BAREKENG: Journal of Mathematics and Its Applications
September 2023 Volume 17 Issue 3 Page 1543–1552
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol17iss3pp1543-1552

ON RAINBOW ANTIMAGIC COLORING OF SNAIL GRAPH(S_n), COCONUT ROOT GRAPH ($Cr_{n,m}$), FAN STALK GRAPH (Kt_n) AND THE LOTUS GRAPH(Lo_n)

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Article History:
Received: 14th March 2023
Revised: 6th August 2023
Accepted: 14th August 2023

Keywords:
Rainbow antimagic Connection number; Snail graph; Coconut root graph; Fan stalk graph; Lotus graph

ABSTRACT

Rainbow antimagic coloring is a combination of antimagic labeling and rainbow coloring. Antimagic labeling is labeling of each vertex of the graph G with a different label, so that each the sum of the vertices in the graph has a different weight. Rainbow coloring is part of the rainbow-connected edge coloring, where each graph G has a rainbow path. A rainbow path in a graph G is formed if two vertices on the graph G do not have the same color. If the given color on each edge is different, for example in the function f it is colored uv with a weight $w(uv)$, it is called rainbow antimagic coloring. Rainbow antimagic coloring has a condition that every two vertices on a graph cannot have the same rainbow path. The minimum number of colors from rainbow antimagic coloring is called the rainbow antimagic connection number, denoted by $rac(G)$. In this study, we analyze the rainbow antimagic connection number of snail graph (S_n), coconut root graph ($CR_{n,m}$), fan stalk graph (Kt_n) and lotus graph (Lo_n).

How to cite this article:

R. Adawiyah, I. I. Makhfudloh, Dafik, R. M Prihandini and A. C. Prihandoko., "ON RAINBOW ANTIMAGIC COLORING OF SNAIL GRAPH(S_n), COCONUT ROOT GRAPH ($Cr_{n,m}$), FAN STALK GRAPH (Kt_n) AND THE LOTUS GRAPH(Lo_n)," *BAREKENG: J. Math. & App.*, vol. 17, iss. 3, pp. 1543-1552, September, 2023.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article • Open Access

1. INTRODUCTION

Graph G is a set of $(V(G), E(G))$ with $V(G)$ is non empty set which called as vertex and $E(G)$ is set from unordered pairs of different vertices of V (possible empty) called the edge [1]. The number of vertices on the graph G called the order denoted by $|V(G)|$ [2], [3]. While the number of edges in the graph G called the size denoted by $|E(G)|$. The degree of the graph G is the number of the adjacent edges to the vertex x on graph G. The maximum degree on graph $G$is denoted by $\Delta(G)$. Whereas the smallest degree from graph G is denoted with $\delta(G)$ [4].

Graph labeling is a mapping of the set of elements in a graph such as vertices, edges or both into the set of positive integers or natural numbers [5]. The total labelling is the labelling of edge labeling and vertex labeling. If there is a bijective function on the graph $G, f: E(G) \to \{1, 2, \dots, |E(G)|\}$ called antimagic labeling with the number of vertices having different weight values. The sum of the label from every edge is called as edge weight, denoted by $w(v)$.

The graph coloring is the procedure of assigning color on vertices and edges on graph. The rainbow coloring is a part of edge coloring in graph G. Suppose that graph G is a connected graph with edge coloring $c: E(G) \to \{1, 2, 3, \dots, k\}$, under the condition k is a part of the natural number where the neighboring edges can have the same color. The rainbow path in a graph G formed if two vertices in graph has no path with the same color [6]. Rainbow coloring is also called edge coloring in the graph G which is rainbow connected. The minimum colors of rainbow coloring is called the rainbow connection number and it is notated with $rc(G)$ [7].

