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GLOBAL STABILITY OF DISEASE-FREE EQUILIBRIA IN COVID-19 SPREAD THROUGH LIVING AND INANIMATE OBJECTS MATHEMATICAL MODEL

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ABSTRACT

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Covid-19 is a dangerous disease that is easily transmitted, both through living media in the form of interactions with infected human, as well as through inanimate objects in the form of surfaces contaminated with the Coronavirus. Various preventive and repressive efforts have been made to prevent the spread of this disease, such as isolating and recovering the infected human. In this study, the authors construct and analyze a new mathematical model in the form of a three-dimensional differential equations system that represent the interactions between subpopulations of coronavirus living on inanimate objects, susceptible human, and infected human within a population. The purpose of this study is to investigate the criteria that must be met in order to create a population free from Covid-19 by considering inanimate objects as a medium for its spread besides living objects. The model solution that represents the number of each subpopulation is non-negative and bounded, so it is in accordance with the biological condition that the number of subpopulations cannot be negative and there is always a limit for its value. The eradication rate of Coronavirus living on inanimate objects, the recovery rate of infected human, and the interaction rate between susceptible human and infected human such that the population is free from Covid-19 for any initial conditions of each subpopulation were investigated in this study through global stability analysis of the disease-free equilibrium point of the model.

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1. INTRODUCTION

Covid-19 is a dangerous infectious disease that has been a pandemic since 2020 [1]. The disease was first discovered in Wuhan, China in late December 2019 [2] [3] and is caused by a virus infection called *Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2)* or Coronavirus [4]. As of February 19th, 2023, more than 757 million positive cases and more than 6.8 million deaths cases from this disease have been detected in the world [5].

The spread of Covid-19 can occur through interactions between animals and human or between human who carrying the Coronavirus [6] [7]. Furthermore, inanimate objects can also mediate the spread of Coronavirus [8]. Coronaviruses can live for several hours to days in inanimate objects, such as aerosols and surfaces of objects [9]. Coronavirus can live for three hours in aerosols, 4 hours in copper, 1 day in cardboard [9]. Moreover, Coronavirus can live more stably in plastic and steel than copper and cardboard for 72 hours [9]. Even Coronavirus can live on paper, glass, PVC, metal, ceramics, and teflon for up to 5 days [10]. It shows that the Coronavirus can survive on various surfaces of objects, making it possible for Coronavirus to infect human who touch it. Therefore, inanimate objects can mediate the spread of Covid-19 other than through direct interaction between human.

Various studies have been carried out to form a model for the spread of Covid-19, such as the SIR model. However, the dissemination medium taken in the previous SIR model such as in [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], and [22] still focuses on human-to-human interactions and has not considered the possibility of Coronavirus infection through inanimate objects . Thus, in this study we construct a new mathematical model in the form of a three-dimensional differential equation system that considers those two facts, i.e. the Coronavirus-to-human besides human-to-human interactions and the existence of Coronavirus in inanimate objects which can infect humans. The model represents interaction between Coronavirus living on inanimate objects, susceptible human, and infected human subpopulation. We analyze the model to identify the criteria that must be met to make the population free from Covid-19, so that it can be used as a recommendation to prevent or overcome Covid-19 in the population.

2. RESEARCH METHODS

The type of this research is quantitative. This research is done by following the steps below.

1. Conduct a literature study on the characteristics of Covid-19, especially the transmission media, so that the factors related to it can be identified.
2. Determine some assumptions to make this research focus. The assumptions generated in this research are:
    a. Neglect the possibility that susceptible human subpopulation can carry Coronavirus besides infected subpopulation.
    b. Infection of Coronavirus living on inanimate objects is more massive than its eradication by susceptible human subpopulation.
    c. Consider that every baby born is not infected by Coronavirus.
    d. Only investigate the conditions for the population to be free from Covid-19.
3. Define model variables dan parameters based on the literatures and assumptions that have been made in order to construct the mathematical model.
4. Construct the mathematical model which represents the interactions between the variables with the parameters as the rate. The model is a differential equation system with three variable dimensions and eight parameters' dimensions.
5. Analyze the mathematical model by using mathematical theory. The analysis includes non-negativeness and boundedness of the model solution, disease-free equilibria, basic reproduction number, local and global stability of the disease-free equilibria.
6. Simulate the solution of the model by setting the parameter values based on the literature and assumption such that they meet the global stability of disease-free equilibria requirements in order to confirm the correctness of the analysis results and illustrate the dynamics of Coronavirus, susceptible, and infected subpopulation related to the disease-free equilibria.

7. Interpret the mathematical results in medical or biological terms in order to obtain recommendations in solving the existing problems.

