventory models with constant deterioration, partial backlogging, with various demand functions are developed. We consider inventory-dependent demand, time-dependent demand, and exponential demand functions in each model. In addition, we also consider the replenishment policies for those three items, viz. individual replenishment, joint replenishment, and a combination of both individual and joint replenishments. A sensitivity analysis of the models is also performed, and we found that the ordering cost greatly affects the total inventory cost when comparing the available replenishment policies.

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1. INTRODUCTION

Inventories are stored goods for future use or sale. Good inventory management is needed by companies to maintain the continuity of their business through managing inventory costs. According to [1], inventory costs generally consist of purchasing costs, ordering costs, holding costs, and shortage costs. In addition, determining the optimal order time and order quantity are two things that need to be given special attention. Both of these are related to the availability of goods to fulfill demands from customers. If there is too much inventory, then the holding costs will increase. Conversely, if there is too little inventory, there can be a shortage of goods, which will result in lost sales and a loss of potential profits.

There are also external factors that should be considered in inventory management, and one of them is demand. In the mathematical models for inventory systems, the demand factor has been included and assumed to be constant or depending on other factors such as time, inventory, or selling price. The inventory management model with constant demand was first introduced by [2] and called the Economic Order Quantity (EOQ) model. In the last few decades, many papers have developed mathematical models for inventory control with non-constant demand, such as time-dependent demand [3], [4], [5], price-dependent demand [6], [7], [8], [9], [10], [11], [12], [13], inventory-dependent demand [14], [15], [16], [17], [18], or a combination of them [19], [20], [21], [22], [23], [24], [25], [26], [27].

Other than demand factors, the deterioration of goods is one of the important factors that need to be considered. Deterioration is a decrease in the quality of an item that results in the item being unusable or having no selling value. Deterioration will occur if goods are stored too long in the warehouse, especially medicines, food, vegetables, and fruit. If the inventory stays in the warehouse for too long, there can be a reduction in quality, loss, or damage to the goods when they arrive to consumers. Therefore, companies shouldn't store deteriorating goods for a long period and in large quantities, as it will be detrimental to the company itself. Several mathematical models have been developed involving deterioration factors (see, for example, [21], [28], [29], [30], [31], [32]).

Inventory management generally aims to fulfill consumer demand. In reality, there are conditions where consumer demand cannot be fulfilled. When the demand comes, the company's inventory has run out. In these conditions, there are a number of consumers who want to wait until the goods arrive (backorder), but there are also those who do not want to wait and look for other companies (lost sales). Related to this, companies can take a policy called partial backlogging. Partial backlogging is a policy of fulfilling a certain amount of consumer demand even though inventory has run out. It is assumed that not all consumers are willing to wait until the inventory is available again so that only part of the consumer demand is fulfilled. Several studies have included this partial backlogging in the inventory model developed, including [5], [8], [23], [31], [32].

[31], [32] have developed an inventory model for deteriorating goods with time-dependent demand and holding costs and considering partial backlogging. This paper is an extension of the model in [31], [32] by considering three types of goods with different demand functions for constant-deterioration goods and considering partial backlogging. Of the three types of goods, the ordering policy that minimizes the total cost of inventory will also be determined. This ordering policy is related to multi-item inventory management, such as joint order policy, individual order policy, or a combination of the two policies.

There are three models developed, namely: (1) **Model I**, which is an inventory model with the inventory-dependent demand function, (2) **Model II**, which is an inventory model with a time-dependent demand function, (3) **Model III**, which is an inventory model with an exponentially decreasing demand function. The decision variable of the developed model is to determine the time when to reorder and the time when the inventory runs out. From these decision variables, the optimal order quantity and total inventory cost will be determined by choosing the right ordering policy.

2. RESEARCH METHODS

The mathematical model in this paper is developed on the basis of the following notations and assumptions.