Rainbow antimagic coloring is a combination of antimagic labeling and rainbow coloring. The definition of rainbow antimagic coloring is as follow: suppose a graph G is a simple graph, the bijective function $f : V(G) \to \{1, 2, \dots, |V(G)|\}$ of the edge weight for each $uv \in E(G)$ on f is $w(uv) = f(u) + f(v)$. If there are two edges $uv, u'v' \in E(G)$, where the color of each edge is represented by the edge weights $w(uv), w(u'v')$ on the track P on labeling graph vertex G called the rainbow trajectory. A function on a graph$G$ can be said rainbow antimagic labelling (labeling rainbow antimagic), If every two vertices $u, v \in V(G)$there is a rainbow trail uv. *Rainbow antimagic coloring* is the coloring of each edge with a different color, say in a function f colored on uv with weights $w(uv)$ [8], [9]. Some relevance research can be seen at [10]–[13]. The definition of rainbow antimagic is the smallest number of colors needed to create a graph G connected to the antimagic rainbow is called the rainbow antimagic connection number. In previous studies, several graphs have been studied rainbow antimagic coloring, namely path (P_n)graphs, *dragonfly graphs*, (Dg_n)*octopus* (O_n)graphs and others.This study also examines the topic of *rainbow antimagic coloring*, but on a different graph. In this paper, we focused on analyzing the rainbow antimagic coloring of Snail graph (S_n), Coconut Root graph $(CR_{n,m})$, Stalk Fan graph (Kt_n) and lotus graph (Lo_n).

2. RESEARCH METHODS

There are 2 methods that will be used in this research, namely the axiomatic deductive method and pattern recognition. The axiomatic deductive method is a method that uses deductive proofs that apply in mathematical logic [14], [15]. This method uses existing theorems to solve the problem of the topic to be studied. While the pattern recognition method is a method used to determine patterns, cardinality and look for rainbow antimagic connection numbers.

In order to prove some theorem in this paper, we use the lemma related to the rainbow antimagic connections number lower bound. The lemma is as follow:

Lemma 1.[13] *The lower bound of rainbow antimagic connection on graph G is maximum number of rainbow connection and the maximum degree of graph G, $rac(G) \geq max\{rc(G), \Delta(G)\}$*

Based on the **Lemma 1,** in order to prove the lower bound of rainbow antimagic connection number, we can consider the rainbow connection number and the maximum degree of graph G. We can consider the following step in order to prove the theorem of rainbow antimagic connection:

1. We can choose the biggest number between the rainbow connection number and the maximum degree of graph G to be the lower bound of rainbow antimagic connections.

2. After analyzing the lower bound, then we analyze the upper bound of rainbow antimagic connection number by determining the vertex labelling and the edge weight of each graph.
3. By proving the lower bound and upper bound of rainbow antimagic connection number, we can say that the theorem is proved.

3. RESULTS AND DISCUSSION

This research produces four theorems about the rainbow antimagic connection number . The following are the result of the theorem along with the proof regarding the rainbow antimagic connection number on snail (S_n) graph, coconut root $(CR_{n,m})$ graph, fan stalk graph (Kt_n) and graph lotus (Lo_n).

Theorem 1. *Rainbow antimagic connection number on a snail graph* (S_n), *for* $n \geq 3$ *is* $rac(S_n) = n + 3$.

Proof. A snail graph (S_n) has a set of vertices $V(S_n) = \{x_i; 1 \leq i \leq 6\} \cup \{x_{3,j}; 1 \leq j \leq 2n + 1\}$ and a set of edges $E(S_n) = \{x_i x_{i+1}; i = 1,2,3,5\} \cup \{x_3 x_5\} \cup \left\{x_3 x_{3,j}; 2 \leq j \leq 2n, j \text{ is even}\right\} \cup \{ x_{3,j} x_{3,j+1}; 1 \leq j \leq 2n\} \cup \{x_6 x_{3,1}\} \cup \{x_2 x_{3,2n+1}\}$. The cardinality of edge sets and vertices on a snail graph are $|E(Sn)| = 3n + 7$ and $|V\ (S_n)| = 2n + 7$. We will show that $rac(S_n) = n + 3$ using the lower bound and upper bound. First, we will prove that lower bound of a snail graph $(S_n)$with $n \geq 3,\ rac(S_n) \geq n + 3$. Based on **Lemma 1** obtained:

$$\begin{aligned} rac(S_n) &\geq max\ \{rc(Sn), \Delta(S_n)\} \\ n + 3\ &\geq\ max\{5, n + 3\} \\ n + 3\ &\geq\ n\ +\ 3 \end{aligned}$$

So, the lower bound of the rainbow antimagic connection number of the snail graph (S_n) is $rac(S_n) \geq n + 3$. Next, we will prove the upper bound of the snail graph (S_n) with $n \geq 3,\ rac(S_n) \leq n + 3$, for example $f: V\ (Sn) \to \{1, 2,\ \ldots, 2n + 7\}$ as follows.