3. RESULTS AND DISCUSSION

In this section, we will discuss about the formation of mathematical models, the analysis of the non-negativeness and boundedness of solutions, the investigation of disease-free equilibria, along with the analysis of its local and global stability.

3.1 Mathematical Models

Covid-19 infection can occur through interaction between human and inanimate objects as the medium. Therefore, in the population, three subpopulations are defined, that is the subpopulation of Coronavirus living on inanimate objects, susceptible human, and infected human.

Some interactions occur between subpopulations. The subpopulation of Coronavirus living on inanimate objects increases due to the droplets from infected human subpopulations that attached to them. Coronaviruses that attached to inanimate objects decreases because the Coronavirus can only survive at a certain period of time in inanimate objects [9], [10]. Susceptible human subpopulations are prone to increase due to the natural birth and the presence of infected human who recover. This subpopulation also increases indirectly due to the efforts in eradicating Coronavirus living on inanimate objects by susceptible human. This subpopulation is reduced because it is being infected with Coronavirus living on inanimate objects and infected human. The subpopulation of infected human increases because there are susceptible humans who are infected by Coronavirus due to the infections that occur both through Coronavirus living on inanimate objects, as well as human who are infected. The subpopulation of infected human is reduced because this subpopulation may die naturally or recover. The subpopulation of infected human is also reduced because Coronavirus infection.

Based on the subpopulations involved in the spread of Covid-19 that have been described, several variables and parameters are defined to construct the mathematical models. Some model variables representing subpopulations are written in Table 1.

Table 1. Model Variable

Variable	Interpretation	Unit
V	Coronavirus living on inanimate objects subpopulation	Virion
S	Susceptible human subpopulation	Person
I	Infected human subpopulation	Person
t	Time	Day

In accordance with the biological facts of Coronavirus living on inanimate objects, susceptible human, and infected human are always more than or equal to zero, so the value $V, S,$ and I are non-negative. Model parameters representing the level of interaction between subpopulations are written in Table 2.

Table 2. Model parameter

Parameter	Interpretation	Unit
a	Addition rate of Coronavirus living on inanimate objects due to droplets from infected human	Virion per day
b	Natural mortality rate of Coronavirus living on inanimate objects	Virion per day
c	Natural birth rate of susceptible human	People per day
p	Infection rate of Coronavirus living on inanimate objects against susceptible human	Virion per day
q	Eradication rate of Coronavirus living on inanimate objects by susceptible human	Virion per day
e	Natural mortality rate of susceptible human	Per day
f	Recovery rate of infected human	Per day
g	Interaction rate between susceptible human and infected human	People per day
h	Mortality rate of infected human due to Coronavirus infection	Per day

All parameters are positive, since all parameters represent interactions that occur between subpopulations. Specifically, $0 \leq b, e, h \leq 1$, because they represent proportion and $p > q$ based on the assumption.

The following compartment diagram represents the interaction between subpopulations with the level of interaction of each as defined.

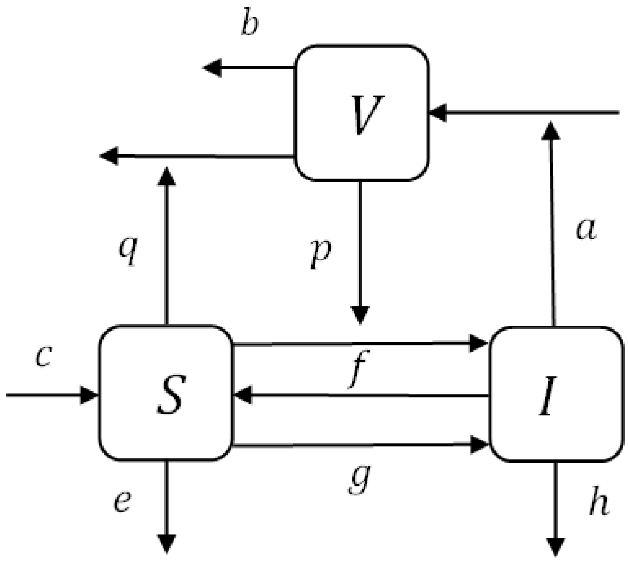

Figure 1. Compartment Diagram Of The Interaction Between Subpopulations

Based on the compartment diagram, we constructed a new mathematical model in the form of a system of differential equations with three dimensions of variable as follows.

$$\frac{dV}{dt} = aI - bV \quad (1)$$

$$\frac{dS}{dt} = c - pSV + qSV - eS + fI - gSI \quad (2)$$

$$\frac{dI}{dt} = pSV - qSV + gSI - fI - hI \quad (3)$$

Equation (1) represents the rate of change of Coronavirus living on inanimate objects subpopulations with respect to time. The first term denotes the addition of Coronavirus living on inanimate objects as a result of infected human droplets attached to inanimate objects with a as the addition rate. The second term denotes the reduction of Coronavirus due to the natural death of Coronavirus with b as the mortality rate.