2.1 Notations

The notations used in the development of this model are:

HC	: holding cost per cycle,
DC	: deterioration cost per cycle,
SC	: shortage cost per cycle,
OC	: ordering cost per order,
LSC	: lost sales cost per cycle,
D_i	: deterioration cost per unit for the i^{th} item,
S_i	: shortage cost per unit per cycle for the i^{th} item,
H_i	: holding cost per unit per cycle for the i^{th} item,
C_i	: ordering cost per unit per cycle for the i^{th} item,
L_i	: lost sales cost per unit for the i^{th} item,
θ	: percentage of deteriorating item,
δ	: percentage of partial backlogging item,
T	: reorder time,
t_1	: time when the inventory runs out,
W	: the maximum inventory level for one cycle,
D_B	: maximum amount of demand backlogged per cycle,
β	: the factor of increasing demand for item depends on inventory,
λ	: the factor of increasing demand for items depends on time,
ϕ	: the factor of increasing demand for items decreases exponentially,
Q	: order quantity,
TC	: total cost,
$I(t)$	: inventory level at time $t \in [0, T]$,
$I_1(t)$	: inventory level at time $t \in [0, t_1]$,
$I_2(t)$	: inventory level at time $t \in [t_1, T]$.

2.2 Assumptions

The following assumptions are used in this model.

1. There is no lead time for ordering items, meaning that inventory will be replenished immediately when inventory runs out and an order is placed.
2. The deterioration factor is constant, $\theta \in (0,1)$, and there is no replacement or repair for deteriorated items.
3. Inventory-dependent demand function is expressed as follows:
$$D(t) = \begin{cases} a + \beta I(t), & \text{if } 0 \le t < t_1; \\ b, & \text{if } t_1 \le t \le T, \end{cases} \tag{1}$$
where a is the initial demand and b is the demand during backlogging, with $a > 0, b > 0$, and $0 < \beta < 1$.
4. Time-dependent demand function is expressed as follows:
$$D(t) = \begin{cases} \alpha - \lambda t, & \text{if } 0 \le t < t_1; \\ k, & \text{if } t_1 \le t \le T, \end{cases} \tag{2}$$
where α is the initial demand and k is the demand during backlogging, with $\alpha > 0, k > 0$, and $0 < \lambda < 1$.
5. The demand function for an exponentially decreasing item is expressed as follows:
$$D(t) = \begin{cases} ce^{-\phi t}, & \text{if } 0 \le t < t_1; \\ h, & \text{if } t_1 \le t \le T, \end{cases} \tag{3}$$
where c is the initial demand and h is the demand during backlogging, with $c > 0, h > 0$, and $0 < \phi < 1$.
6. When the inventory runs out but the demand is still there or there is a shortage of items, at time $t \in [t_1, T]$, a partial backlogging policy will be used. The partial backlogging function is expressed as follows [31], [32]:
$$B(t) = \frac{1}{1 + \delta(T - t)} \tag{4}$$

where $0 < \delta < 1$.

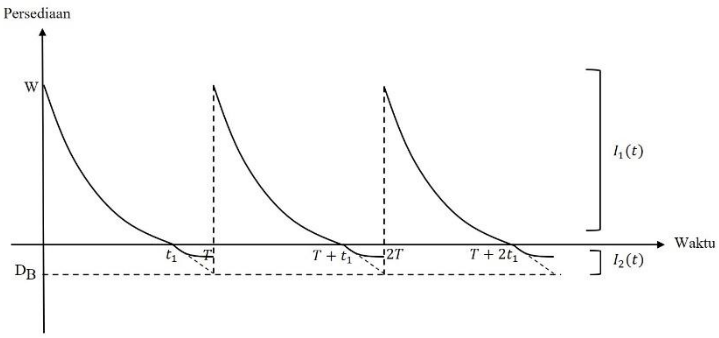

Figure 1. Inventory Model with Partial Backlogging Factor

Figure 1 is an illustration of an inventory model with a partial backlogging factor. At the beginning of the cycle ($t = 0$) there is an inventory of W units. Inventory will decrease due to demand and deterioration factors and finally the inventory will run out at t_1. When the inventory runs out but there is still demand, partial backlogging policy will be applied until the items are available for the next cycle.