$$f(x_i) = \begin{cases} \dfrac{\mathrm{i} + 1}{2} & ,\text{for i} = 1,3 \\ 2\mathrm{n}\ -\ \mathrm{i}\ +\ 11, & \text{for i} = 4,5 \end{cases}$$

$$f(x_6)\ =\ 3$$

$$f(x_2)\ =\ n\ +\ 4$$

$$f(x_{3,j}) = \begin{cases} \dfrac{4\mathrm{n} - \mathrm{i} + 11}{2}, & \text{for j is odd} \\ \dfrac{6 + \mathrm{i}}{2} & ,\text{for j is even} \end{cases}$$

Based on the vertex function above, the edge weights on the Snail graph (S_n) are obtained as follows

$$w(x_i x_{i+1})\ =\ n\ +\ 4\ +\ i;\ \text{for}\ i\ =\ 1,2$$

$$w(x_i x_{i+1}) =\ 2n\ +\ 9;\ \text{for}\ i\ =\ 3,5$$

$$w(x_3 x_5)\ =\ 2n\ +\ 8$$

$$w\left(x_3 x_{3j}\right) = \frac{j}{2},\qquad for\ 2 \leq j \leq 2n\,,\qquad j\ is\ \text{even}$$

$$w(x_{3,j} x_{3,j+1}) = \begin{cases} 2n\ +\ 9, for\ j\ is\ \ odd \\ 2n\ +\ 8, for\ j\ is\ even \end{cases}$$

$$w\left(x_6 x_{3,1}\right) =\ 2n\ +\ 8$$

$$w(x_2 x_{3,2n+1})\ =\ 2n\ +\ 9$$

Then, we analyze the set of different edge weights. The set $w\left(x_{3,j}, x_{3,j+1}\right);\ j\ is\ odd\ number$ and $w(x_i x_{i+1}); i = 1,5$ is a subset of $w(x_2 x_{3,2n+1})$. The set $w(x_{3,j}, x_{3,j+1});\ j$ is even number and $w(x_3 x_5)$ is a subset of $w(x_6 x_{3,1})$. So that the set of different edge weights is the set of $w(x_2 x_{3,2n+1}), w(x_6 x_{3,1}), w(x_2 x_3)$ and $w(x_3 x_{3j})$. The following describes a set of different edge weights in a row.

$$W = \{6,7,8,\ldots,n+5,2+8,2n+9,n+6\}$$
$$|W| = n+3$$

Based on the set of edge weights, the upper bound is obtained $rac(S_n) \leq n+3$. Based on the lower and upper bounds, the *rainbow antimagic connection number* on the Snail graph (S_n) is $rac(S_n) = n+3$. ■

Here 's *the rainbow path* of the Snail graph:

Table 1. Rainbow Path on graph snail (S_n)

Case	Condition	x	y	Rainbow Path	Condition
1	$n \geq 3$	x_i	x_j	x_i, x_j	i = 1, 2, 3, 5; j = i + 1
2	$n \geq 3$	x_i	x_k	x_i, x_j, x_k	1 ≤ i ≤ 3; j = 1 + 1; k = i + 2
3	$n \geq 3$	x_i	x_l	x_i, x_j, x_k, x_k	a = 1, 2; b = 2 3; c = 3, 5; d = 4, 5, 6
4	$n \geq 3$	x_i	x_m	x_i, x_j, x_k, x_k, x_n	a = 1; b = 2; c = 3; d = 5; e = 6
5	$n \geq 3$	x_3	$x_{3,j}$	$x_3, x_{3,i}, x_{3,j}$	2 ≤ i ≤ 2n, j ∈ even number; 1 ≤ j ≤ 2n, j ∈ odd number
6	$n \geq 3$	$x_{3,i}$	$x_{3,k}$	$x_{3,i}, x_{3,j}, x_{3,k}$	2 ≤ j ≤ 2n; 1 ≤ i ≤ 2n; 3 ≤ k ≤ 2n

Figure 1 shows the rainbow antimagic coloring graph snail (S_5).