Equation (2) represents the rate of change of susceptible human subpopulations with respect to time. The first term denotes the addition of susceptible human due to the natural birth with c as the birth rate. The second term denotes the reduction of susceptible human due to the Coronavirus infections living on inanimate objects with p as the infection rate. The third term denotes the addition of susceptible human indirectly due to the eradication of Coronavirus living on inanimate objects by susceptible human with q as the eradication rate. The fourth term denotes the reduction of susceptible human due to the natural death with e as the mortality rate. The fifth term denotes the rate of addition of susceptible human due to the recovery of infected human with f as the recovery rate. The sixth term denotes the reduction of susceptible human due to their interactions with infected human, in such that susceptible human are becoming infected with g as the interaction rate.

Equation (3) represents the rate of change of infected human subpopulations with respect to time. The first term denotes the addition of human infected due to the Coronavirus infections living on inanimate objects with p as the infection rate. The second term denotes the reduction of infected human indirectly due to the eradication of Coronavirus living on inanimate objects by susceptible human with q as the eradication rate. The third term denotes the addition of infected human due to their interactions with susceptible human, in such that susceptible human become infected with g as the interaction rate. The fourth term denotes the rate of infected human reduction due to the recovery with f as the recovery rate. The fifth term denotes the death rate of infected human due to the infection with h as the mortality rate.

3.2 Non-negativeness and Boundedness of the Solution

Non-negativeness of the model solution guarantees that for any time, the subpopulation of Coronavirus living on inanimate objects, susceptible human, and infected human have non-negative value which corresponds to the biological fact that subpopulations cannot be negative [23]. The non-negativeness of the model solution is expressed in the following theorem.

Theorem 1. *For each initial condition,* $V(0) \geq 0, S(0) \geq 0,$ *and* $I(0) \geq 0,$ *then the model solution is always non-negative for each* $t > 0$.

Proof. By subtracting the right field of Equation (1) by aI, we obtain

$$\frac{dV}{dt} \geq -bV \Leftrightarrow V(t) \geq V(0)e^{-bt} \geq 0 \tag{4}$$

Then, by subtracting the right field of Equation (2) by $c + qSV + fI$, we obtain

$$\frac{dS}{dt} \geq -pSV - eS - gSI \Leftrightarrow S(t) \geq S(0)e^{-\int_0^t (dV(t)+gI(t))dt+et} \geq 0 \tag{5}$$

Moreover, by subtracting the right field of Equation (3) by $pSV - qSV$, we obtain

$$\frac{dI}{dt} \geq gSI - fI - hI \geq I(0)e^{\int_0^t (gI(t))dt-(f+h)t} \geq 0 \tag{6}$$

Based on the inequality (4), (5), and (6), we obtain $V(t) \geq 0, S(t) \geq 0,$ and $I(t) \geq 0$ which means that the model solution is always of non-negative value for each $t > 0$. ∎

Boundedness of the model solution guarantees that for any time, the subpopulation of Coronavirus living on inanimate objects, susceptible human, and infected human is bounded which corresponds to the biological fact that there is a maximum or minimum limit of subpopulation [23]. Boundedness of the model solution is expressed in the following theorem.

Theorem 2. *The model solution is bounded for each* $t > 0$.

Proof. We will show that there is $M > 0$ so that $V(t) \leq M,\ S(t) \leq M$, and $I(t) \leq M$ for each $t > 0$. Based on Equations (2) and (3), we obtain

$$\frac{d(S+I)}{dt} = c - eS - hI \leq c - k(S + I),$$

with $k = \min\{e, h\}$. Furthermore, note that

$$\lim_{t\to\infty}(S(t) + I(t)) \leq \frac{c}{k} = M_1.$$

Consequently $S(t) + I(t)$ bounded for each $t > 0$. $S(t)$ and $I(t)$ is also bounded for each $t > 0$, because $S(t) + I(t)$ is bounded for each $t > 0$. Then, based on Equation (1), we obtain

$$\frac{dV}{dt} = aI - bV \leq aM_1 - bV,$$

so that

$$\lim_{t\to\infty} V(t) \leq \frac{aM_1}{b} = M_2.$$

Consequently $V(t)$ is bounded for each $t > 0$. Then, we choose $M = maks\{M_1, M_2\}$, so that we obtain $V(t) \leq M,\ S(t) \leq M$, and $I(t) \leq M$ for each $t > 0$. ∎

3.3 Disease-Free Equilibria

The non-linearity of the model makes the model solution difficult to find exactly. Therefore, the dynamics of the model solution will be investigated related to the equilibria representing condition that have not changed for a very long time. We especially investigate the disease-free equilibria because it represents the condition of a population that is free from Covid-19. Based on this, investigation of the disease-free equilibria is written in the following theorem.