2.3 Development of Model I

The inventory decreases due to demand and deterioration factors in the interval $[0, t_1]$ can be modeled through the following differential equation:

$$\frac{dI_1(t)}{dt} = -D(t) - \theta I_1(t), \qquad 0 < t \leq t_1 \tag{5}$$

where $D(t)$ is as in (1). By using the boundary condition $I_1(t_1) = 0$ in **Equation (5)**, we obtain

$$I_1(t) = \frac{-a}{\beta+\theta} + \frac{ae^{(\beta+\theta)(t_1-t)}}{\beta+\theta}. \tag{6}$$

By using the initial condition $I_1(0) = W$ in equation (6), the maximum inventory level (W) can be determined as follows:

$$I_1(0) = W = \frac{-a}{\beta+\theta} + \frac{ae^{(\beta+\theta)t_1}}{\beta+\theta}.$$

At the time $t = t_1$, the items run out, so there is a shortage in the interval $[t_1, T]$. Thus, the inventory level is expressed as follows:

$$\frac{dI_2(t)}{dt} = \frac{-b}{1+\delta(T-t)}, \qquad t_1 \leq t \leq T. \tag{7}$$

By using the boundary condition $I_2(t_1) = 0$ in **Equation (7)**, we obtain

$$I_2(t) = \frac{b}{\delta}\ln\big(1+\delta(T-t)\big) - \frac{b}{\delta}\ln\big(1+\delta(T-t_1)\big). \tag{8}$$

Let $t = T$ in **Equation (8)**, we obtain the maximum amount of backlogged demand per cycle (D_B) as follows:

$$D_B = -I_2(T) = \frac{b}{\delta}\ln\big(1+\delta(T-t_1)\big).$$

So, the order quantity per cycle is given by

$$Q = W + D_B = \frac{-a}{\beta+\theta} + \frac{ae^{(\beta+\theta)t_1}}{\beta+\theta} + \frac{b}{\delta}\ln\big(1+\delta(T-t_1)\big). \tag{9}$$

There are five cost components for the total cost of inventory for one year, that is deterioration cost (*DC*), shortage cost (*SC*), holding cost (*HC*), ordering cost (*OC*), and lost sale cost (*LSC*), which each are given below.

Deterioration Cost (*DC*)

$$DC = D_1\,\theta\,W. \tag{10}$$

Shortage Cost (*SC*)

$$SC = S_1 b\left[\frac{T-t_1}{\delta} - \frac{\ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right]. \tag{11}$$

Holding Cost (*HC*)

$$HC = H_1\left[\frac{-at_1}{\beta+\theta} + \frac{ae^{t_1(\beta+\theta)}}{(\beta+\theta)^2} - \frac{-a}{(\beta+\theta)^2}\right]. \tag{12}$$

Ordering Cost (*OC*)

$$OC = C_1. \tag{13}$$

Lost Sale Cost (*LSC*)

$$LSC = L_1 b\left[T - t_1 - \frac{\ln\left(1+\delta(T-t_1)\right)}{\delta}\right]. \tag{14}$$

Total Cost of Inventory for One Year

$$TC(t_1,T) = \frac{D_1\,\theta\,W + S_1 b\left[\frac{T-t_1}{\delta} - \frac{\ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right] + H_1\left[\frac{-at_1}{\beta+\theta} + \frac{ae^{t_1(\beta+\theta)}}{(\beta+\theta)^2}\right.}{T} \frac{\left. - \frac{-a}{(\beta+\theta)^2}\right] + C_1 + L_1 b\left[T - t_1 - \frac{\ln\left(1+\delta(T-t_1)\right)}{\delta}\right]}{T} \tag{15}$$

To find the values of t_1 and T that minimize the total inventory cost for one year, **Equation (15)** must satisfy the following conditions:

1. The first partial derivative test, to find the stationary point (t_1, T), is:

$$\frac{\partial TC}{\partial t_1} = 0 \qquad \text{and} \qquad \frac{\partial TC}{\partial T} = 0.$$

2. Second partial derivative test, to determine the nature of the stationary point (t_1, T) or the convexity of (15), through the determinant of the positive-valued Hessian matrix, as follows:

$$H = \begin{vmatrix} \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{vmatrix} = \left(\frac{\partial^2 TC}{\partial t_1^2}\cdot\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\cdot\frac{\partial^2 TC}{\partial T \partial t_1}\right) > 0,$$

and $\frac{\partial^2 TC}{\partial t_1^2} > 0$.