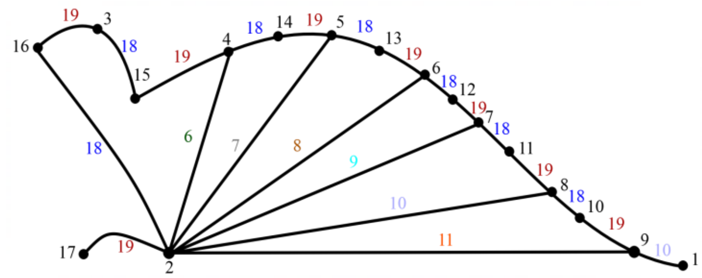

Figure 1. RAC graph snail S_5

Theorem 2. *Rainbow antimagic connection number on coconut roots graph* $(CR_{n,m})$, *for every integer* $n = 5$ *and* $m \geq 2$ *is* $rac(CR_{n,m}) = m+3$.

Proof. The Coconut root graph $(CR_{n,m})$ has a set of vertices $V(CR_{n,m}) = \{x_i, y_j, z;\ 1 \leq i \leq n, 1 \leq j \leq m\}$and a set of edges $E(CR_{n,m}) = \{x_n x_1\} \cup \{x_i x_{i+1};\ 1 \leq i \leq n-1\} \cup \{x_n y_j;\ 1 \leq j \leq m\} \cup \{y_j y_{j+1};\ 1 \leq j \leq m-1\} \cup \{x_n z\}$. The cardinality of the edge set and vertex set on the coconut root graph $(CR_{n,m})$ are $|V(CR_{n,m})| = n+m+1$ and $|E(CR_{n,m})| = n+2m$. In order to prove the $rac(CR_{n,m}) = m+3$, we should show that $rac(CR_{n,m}) \leq m+3$ and $rac(CR_{n,m}) \geq m+3$. First, we prove the lower bound of the coconut root graph $(CRn,m)$with $n = 5\ dan\ m \geq 2, rac(CR_{n,m}) \geq m+3$. Based on Definition 2.7.2, we obtained:

$$rac(CR_{n,m}) \geq \{rc(CR_{n,m}), \Delta(CR_{n,m})\}$$
$$m+3 \quad \geq\ max\{4, m+3\}$$
$$m+3 \quad \geq m+3$$

So, the lower bound of the rainbow antimagic connection number of coconut roots $(CR_{n,m})$is $rac(CR_{n,m}) \geq m+3$. Then prove the upper bound of the coconut root graph $(CR_{n,m})$ with $n = 5\ dan\ m \geq 2, rac(CR_{n,m}) \leq m+3$, for example $f : V(CR_{n,m}) \to \{1, 2, \ldots, n+m+1\}$ as follows

$$f(z) = m + 5, \text{for } m \geq 6$$

$$f(x_1) = m + 3, \text{for } m \geq 6$$

$$f(x_2) = m - 1, \text{for } m \geq 6$$

$$f(x_3) = m, \text{for } m \geq 6$$

$$f(x_4) = m + 4, \text{for } m \geq 6$$

$$f(x_5) = m - 2, \text{for } m \geq 6$$

$$f(y_m) = \begin{cases} \text{m} + 5, \text{for } 9 \leq \text{m} \leq 18 \\ \text{m} - 6, \text{for m} \geq 19 \end{cases}$$

$$f(y_m) = \begin{cases} \text{m} + 5, \text{for } 6 \leq \text{m} \leq 8 \text{ and m} \geq 19 \\ \text{m} - 6, \text{for } 13 \leq \text{m} \leq 18 \\ \text{m} - 5, \text{for } 10 \leq \text{m} \leq 12 \end{cases}$$

$$f(y_j) = \begin{cases} j - 1, for\ 4 \leq j \leq 6 \\ j - 3, for\ j = 7,8\ m = 9 \\ 1, for\ j = 1 \\ m - 5, for\ m = j\ m = 6,7 \\ m - 6, for\ m + j = 8, m = 7; m = j, m = 8; \\ m - 6, for\ m - j = 5; m - j = 7, m = 10; \\ m - 6, for\ m + j = 9, m = 11, m = 9 \\ m + j, for\ j = 1,2\ m = 6 \\ m - j, for\ j = 3,4\ m = 6 \\ m + j - 1, for\ j = 2,3\ m = 7,8 \\ m - j + 1, for\ j = 4,5\ m = 7 \end{cases}$$