Theorem 3. *The disease-free equilibria in the model is* $E_0 = (V^*, S^*, I^*) = \left(0, \frac{c}{e}, 0\right)$ *which exists for each condition.*

Proof. The equilibria is obtained by solving $\frac{dV}{dt}=\frac{dS}{dt}=\frac{dI}{dt}=0$ [24], [25], [26], [27]. Therefore, based on Equations (1), (2), and (3) we obtain

$$\frac{dV}{dt}=aI-bV=0 \quad (7)$$

$$\frac{dS}{dt}=c-pSV+qSV-eS+fI-gSI=0 \quad (8)$$

$$\frac{dI}{dt}=pSV-qSV+gSI-fI-hI=0 \quad (9)$$

Based on Equation (1) we obtain $V=\frac{a}{b}I$. Furthermore, by eliminating Equations (8) and (9) we obtain $c-eS-hI=0$ which is equivalent to $S=\frac{-hI+c}{e}$. Then, by substituting $V=\frac{a}{b}I$ and $S=\frac{-hI+c}{e}$ to Equations (9) we obtain

$$I(AI-B)=0 \quad (10)$$

with $A=\frac{(p-q)ah+bgh}{be}$ and $B=\frac{bef+beh+(p-q)ac+bgc}{be}$. Based on Equation (10), we obtain $I=0$ or $I=\frac{B}{A}=\frac{bef+beh+(p-q)ac+bgc}{(p-q)ah+bgh}=\frac{be(f+h)+c[(p-q)a+bg]}{h[(p-q)a+bg]}$. We choose $I=0$, because this study only focused on the analysis of an equilibria with zero value of infected human subpopulations which represents the condition of a Covid-19-free population. If $I=0$, then $V=\frac{a}{b}(0)=0$ and $S=\frac{-h(0)+c}{e}=\frac{c}{e}$. Consequently, we found an equilibria $E_0=(V^*,S^*,I^*)=\left(0,\frac{c}{e},0\right)$. $E_0=(V^*,S^*,I^*)=\left(0,\frac{c}{e},0\right)$ exists for each condition, because $V^*=0, S^*=\frac{c}{e}>0$, and $I^*=0$, thus met the biological fact that each subpopulation has non-negative value. E_0 is called the disease-free equilibria, because the value of infected human subpopulation on E_0, i. e. I^* is zero which means that there is no human infected by Coronavirus in the population, so the population is free from Covid-19. Hence, the disease-free equilibria in the model is $E_0=(V^*,S^*,I^*)=\left(0,\frac{c}{e},0\right)$ which exists for each condition. ∎

3.4 Basic Reproduction Numbers (R_0)

The basic reproduction number is an indicator that determines condition of the population that is free from Covid-19 or the spread of Covid-19 [28]. The basic reproduction number of the model is expressed in the following theorem.

Theorem 4. *The basic reproduction number of the model is* $R_0=\frac{gc}{e(f+h)}$.

Proof. The basic reproduction number of the model is calculated using the *Next Generation* Matrix [29]. The matrix can be determined by an equation that represents the rate of change of the infected human subpopulation, namely Equation (9) which is equivalent to

$$\frac{dI}{dt}=\sigma(V,S,I)-\mu(V,S,I),$$

with $\sigma(V,S,I)=pSV+gSI$ and $\mu(V,S,I)=qSV+fI+hI$. By linearizing $\sigma$and μ with respect to I, we get $\omega=gS$ and $\delta=f+h$. Next, we define the *Next Generation* Matrix of the model as H, so that

$$H=\omega\delta^{-1}=gS(f+h)^{-1} \quad (11)$$

Then, by substituting $S=S^*$ to Equation (11) is obtained

$$H=\omega\delta^{-1}=\frac{gc}{e(f+h)}=R_0. \blacksquare$$

3.5 Local Stability of Disease-Free Equilibria

The dynamics of the model solution around the disease-free equilibria E_0 were analyzed using the local stability of the equilibria. The local stability analysis of disease-free equilibria E_0 shows the criteria that must be met in order to make the population free from Covid-19 for the initial conditions of each subpopulation around the equilibria as time goes by. The criterion is written in the following theorem.