2.4 Development of Model II

Using the same approach as the **development of Model I**, the maximum inventory level (W) and the order quantity per cycle (Q) for **Model II** are given as follows:

$$W = \frac{-\alpha}{\theta} - \frac{\lambda}{\theta^2} + \left(\frac{\alpha}{\theta} - \frac{\lambda t_1}{\theta} + \frac{\lambda}{\theta^2}\right)e^{\theta t_1}$$

and

$$Q = W + D_B = \frac{-\alpha}{\theta} - \frac{\lambda}{\theta^2} + \left(\frac{\alpha}{\theta} - \frac{\lambda t_1}{\theta} + \frac{\lambda}{\theta^2}\right)e^{\theta t_1} + \frac{k}{\delta}\ln\left(1+\delta(T-t_1)\right). \tag{16}$$

The deterioration cost (*DC*), shortage cost (*SC*), holding cost (*HC*), ordering cost (*OC*), and lost sale cost (*LSC*) for Model II are given below.

Deterioration Cost (*DC*)

$$DC = D_2\,\theta\,W. \tag{17}$$

Shortage Cost (*SC*)

$$SC = S_2 k\left[\frac{T-t_1}{\delta} - \frac{ln\big(1+\delta(T-t_1)\big)}{\delta^2}\right]. \tag{18}$$

Holding Cost (*HC*)

$$HC = H_2\left[\frac{\lambda t_1^2\theta^2 - 2e^{\theta t_1}\lambda t_1\theta - 2\alpha t_1\theta^2 + 2e^{\theta t_1}\alpha\theta + 2e^{\theta t_1}\lambda - 2\alpha\theta - 2\lambda}{2\theta^3}\right]. \tag{19}$$

Ordering Cost (*OC*)

$$OC = C_2. \tag{20}$$

Lost Sale Cost (*LSC*)

$$LSC = L_2 k\left[T - t_1 - \frac{ln\big(1+\delta(T-t_1)\big)}{\delta}\right]. \tag{21}$$

Total Cost of Inventory for One Year

$$TC(t_1,T) = \frac{D_2\,\theta\,W + S_2 k\left[\frac{T-t_1}{\delta} - \frac{ln\big(1+\delta(T-t_1)\big)}{\delta^2}\right] + H_2\left[\frac{\lambda t_1^2\theta^2 - 2e^{\theta t_1}\lambda t_1\theta - 2\alpha t_1\theta^2}{2\theta^3}\right.}{T} + \frac{\left.\frac{+2e^{\theta t_1}\alpha\theta + 2e^{\theta t_1}\lambda - 2\alpha\theta - 2\lambda}{2\theta^3}\right] + C_2 + L_2 k\left[T - t_1 - \frac{ln\big(1+\delta(T-t_1)\big)}{\delta}\right]}{T} \tag{22}$$

The same conditions as of the Model I are also applied in order to find the values of t_1 and T that minimize the total cost of inventory for one year in **Equation (22)**.

2.5 Development of Model III

In the same way as the **development of Model I**, the maximum inventory level (W) and the order quantity per cycle (Q) for **Model III** are as follows:

$$W = c\left(\frac{e^{(\theta-\phi)t_1}}{\theta-\phi} - \frac{1}{\theta-\phi}\right)$$

and

$$Q = W + D_B = c\left(\frac{e^{(\theta-\phi)t_1}}{\theta-\phi} - \frac{1}{\theta-\phi}\right) + \frac{h}{\delta}ln\big(1+\delta(T-t_1)\big). \tag{23}$$

There are five cost components for the total cost of inventory for one year are given below.

Deterioration Cost (*DC*)

$$DC = D_3\,\theta\,W. \tag{24}$$

Shortage Cost (*SC*)

$$SC = S_3 h\left[\frac{T-t_1}{\delta} - \frac{ln\big(1+\delta(T-t_1)\big)}{\delta^2}\right]. \tag{25}$$

Holding Cost (*HC*)

$$HC = H_3 c\left(\frac{e^{(\theta-\phi)t_1} - e^{-\phi t_1}}{(\theta-\phi)\theta} + \frac{e^{-\phi t_1} - 1}{(\theta-\phi)\phi}\right). \tag{26}$$

Ordering Cost (*OC*)

$$OC = C_3. \tag{27}$$

Lost Sale Cost (LSC)

$$LSC = L_3 h\left[T - t_1 - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta}\right]. \tag{28}$$

Total Cost of Inventory for One Year

$$TC(t_1, T) = \frac{D_3\,\theta\,W + S_3 h\left[\frac{T-t_1}{\delta} - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right] + H_3 c\left(\frac{e^{(\theta-\phi)t_1} - e^{-\phi t_1}}{(\theta-\phi)\theta} + \frac{e^{-\phi t_1} - 1}{(\theta-\phi)\phi}\right)}{T} + \frac{C_3 + L_3 h\left[T - t_1 - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta}\right]}{T} \tag{29}$$

Equation (29) must satisfy the same conditions as the Model I in order to find the optimal values of t_1 and T that minimize the total cost of inventory for one year.