Based on the vertex function above, the edge weights on the coconut root graph are obtained as follows

$$w(x_1x_5) = 2m + 1, for\ m \geq 6$$

$$w(x_2x_3) = 2m - 1, for\ m \geq 6$$

$$w(x_3x_4) = 2m + 4, for\ m \geq 6$$

$$w(x_jx_{j+1}) = 2m + 2, for\ m \geq 6, j = 1,4$$

$$w(x_5z) = 2m + 4, for\ m \geq 6$$

$$w(y_mx_5) = \begin{cases} 2m + 3, for\ 9 \leq m \leq 18 \\ 2m - 8, for\ m \geq 19 \end{cases}$$

$$w(x_5y_1) = \begin{cases} 2m - 1, for\ m = 6 \\ m - 1, for\ m \geq 7 \end{cases}$$

$$w(y_{m-1}x_5) = \begin{cases} 2m + 3, for\ 6 \leq m \leq 9; m \geq 19 \\ 2m - 8, for\ 13 \leq m \leq 18 \\ 2m - 7, for\ 10 \leq m \leq 12 \end{cases}$$

$$w(y_j x_5) = \begin{cases} m+j-3, for\ 4 \le j \le 6;\ j = 7,8, m = 9 \\ 2m+j-2, for\ m = 6, j = 1,2 \\ 2m-j-2, for\ m = 6, j = 3,4 \\ 2m-j-1, for\ m = 7, j = 4,5 \\ 2m+j-3, for\ m = 7,8, j = 2,3 \\ 2m-8, for\ m+j = 8, m = 7;\ m = j, m = 8;\ m-j = 5 \\ 2m-8, for\ m = 9;\ m-j = 7, m = 10 \\ 2m-7, for\ m = j, m = 6,7 \\ \frac{j+2m-3}{2}, for\ j\ is\ odd\ ;\ 1 \le j \le 11, m \ge \frac{j+15}{2};\ 13 \le j \le n \\ \frac{j+2m-3}{2}, for\ m \ge j+2 \\ \frac{4m-j+2}{2}, for\ j = 2,4, m \ge 9 \\ \frac{4m-j+2}{2}, for\ j = 2,4, m \ge 9 \\ \frac{4m-j}{2}, for\ 10 \le j \le n, m \ge j+2 \end{cases}$$

$$w(y_j y_{j+1}) = \begin{cases} m+3, for\ j = 1,3, m \ge 9 \\ m+4, for\ j = 2,4, m \ge 9 \\ m, for\ j = 5,7, m \ge 9;\ j = 9, m \ge 13;\ j \text{ is even number} \\ m, for\ 12 \le j \le n\ m \ge j+2 \\ m+1, for\ j = 6,8, m \ge 9;\ j = 10, m \ge 13;\ j = 5,6 \\ m-1, for\ 11 \le j \le n, j \in odd\ number, m \ge j+2, m = 8,9 \\ 2m-1, for\ j = 2,3, m = 6,7;\ m \ge 13 \\ 2m+1, for\ j = 4,5, m = 6,7;\ j = 8, m = 9 \\ 2m+1, for\ 12 \le j \le 15, 14 \le m \le 17 \\ 2m+2, for\ j = 6, m = 8 \\ 2m+3, for\ j = 1,2, m = 6,7 \\ 2m, for\ j = 5, m = 6 \\ 2m-6j+15, for\ j = 2,3,4, m = 8 \\ m+2, for\ j = 1, m = 7,8 \\ m+j+1, for\ j = 6,7, m = 7,8 \\ 2m+3, for\ 6 \le m \le 9;\ m \ge 19 \\ m-8, for\ 13 \le m \le 18 \\ 2m-7, for\ 10 \le m \le 12 \\ \frac{3m-12}{2}, for\ j = 16, m = 18 \\ \frac{3m+9}{2}, for\ j = 17, m = 19 \\ \frac{4m-j+7}{2}, for\ 19 \le j \le 27 \\ \frac{4m-j+8}{2}, for\ j \text{ is even number}, 18 \le j \le 26 \end{cases}$$

Then, we analyze the set of different edge weights. The set of different edge weights is the set $w(x_1x_5), w(x_4x_5), w(x_5z)$and the $w(y_j\ x_5)$.following is a description of the set of different edge weights respectively.