Theorem 5. *E_0 is locally asymtotically stable if $R_0 < 1$.*

Proof. Local stability E_0 was analyzed by linearizing the model using Jacobian Matrix [24], [25], [26], [27]. The Jacobian matrix of the model evaluated at E_0 is

$$J\left(0,\frac{c}{e},0\right)=\begin{pmatrix} -b & 0 & a \\ -\frac{(p-q)c}{e} & -e & f-\frac{gc}{e} \\ \frac{(p-q)c}{e} & 0 & \frac{gc}{e}-f-h \end{pmatrix}.$$

Consider λ is the eigen value of the Jacobian matrix. Hence, the characteristic equation of the Jacobian matrix is obtained as follows.

$$(\lambda+e)[\lambda^2+A\lambda+B]=0$$

where $A=b+f+h-\frac{gc}{e}$ and $B=b(f+h)-\frac{c[a(p-q)+bg]}{e}$. Therefore, we obtain $\lambda_1=-e<0$ and $\lambda^2+A\lambda+B=0$. The real part of the roots in equation $\lambda^2+A\lambda+B=0$ is negative if $A>0$ and $B>0$ which are equivalent to

$$b+f+h>\frac{gc}{e} \tag{12}$$

and

$$b(f+h)>\frac{c[a(p-q)+bg]}{e} \tag{13}$$

respectively. Based on Inequation (13) and assumption that $p>q$, we obtain $eb(f+h)>cbg$ which is equivalent to $R_0=\frac{gc}{e(f+h)}<1$. If $R_0=\frac{gc}{e(f+h)}<1$, we obtain $f+h>\frac{gc}{e}$, so that $b+f+h>\frac{gc}{e}$ which equivalent to Inequation (12). Hence, E_0 is locally asymtotically stable if $R_0<1$. ∎

3.6 Global Stability of the Disease-Free Equilibria

The dynamics of the model solution under any initial subpopulation conditions were analyzed using the global stability of the equilibria. The global stability of disease-free equilibria E_0 indicates the criteria that must be met in order to make the population free from Covid-19 for any initial condition of each subpopulation as time goes by. The criterion is written in the following theorem.

Theorem 6. *E_0 is globally asymptotically stable if these two conditions are fulfilled, i.e.*

1) $q>p-\frac{be}{c}$
2) *At least one of these three conditions is fulfilled, i.e.*
   i. $a<f+h-\frac{gc}{e}$
   ii. $f>a+\frac{gc}{e}-h$
   iii. $g<\frac{(f+h-a)e}{c}$

Proof. The criteria for global stability of disease-free equilibria E_0 were investigated using the LaSalle-Lyapunov Theorem [30]. We define Lyapunov function

$$L=V+I,$$

so that,

$$\frac{dL}{dt}=aI-bV+pSV-qSV+gSI-fI-hI$$

$$\leq aI+\frac{gc}{e}I-fI-hI-bV+\frac{pc}{e}V-\frac{qc}{e}V$$

$$= -\left(-a - \frac{gc}{e} + f + h\right) I - \left(b - \frac{pc}{e} + \frac{qc}{e}\right) V$$
$$< 0$$

if $b - \frac{pc}{e} + \frac{qc}{e} > 0$ which is equivalent to $q > p - \frac{be}{c}$ and $-a - \frac{gc}{e} + f + h > 0$ which is equivalent to $a < f + h - \frac{gc}{e}$ or $f > a + \frac{gc}{e} - h$ or $g < \frac{(f+h-a)e}{c}$. Moreover, $\frac{dL}{dt} = 0$ if and only if $I = 0$ and $V = 0$. Therefore, the maximum invariant compact set is the field $I = 0$ and $V = 0$. Hence, the solutions which are initiated in the field will converge to E_0. Therefore, E_0 is globally asymptotically stable if $q > p - \frac{be}{c}$ and at least one of these three conditions is satisfied, i.e. $a < f + h - \frac{gc}{e}$ or $f > a + \frac{gc}{e} - h$ or $g < \frac{(f+h-a)e}{c}$. ∎

Based on the conditions that must be met in order to make the disease-free equilibria E_0 become globally asymptotically stable have been investigated in **Theorem 6**, we found that the population will be free from Covid-19 for any initial condition of each subpopulation as time goes by if these following two conditions are fulfilled:

1) The eradication rate of Coronavirus living on inanimate objects (q) is greater than $p - \frac{be}{c}$.
2) At least one of these three conditions is satisfied, i.e.
   i. the addition rate of Coronavirus living on inanimate objects due to the droplets from infected human (a) is smaller than $f + h - \frac{gc}{e}$,
   ii. the recovery rate of infected human (f) is greater than $a + \frac{gc}{e} - h$,
   iii. the interaction rate between susceptible human and infected human (g) is smaller than $\frac{(f+h-a)e}{c}$.

3.7 Numerical Simulation

Dynamics of the population which is free from Covid-19 for any initial condition are described by the fluctuation of Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations when they meet the criteria for global stability of disease-free equilibria (E_0). Those phenomena are illustrated by numerical simulation.