2.6 Ordering Policy for Multi-Item Inventory Model

In ordering items, there are three types of replenishment policies that can be applied, namely individual order policy, joint order policy, and the combination of the two policies.

1. Individual Order Policy
   The individual order policy is a policy where the company places orders for each type of item separately. Suppose n is the number of types of items and TC_i is the total cost of inventory for the i^{th} item, then the total cost of the individual order policy is

$$TC_{individual} = \sum_{i=1}^{n} TC_i \tag{30}$$

2. Joint Order Policy
   The joint order policy is a policy where the company places orders for all types of items together from the same supplier. This means that the order is only made once so the ordering cost is only charged once. Suppose n is the number of types of items and TC_i is the total cost of inventory for the i^{th} item, then the total cost of the joint order policy is

$$TC_{joint} = OC + \sum_{i=1}^{n} TC_i \tag{31}$$

3. Combined Order Policy between Individual and Joint Order
   This policy is a combination of individual and joint order policies and can only be used if the company orders more than two types of items. Suppose, the company places an order for three types of items, then there are three alternatives for the replenishment policy.
   a. Order the first item and the second item together, while the third item separately. The total cost for the first alternative is

$$TC = 0C_{1,2} + \sum_{i=1}^{2}(DC_i + SC_i + HC_i + LSC_i) + (DC_3 + SC_3 + HC_3 + OC_3 + LSC_3) \tag{32}$$

   b. Order the first item and the third item together, while the second item separately. The total cost for the second alternative is

$$TC = 0C_{1,3} + \sum_{i\in\{1,3\}}(DC_i + SC_i + HC_i + LSC_i) + (DC_2 + SC_2 + HC_2 + OC_2 + LSC_2 \tag{33}$$

   c. Order the second item and the third item together, while the first item separately. The total cost for the third alternative is

$$TC = OC_{2,3} + \sum_{i=2}^{3}(DC_i + SC_i + HC_i + LSC_i) + (DC_1 + SC_1 + HC_1 + OC_1 + LSC_1) \quad (34)$$

3. RESULTS AND DISCUSSION

3.1. Results

Suppose a company sells three types of items with parameter values shown in **Table 1** and the ordering cost of joint order policy and combined order policy between individual and joint order given in **Table 2**.

If the company orders items separately (individual orders), then using equation (30), the time when the inventory runs out (t_1), the reorder time (T), the order quantity (Q), and the total cost (TC) for each of the three items is shown in **Table 3**.

If the company orders items for the three types of items individually, the total cost that must be incurred by the company for 1 year is IDR 416,383.

Table 1. Parameter Values for Three Types of Items

Parameter	Item 1	Item 2	Item 3
Initial demand	120	120	120
Demand during backlogging	100	100	100
The factor of increasing demand	0.5	0.5	0.005
Percentage of deteriorating item	0.1	0.1	0.1
Percentage of partial backlogging item	0.8	0.8	0.8
Holding cost	1,200	1,200	1,200
Deterioration cost	8,000	8,000	7,000
Ordering cost	15,000	15,000	15,000
Shortage Cost	8,000	8,000	8,000
Lost sale cost	5,000	5,000	5,000

Table 2. The Ordering Cost of Joint Order Policy and Combined Order Policy between Individual and Joint Order

Parameter	*Joint Order*	Combined	
		Joint Order	*Individual Order*
Ordering cost	30,000	20,000	15,000

Table 3. Total cost for each of the three items

	t_1	T	Q	$TC_{individual}$
Item 1	0.2262	0.3598	42	144.850
Item 2	0.2838	0.4126	47	140.056
Item 3	0.3081	0.4282	49	131.477
Total cost (TC)				**416.383**

Furthermore, the total cost incurred if joint order policy is applied will be analyzed. To analyze the total cost, T_{joint} is selected based on the optimal T value of each of the three items and the optimal T value of the joint order policy.

If the company orders items jointly (joint order), then using **Equation (31)**, the total cost incurred for each selected T_{joint} is shown in **Table 4**, while the time when the inventory of the three items runs out and the order quantity for each of the three items of the minimum total cost is shown in **Table 5**.