$$W = \left\{2m+1, 2m+2, 2m+4, \frac{2m-1}{2}, \dots, \frac{n+2m-3}{2}\right\}$$

$$|W| = m+3$$

Based on the set of edge weights, the upper bound is obtained $rac(CR_{n,m}) \le n+3$. Based on the lower and upper bounds, the rainbow antimagic connection number on the coconut root graph $(CR_{n,m})$ is $rac(CR_{n,m}) = m+3$. ∎

The following is the rainbow path of the coconut root graph:

Table 2. **Rainbow Path on the coconut root graph**$(CR_{n,m})$

Case	Condition	x	y	Rainbow Path	Condition
1	$n=5$ $m\geq 2$	x	z	x, z	$-$
2	$n=5$ $m\geq 2$	x_i	x_j	x_i, x_j	$1\leq i\leq 4$ $j=i+1$
3	$n=5$ $m\geq 2$	x_i	x_j	x_i, x_{i_+1}, x_j	$1\leq i\leq 3$ $j-i=2$
4	$n=5$ $m\geq 2$	x_5	y_j	x_5, y_j	$1\ \leq\ j\ \leq\ m$
5	$n=5$ $m\geq 2$	x_i	y_j	x_i, x_k, y_j	$i=1,4;\ k=5$ $1\leq j\leq m$
6	$n=5$ $m\geq 2$	x_i	y_j	x_i, x_k, x_5, y_j	$i\ =\ 2,3;\ k\ =\ 1,4$ $1\leq j\leq m$
7	$n=5$ $m\geq 2$	y_i	y_j	y_i, y_j	$1\leq i\leq m-1$ $1\leq j\leq m$

Figure 2 and **Figure 3** are illustrations of rainbow antimagic coloring coconut root graph $(CR_{5,2}, CR_{5,3}, CR_{5,4}, dan\ CR_{5,5})$:

Figure 2. RAC graph roots coconut $(CR_{5,2}\ And\ CR_{5,3})$

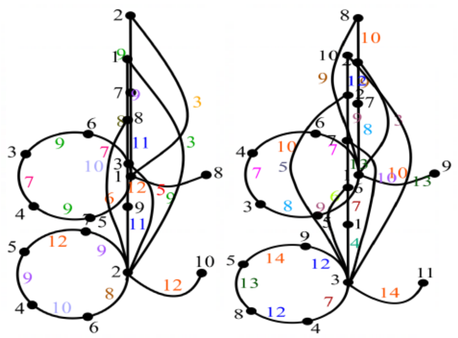

Figure 3. RAC graph roots coconut $(CR_{5,4}\ And\ CR_{5,5})$

Theorem 3. *Rainbow antimagic connection number* on a lotus graph (Lo_n) for $n\ \geq\ 2$ is $rac(Lo_n)=n+1$.

Proof. A lotus graph (Lo_n) has a set of vertices $V(Lo_n)=\{x\}\cup\{y\}\cup\{x_i;\ 1\leq i\leq 2n-1\}$ and a set of edges $E(Lo_n)=\{xy\}\cup\{xx_i; 1\leq i\leq 2n-1; i\in Odd\ number\}\cup\{x_ix_{i+1}; 1\leq i\leq 2n-2\}$. The cardinality of edge sets and vertices on a lotus graph are $(Lo_n)$and $|E(Lo_n)|=4n-2$, respectively $|V\ (Lo_n)|=2n+1$. Proved $rac(Lo_n)=n+1$using the lower bound and upper bound. First, we prove the lower bound of the lotus graph(Lo_n) with $n\ \geq\ 2$, $rac(Lo_n)\geq n+1$based on Definition 2.7.2, we obtained

$$rac(Lo_n)\geq max\{rc(Lo_n),\Delta(Lo_n)\}$$

$$n+1\quad\geq max\{3,n+1\}$$

$$n+1\quad\geq n+1$$

So, the lower bound of the *rainbow antimagic connection number* graph lotus (Lo_n) is $rac(Lo_n)\geq n+1$. Next, we prove the upper bound of the lotus graph $(Lo_n)$with $n\geq 2, rac(Lo_n)\leq n+1$, for example $f:V(Lo_n)\rightarrow\{1,2,\ldots,2n+1\}$as follows.

$$f(x)\ =1$$

$$f(y)\ =2n+1$$

$$f(xi)\ =\begin{cases}\dfrac{4n-i+1}{2}, for\ i \text{ is odd number}\\ \dfrac{i+2}{2}, for\ i \text{ is even number}\end{cases}$$