The numerical simulation scheme is written as follows. The criteria of disease-free equilibria (E_0) global stability which have been carried out in **Theorem 6** should be fulfilled, i.e. $q > p - \frac{be}{c}$ and at least one of these three conditions is satisfied, i.e. $a < f + h - \frac{gc}{e}$ or $f > a + \frac{gc}{e} - h$ or $g < \frac{(f+h-a)e}{c}$ in order to create the population condition which is free from Covid-19. We assign the values of the model parameters, namely $a = 0.1, b = 0.05, c = 5, p = 0.1, q = 0.095, e = 0.62, f = 1$ **[31]**, $g = 0.0707$ **[32]**, dan $h = 0.02$ **[11]** such that the global stability of disease-free equilibria E_0 criteria are fulfilled, because we want to illustrate the dynamics of the population which is free from Covid-19.. Based on the selected parameters value, a disease-free equilibria is found, i.e. $E_0 = (0,8.065,0)$. Next, we set some arbitrary initial conditions of each subpopulation, both around, and quite far from $E_0 = (0,8.065,0)$, i.e. $A = (20,10,20)$, $B = (10,20,5)$, $C = (30,40,20)$, and $D = (40,10,50)$, because we want to illustrate that the Coronavirus living on inanimate objects and infected human subpopulation will go to 0, while the susceptible human subpopulation will go to 8.065, i.e. the population is free from Covid-19 for every initial condition of the subpopulations, both around E_0 and far from E_0. Then, we plot the solution of the model in two perspective ways. The first perspective is the plot of each subpopulation with respect to time which shows the convergence of each subpopulation as time goes by. The second perspective is the plot of each subpopulation with respect to the other subpopulations which shows the impact of one subpopulation fluctuation to the other subpopulations.

The dynamics of Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations with respect to time with the initial conditions of subpopulations and model parameter values that have been set in such a way to fulfill the criteria for global stability of disease-free equilibria E_0 are illustrated in **Figure 2** below.