From **Table 4**, the minimum total cost with a joint order policy is when $T_{joint} = 0.3$, which is IDR 374,043. From **Table 5**, it is obtained that the company must place an order quantity for the first item is 42 units, while the second and third items are 41 units each, with the time when the inventory of the three items runs out being 0.2262, 0.2383, and 0.2489 years (about 83, 87, and 91 days), respectively.

Table 4. Total Cost for Various T_{joint}

	TC_{joint} $T_{joint}=0.3598$ (item 1)	TC_{joint} $T_{joint}=0.4126$ (item 2)	TC_{joint} $T_{joint}=0.4282$ (item 3)	TC_{joint} $T_{joint}=0.3$ (optimum)
Item 1 **Item 2** **Item 3**	376,094	380,700	382,418	**374,043**

Table 5. Values of t_1, t_2, t_3 and Q_1, Q_2, Q_3 for $T_{joint}=0.3$

	Notation	Value
Time when the inventory runs out	t_1	0.1776
	t_2	0.1865
	t_3	0.1970
The order quantity	Q_1	34
	Q_2	33
	Q_3	34

Next, we will analyze the total cost incurred for a combination of individual and joint orders, where this policy has three alternatives.

If the company orders items in combination, then using equations (32), (33), (34), the total cost incurred for the first and third alternatives is shown in Table 6 and for the second alternative is shown in Table 7. The time when the inventory runs out, the reorder time, and the orders quantity for each of the three items of the minimum total cost is shown in Table 8.

From Table 6 and Table 7, the minimum total cost obtained by using the first alternative is IDR 386,966. From Table 8, the company must place an order for the first and second items every 0.2902 years (about 106 days), while the third item every 0.4282 years (about 156 days), with the time until the inventory of the first, second, and third items runs out being 0.1696, 0.1780, and 0.3081 years (about 62, 65, and 112 days), respectively. The orders quantity for each of the three items are 33, 32, and 49 units, respectively.

Table 6. Total Cost for Alternative Policies 1 and 3

	TC_{joint} Alternative 1	TC_{joint} Alternative 3
Item 1	255,489	144,850
Item 2		244,562
Item 3	131,477	
Total Cost (TC)	**386,966**	389,412

Table 7. Total Cost for Alternative Policy 2

	TC_{joint} Alternative 2
Item 1 **Item 3**	247,792
Item 2	140,056
Total Cost (TC)	387,848

Table 8. Values of $t_1, t_2, t_3, T_{joint}, T_{individual}$ and Q_1, Q_2, Q_3 for Alternative Policy 1

	Notation	Value
Time when the inventory runs out	t_1	0.1696
	t_2	0.1780
	t_3	0.3081
The reorder time	T_{joint}	0.2902
	$T_{individual}$	0.4282
The order quantity	Q_1	33
	Q_2	32
	Q_3	49

3.2. Discussion

Based on the numerical example above, the total cost generated by the joint order policy is cheaper than the total cost generated by the individual order policy or the combined individual and joint order policies. This makes sense because the ordering cost when ordering all three types of items at once from the same supplier is only charged once; this can save the company expenses.

Furthermore, a sensitivity analysis will be carried out to determine the effect of changes in ordering costs for the joint order policy and the combined policy between individual and joint orders on the time when the inventory runs out, the reorder time, and the total cost.

From **Table 9**, it is obtained that the greater the ordering cost for the joint order (OC) policy, the longer the time when the inventory of the three items runs out (t_1, t_2, t_3) and the longer the reorder time (T). This happens because of the increase in the order quantity of the three items (Q_1, Q_2, Q_3) so that the time required during the sales period is getting longer. In addition, the greater the ordering cost, the greater the total cost (TC) that must be incurred because the order quantity is increasing.