Based on the vertex function above, the edge weights on the lotus graph $(Lo_n)$are obtained as follows

$$w(xy) = 2n + 2$$

$$w(xx_i) = \frac{4n-i+3}{2}, for\ i \text{ is odd number}$$

$$w(x_i x_{i+1}) = \begin{cases} 2n+2, for\ i\ is\ odd \\ 2n+1, for\ i\ is\ even \end{cases}$$

Then, we analyze the set of different edge weights. The set $w(x_i x_{i\_+1})$, i is an odd number, is a subset of $w(xx_i)$. The set $w(x_i x_{i\_+1})$, i is an even number, is a subset of $w(xy)$. So that the set of different edge weights is the set $w(xx_i)$and $w(xy)$. The following describes a set of different edge weights in a row.

$$W = \left\{\frac{4n+2}{2}, 4n, \dots, \frac{5n+3}{2}, 2n+2\right\}$$

$$|W| = n + 1$$

Based on the set of edge weights, the upper bound is obtained $rac(Lo_n) \leq n + 1$. Based on the lower and upper bounds, the rainbow antimagic connection number on the lotus graph $(\mathrm{Lo_n})$is $\mathrm{rac(Lo_n) = n + 1}$. ■The following is the rainbow path of the lotus graph:

Table 3. Rainbow Path on a lotus graph($\mathbf{Lo_n}$)

Case	Condition	x	y	Rainbow Path	Condition
1	$n \geq 2$	x	y	x, y	$-$
2	$n \geq 2$	x	x_i	x, x_i	$1 \leq i \leq 2n-1$; i is odd number
3	$n \geq 2$	x_i	x_{i_+1}	x_i, x, x_{i_+1}	$1 \leq i \leq 2n-3$; i is odd number
4	$n \geq 2$	x_i	x_{i_+1}	x_i, x_{i_+1}	$1 \leq i \leq 2n-1$
5	$n \geq 2$	x_i	x	x_i, x_j, x	$1 \leq i \leq 2n-1$; i is odd number; $2 \leq i \leq 2n-2$; i is even number
6	$n \geq 2$	x_i	y	x_i, x_j, x, y	$1 \leq i \leq 2n-1$; i is odd number; $2 \leq i \leq 2n-2$; i is even number

We can see the figure of *rainbow antimagic coloring* graph lotus(Lo_7) in **Figure 4.**

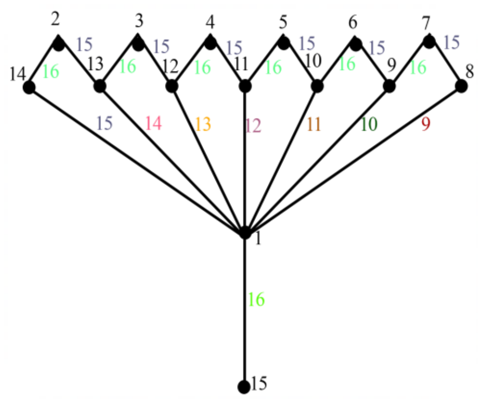

Figure 4. RAC lotus graph(Lo_7)

Theorem 4. *Rainbow antimagic connection number* on a fan stalk graph (Kt_n), for each integer $n \geq 2$is $rac(Kt_n) = n + 1$.

Proof. A stalk fan graph (Kt_n)has a vertex $V(Kt_n) = \{x\} \cup \{y\} \cup \{x_i\ ; 1 \leq i \leq n\}$ set and an edge set $E(Kt_n) = \{xy\} \cup \{xx_i; 1 \leq i \leq n\} \cup \{x_i\ x_{i+1}; 1 \leq i \leq n-1\}$. The cardinality of the edge set and vertex set of the fan stalk graph(Kt_n) respectively are $|V\ (Kt_n)| = n + 2$and $|E(Kt_n)| = 2n$. Proved $rac(Kt_n) =$

$n+1$using the lower bound and upper bound. First, we prove the lower bound of the fan stalk graph(Kt_n) with $n \geq 2, rac(Kt_n) \geq n+1$. Based on **Lemma 1**, we obtained:

$$rac(Kt_n) \geq max\{rc(Kt_n), \Delta(Kt_n)\}$$
$$n+1 \geq max\{3, n+1\}$$
$$n+1 \geq n+1$$