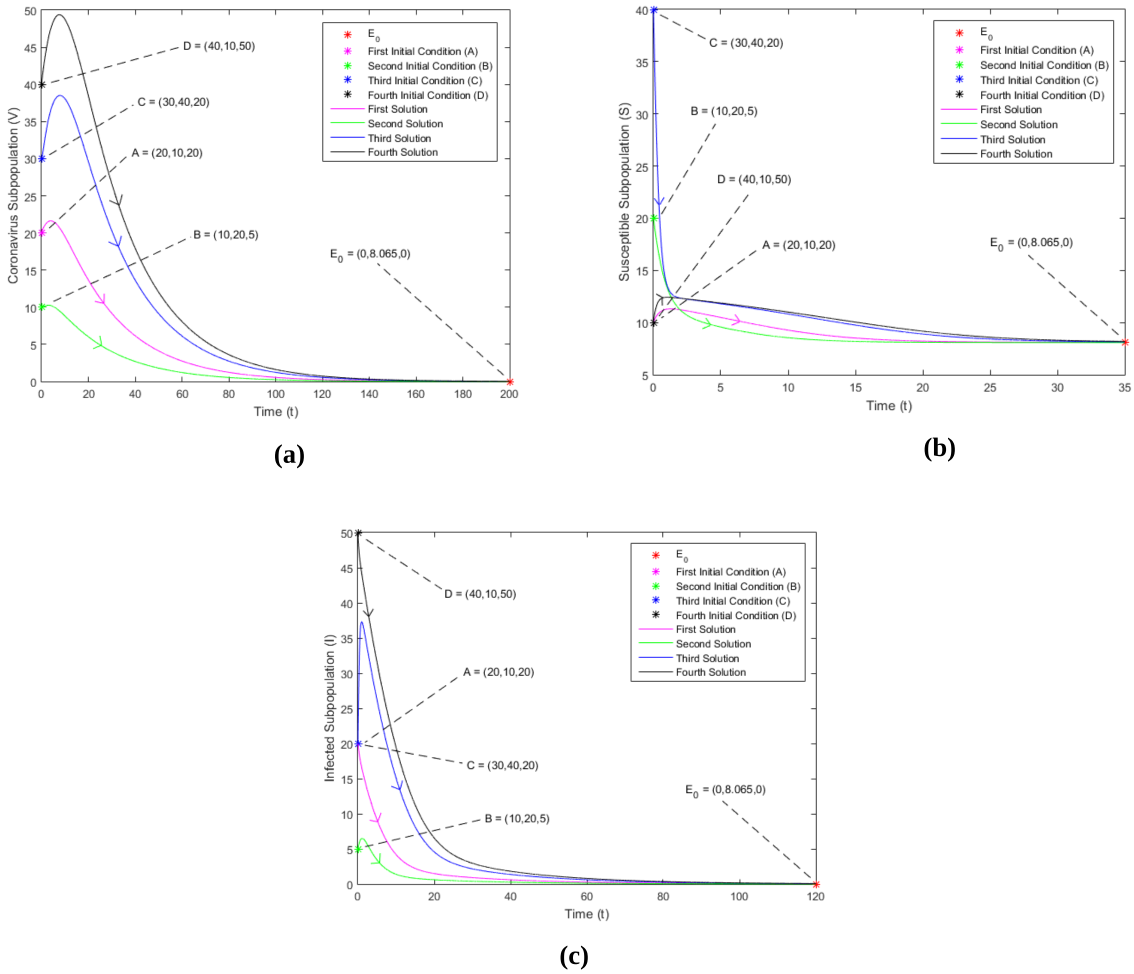

Figure 2. The dynamics of Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations with respect to time when the criteria for global stability of disease-free equilibria E_0 are met. (a) The dynamics of Coronavirus living on inanimate objects subpopulation with respect to time. (b) The dynamics of susceptible human subpopulation with respect to time. (c) The dynamics of infected human subpopulation with respect to time

Based on Figure 2 (a), when the initial conditions of each subpopulation were taken at random condition and the global stability criteria for global stability of the disease-free equilibria E_0 are met, Coronavirus living on inanimate objects subpopulation is convergent to 0 and remains at that condition as time goes by. It represents that Coronavirus living on inanimate objects subpopulation will become extinct or there is no Coronavirus living on inanimate objects in the population.

Based on Figure 2 (b), when the initial conditions of each subpopulation were taken at random condition and the global stability criteria for global stability of the disease-free equilibria E_0 are met, susceptible human subpopulation progresses towards 8.065 and remains at that condition as time goes by. It means that the susceptible human subpopulation will go to a value and remains at that value, so that the susceptible human subpopulation still exists in the population.

Based on Figure 2 (c), when the initial conditions of each subpopulation were taken at random condition and the global stability criteria for global stability of the disease-free equilibria E_0 are met, infected human subpopulation progressively goes towards 0 and remains at that condition as time goes by. It represents that there is no human that infected by Coronavirus in the population.

The dynamics of interaction between Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations with those initial conditions of subpopulations and model parameter values that have been selected in such a way to fulfill the criteria for global stability of disease-free equilibria E_0 are presented in Figure 3 below.