Table 9. Effect of Changes in Ordering Cost for Joint Order Policy

Parameter	Nilai	% Change	T	t_1	t_2	t_3	Q_1	Q_2	Q_3	TC	$\%\Delta TC$
	24,000	−20	0.26	0.1449	0.1519	0.1622	29	29	29	352,543	−5.7480
	27,000	−10	0.28	0.1613	0.1692	0.1796	32	31	31	363,738	−2.7550
OC	30,000	0	0.3	0.1776	0.1865	0.1970	34	33	34	374,043	0
	33,000	+10	0.32	0.1939	0.2039	0.2144	35	34	34	383,624	2.5615
	36,000	+20	0.34	0.2102	0.2212	0.2317	36	36	36	392,611	4.9643

From **Table 10** − **Table 12**, it is obtained that for the three alternative combined policies, the greater the ordering cost for items ordered together and the ordering cost ordered separately, the longer the time when the inventory of the three items runs out (t_1, t_2, t_3) and the longer the reorder time (T). This happens because of the increase in the order quantity of the three items (Q_1, Q_2, Q_3) so that the time required during the sales period is getting longer. In addition, the greater the ordering cost, the greater the total costs (TC) that must be incurred because the order quantity is increasing. This has the same effect as the joint order policy.

Tabel 10. Effect of Changes in Ordering Cost for Alternatif Policy 1

Parameter		% Change	t_1	t_2	t_3	T_{joint}	$T_{individual}$	Q_1	Q_2	Q_3	TC	$\%\Delta TC$
$C_{1,2}$	C_3											
18,000	13,500	−10	0.1510	0.1584	0.2849	0.2675	0.4014	30	30	46	376,177	−2.7879
19,000	14,250	−5	0.1605	0.1684	0.2967	0.2791	0.4150	32	31	47	381,674	−1.3676
20,000	15,000	0	0.1696	0.1780	0.3081	0.2902	0.4282	33	32	49	386,966	0
21,000	15,750	+5	0.1783	0.1873	0.3191	0.3008	0.4409	34	34	51	392,076	1.3205
22,000	16,500	+10	0.1867	0.1962	0.3298	0.3111	0.4533	36	35	52	397,021	2.5985

Tabel 11. Effect of Changes in Ordering Cost for Alternatif Policy 2

Parameter		% Change	t_1	t_2	t_3	T_{joint}	$T_{individual}$	Q_1	Q_2	Q_3	TC	$\%\Delta TC$
$C_{1,3}$	C_2											
18,000	13,500	−10	0.1592	0.2599	0.1774	0.2775	0.3848	31	43	31	371,152	−2.7580
19,000	14,250	−5	0.1683	0.2721	0.1871	0.2887	0.3989	33	45	32	382,600	−1.3533
20,000	15,000	0	0.1771	0.2838	0.1965	0.2994	0.4126	34	47	34	387,848	0
21,000	15,750	+5	0.1855	0.2952	0.2054	0.3097	0.4258	35	48	35	392,291	1.3080
22,000	16,500	+10	0.1937	0.3062	0.2141	0.3197	0.4385	37	50	36	397,834	2.5747

Tabel 12. Effect of Changes in Ordering Cost for Alternatif Policy 3

Parameter		% Change	t_1	t_2	t_3	T_{joint}	$T_{individual}$	Q_1	Q_2	Q_3	TC	$\%\Delta TC$
$C_{2,3}$	C_1											
18,000	13,500	−10	0.2075	0.1835	0.1939	0.2965	0.3367	39	33	33	378,622	−2.7707
19,000	14,250	−5	0.2170	0.1941	0.2046	0.3087	0.3485	40	34	35	384,116	−1.3600
20,000	15,000	0	0.2262	0.2043	0.2148	0.3205	0.3598	42	36	36	389,412	0
21,000	15,750	+5	0.2351	0.2141	0.2247	0.3319	0.3708	43	37	38	394,530	1.3143
22,000	16,500	+10	0.2437	0.2236	0.3242	0.3429	0.3814	45	41	41	399,488	2.5875

From the sensitivity analysis results, it is found that when determining ordering policies for three types of items, the ordering cost (OC) is very influential in contributing to the total cost (TC), that is, the greater the ordering cost, the greater the total cost.

4. CONCLUSIONS

In this paper, three inventory models have been developed with demand functions depending on inventory, time, and the following exponential function. There are three alternative ordering policies available for the company to choose from in order to minimize the total inventory cost. Based on our numerical examples and sensitivity analysis, we conclude that:

a. Compared to other policies, the joint order policy gives the minimum total cost.
b. When the ordering cost for the joint order policy and combined policies is higher, then the time the inventory runs out becomes longer and also the reorder time.
c. The ordering cost has a substantial contribution to the total cost compared to other costs for the three ordering policies.

Using a suitable distribution for demand and considering discount factors from the supplier are some directions for further research.

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