So, the lower bound of the rainbow antimagic connection number of the fan stalk graph $(Kt_n)$is $rac(Kt_n) \geq n+1$. Then prove the upper bound of the fan stalk graph $(Kt_n)$with $n \geq 2, rac(Kt_n) \leq n+1$, for example $f: V(Kt_n) \rightarrow \{1, 2, \ldots, n+2\}$ as follows.

$$f(x) = 1$$
$$f(y) = n + 2$$
$$f(x_i) = \begin{cases} \frac{2n-i+3}{2}, for\ i \text{ is odd number} \\ \frac{i+2}{2}\ , for\ i \text{ is even number} \end{cases}$$

Based on the vertex function above, the edge weights on the fan stalk graph (Kt_n) are obtained as follows

$$w(xy) = n + 3$$
$$w(xx_i) = \begin{cases} \frac{2n-i+5}{2}, for\ i \text{ is odd number} \\ \frac{i+4}{2}\ , for\ i \text{ is even number} \end{cases}$$
$$w(x_i x_{i+1}) = \begin{cases} n+3, for\ i\ is \text{ odd number} \\ n+2, for\ i \text{ is even number} \end{cases}$$

Then, we analyze the set of different edge weights. The set $w(x_i x_{i+1})$; i ∈ even numberis a subset of $w(xx_i)$. The set $w(x_i x_{i+1})$; i ∈ odd numberis a subset of $w(xy)$. So that the set of different edge weights is the set $w(xx_i)$and $w(xy)$. The following describes a set of different edge weights in a row.

$$W = \{3, 4, 5, \ldots, n+2, n+3\}$$
$$|W| = n+1$$

Based on the set of edge weights, the upper bound is obtained $rac(Kt_n) \leq n+1$. Based on the lower and upper bounds, the rainbow antimagic connection number for the fan stalk graph $(Kt_n)$is rac$(Kt_n) = n+1$. ■

Here, we describe the rainbow path of the fan stalk graph:

Table 4. Rainbow Path on the fan stalk graph(Kt_n)

Case	Condition	x	y	Rainbow Path	Condition
1	$n \geq 2$	x	y	x, y	−
2	$n \geq 2$	x_i	y	x_i, x, y	$1 \leq i \leq n$
3	$n \geq 2$	x_i	x	x_i, x	$1 \leq i \leq n$
4	$n \geq 2$	x_i	x_{i+1}	x_i, x_{i+1}	$1 \leq i \leq n$
5	$n \geq 2$	x_i	x_j	x_i, x, x_j	$1 \leq i \leq n-1$; $2 \leq j \leq m$

We can see the figure of rainbow antimagic coloring fan stalk graph (Kt_9)in **Figure 5**.

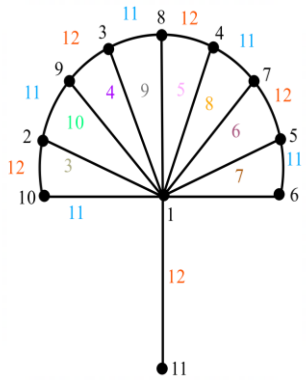

Figure 5. **RAC fan stalk graph(Kt_n)**

4. CONCLUSIONS

Based on the above results, we got a new theorem related to the *rainbow antimagic connection number of* Snail graph (S_n), coconut root ($CR_{n,m}$)graph, fan stalk (Kt_n) graph and lotus graph (Lo_n) as follow:

1. *Rainbow Antimagic Connection Number* on a Snail graph S_n with $n \geq 3$ is $rac(S_n) = n + 3$;
2. *Rainbow Antimagic Connection Number* on a coconut root graph $CR_{n,m}$ with $n = 5$ and $m \geq 2$ is $rac(CR_{n,m}) = m + 3$;
3. *Rainbow Antimagic Connection Number* on a lotus graph $Lo_n$with $n \geq 2$is $rac(Lo_n) = n + 1$;
4. *The Rainbow Antimagic Connection Number* in the fan stalk graph $Kt_n$with $n \geq 2$ is $rac(Kt_n) = n + 1$.

ACKNOWLEDGMENT

Thank you for the support of Mathematics Education Program, PUI PT Kombinatorika and Graph University of Jember year of 2023.

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