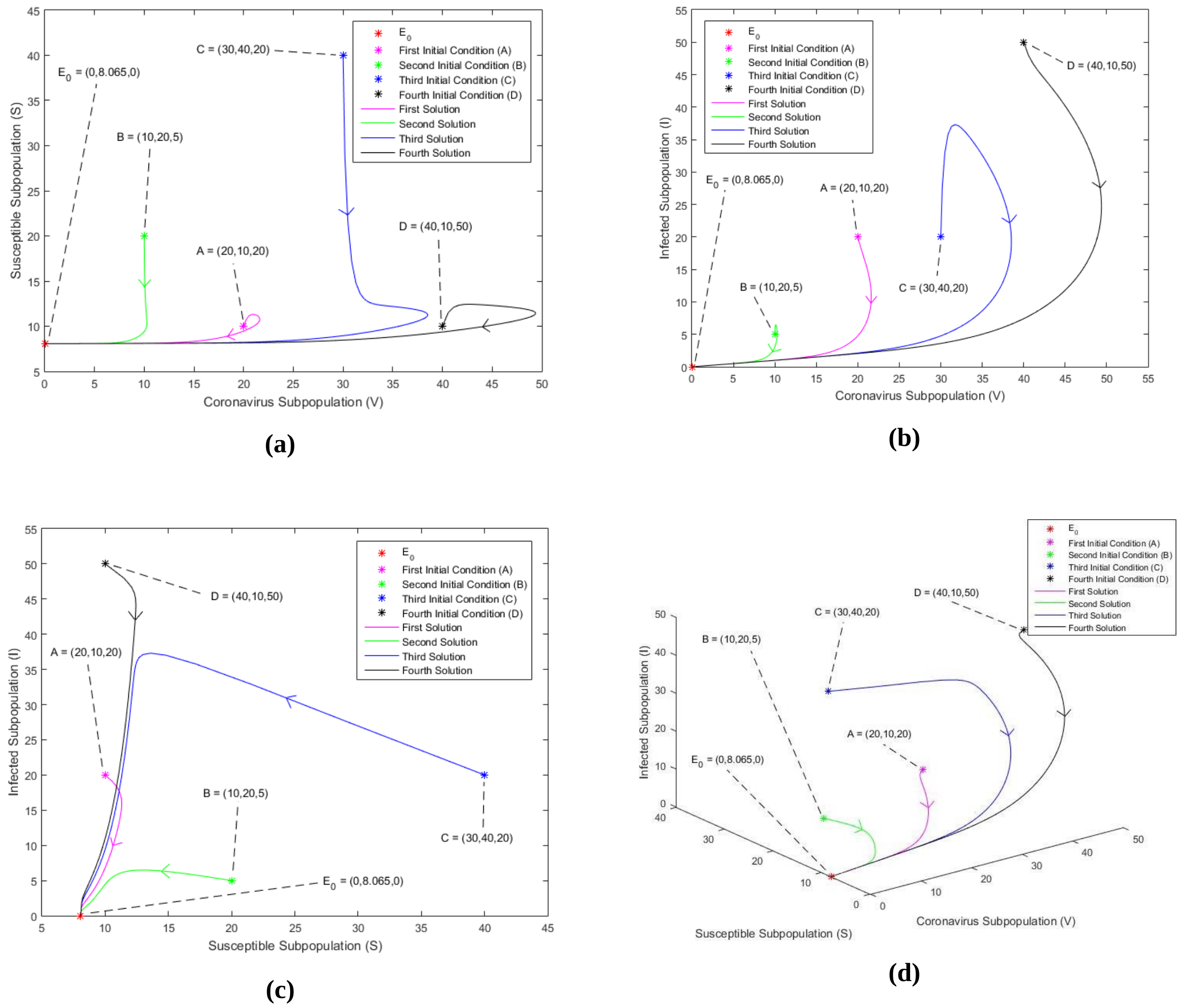

Figure 3. The dynamics of interaction between Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations when the criteria for global stability of disease-free equilibria E_0 are met. (a) The dynamics of interaction between Coronavirus living on inanimate objects with susceptible human subpopulations. (b) The dynamics of interaction between Coronavirus living on inanimate objects with infected human subpopulations. (c) The dynamics of interaction between susceptible human with infected human subpopulations. (d) The dynamics of interaction between Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations.

Based on Figure 3 (a), when the initial conditions of each subpopulation were taken at random condition and the criteria for global stability of disease-free equilibria E_0 are met, Coronavirus living on inanimate objects subpopulation will decrease towards 0, while susceptible human subpopulation will go to 8.065 and remains at that condition as time goes by. It means that Coronavirus living on inanimate objects subpopulation will become extinct, while the susceptible human subpopulation still exists at a value in the population.

Based on Figure 3 (b), when the initial conditions of each subpopulation were taken at random condition and the criteria for global stability of the disease-free equilibria E_0 are met, Coronavirus living on inanimate objects and infected human subpopulations will decrease towards 0, and remains at that condition as time goes by. It represents that there are no Coronavirus living on inanimate objects and infected human in the population.

Based on Figure 3 (c), when the initial conditions of each subpopulation were taken at random condition and the criteria for global stability of disease-free equilibria E_0 are met, infected human subpopulation will decrease towards 0, while susceptible human subpopulation will go to 8.065 and remains at that condition as time goes by. It means that there is no human that infected by Coronavirus, while the susceptible human still exists at a value in the population.

Based on **Figure 3** (d), when the initial conditions of each subpopulation were taken at random condition and the criteria for global stability of the disease-free equilibria E_0 are met, Coronavirus living on inanimate objects and infected human subpopulations will go to 0, while the susceptible human subpopulation will go to 8.065 and remains at that condition as time goes by. It represents that each subpopulation will go towards the value of disease-free equilibria $E_0 = (0{,}8.065{,}0)$. This phenomenon illustrates that when the initial conditions of each subpopulation were taken at random condition, there is only a subpopulation that exists in the population as time goes by, i.e. susceptible human. Beside that, there is no Coronavirus living on inanimate objects or Coronavirus living on inanimate objects subpopulation will become extinct and there is no human that infected by Coronavirus in the population. In other words, the population is free from Covid-19.

4. CONCLUSIONS

A population is free from Covid-19 for any initial condition of Coronavirus living on inanimate objects, susceptible human, and infected human subpopulations as time goes by if these following two conditions are fulfilled:

1) The eradication rate of Coronavirus living on inanimate objects (q) is greater than $p - \frac{be}{c}$.
2) At least one of these three conditions is satisfied, i.e.
   i. the addition rate of Coronavirus living on inanimate objects due to the droplets from infected human (a) is smaller than $f + h - \frac{gc}{e}$,
   ii. the recovery rate of infected human (f) is greater than $a + \frac{gc}{e} - h$,
   iii. the interaction rate between susceptible human and infected human (g) is smaller than $\frac{(f+h-a)e}{c}$.

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