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EDGE IRREGULAR REFLEXIVE LABELING ON ALTERNATE TRIANGULAR SNAKE AND DOUBLE ALTERNATE QUADRILATERAL SNAKE

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ABSTRACT

Let G in this paper be a connected and simple graph with set V(G) which is called a vertex and E(G) which is called an edge. The edge irregular reflexive k-labeling f on G consist of integers $\{1,2,3,\ldots,k_e\}$ *as edge labels and even integers* $\{0,2,4,\ldots,2k_v\}$ *as the label of vertices,* $k=\max\{k_e,2k_v\}$*, all edge weights are different. The weight of an edge xy in G represented by* $wt(xy)$ *is defined as* $wt(xy) = f(x) + f(xy) + f(y)$*. The smallest k of graph G has an edge irregular reflexive k-labeling is called the reflexive edge strength, symbolized by res (G). In article, we discuss about edge irregular reflexive k-labeling of alternate triangular snake* $A(T_n)$ *and the double alternate quadrilateral snake* $DA(Q_n)$*. In this paper, the res of alternate triangular snake* $A(T_n)$ *,* $n \geq 3$ *has been obtained. That is* $\left\lceil\frac{2n-1}{3}\right\rceil$ *for n even,* $2n-1 \not\equiv 2,3 \pmod 6$*,* $\left\lceil\frac{2n-1}{3}\right\rceil+1$ *for n even,* $2n-1 \equiv 2,3 \pmod 6$*,* $\left\lceil\frac{2n-2}{3}\right\rceil$ *for n odd,* $2n-2 \not\equiv 2,3 \pmod 6$*, and* $\left\lceil\frac{2n-2}{3}\right\rceil+1$ *for n odd,* $2n-2 \equiv 2,3 \pmod 6$*. Then, the reflexive edge strength of double alternate quadrilateral snake DA* (Q_n) $\left\lceil\frac{4n-1}{3}\right\rceil$ *for n even,* $4n-1 \not\equiv 2,3 \pmod 6$*,* $\left\lceil\frac{4n-1}{3}\right\rceil+1$ *for n even,* $4n-1 \equiv 2,3 \pmod 6$*,* $\left\lceil\frac{4n-4}{3}\right\rceil$ *for n odd,* $4n-4 \not\equiv 2,3 \pmod 6$*, and* $\left\lceil\frac{4n-4}{3}\right\rceil+1$ *for n odd,* $4n-4 \equiv 2,3 \pmod 6$*.*

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1. INTRODUCTION

Graphs discussed in this paper are connected, undirected, and simple graph. Here using vertex set denoted as $V(G)$ also edge set as $E(G)$. Graph labeling is a mapping with component graph as domain and the co-domain is integers number [1]. There is different types of labeling, which categorized as being vertex labeling, edge labeling, and total labeling. There are many categories of labeling on Gallian research. One type of labeling is irregular total k-labeling [2].

Baca *et al.* [3] the total k-labeling is divided into two types namely edge irregular total k-labeling and vertex irregular total k-labeling. If all edges have different weights, an edge irregular total k-labeling similar with total k-labeling. In 2017, Ryan *et al.* in Baca *et al.* [4] introduced concept of irregular total k-labeling. Namely edge irregular reflexive total k-labeling and vertex irregular reflexive total k-labeling. In graph G, $e = xy$, denoted by $wt(e)$ as the sum of labels vertex x, vertex y and the label of edge e.

An edge irregular reflexive k-labeling is determine as the function $fe : E(G) \rightarrow \{1,2,\ldots,k_e\}$ and $fv : V(G) \rightarrow \{0,2,\ldots,2k_v\}$, where $k = max\{k_e, 2k_v\}$. In a graph, the minimum value of k that can be labeled with an edge irregular reflexive labeling is called the strength of the reflexive edge strength and is denoted by $res(G)$ [5]. For this article, we will discuss about reflexive edge strength of G, denoted as $res\,(G)$. When we prove the result, we will often use the lemma proven by Ryan *et al.* in Baca *et al.*[4].

Lemma 1. *For every graph G*

$$res\,(G) \geq \left\lceil \frac{|E(G)|}{3} \right\rceil for\ |E(G)| \not\equiv 2,3\ (mod\ 6) \text{ and } \left\lceil \frac{|E(G)|}{3} \right\rceil + 1\ for\ |E(G)| \equiv 2,3\ (mod\ 6).$$

The lower bound for $res\,(G)$ appeared from the fact that the minimal edge weight under an edge irregular reflexive labeling is one, and the basis of the maximal edge weights, that is $|E(G)|$ can be achieved only as the sum of three numbers from whose at least two are even.

In the same paper, prove the *res* of prisms graph D_n and wheels [6], generalized friendship graph [4], some tree graph [7]. Guirao *et al.* [8] determined for disjoint union of generalized Petersen graph. Zhang *et al*. [9] defined disjoint union of gear graph and prims graphs. Then 2020, Indriati *et al.* [10] proved the $res(G)$ corona of path and complete graph K in the same year Ibrahim *et al.* [11] determined for star, double star, and caterpillar graphs. In 2021 Setiawan and Indriati proved $res(G)$ Sun graph [12], in the same year Novelia and Indriati determine for Banana tree graphs $B\ 2, n$ and $B\ 3, n$ [13], Nadia *et al*. proved tadpole graphs $T_{m,1}$ and $T_{m,2}$ [14]. This article, we determine the $res\,(G)$ of alternate triangular snake AT_n and double alternate quadrilateral snake $DA(Q_n)$.

2. RESEARCH METHODS

Research explained in this article uses literature studies related to reflexive edge strength for various types about graphs. The materials used in this research are books, journals, papers, and articles. Related to reflexive edge strength, alternate triangular snake, and double alternate quadrilateral snake. The procedure used in this research are as follows,

1. Determine the lower bound $res(G)$ of alternate triangular snake $A(T_n)$ and double alternate quadrilateral snake $DA(Q_n)$ based on lemma proven by Ryan *et al.* in Baca *et al.* [4],
2. Labeling alternate triangular snake $A(T_n)$ and double alternate quadrilateral snake $DA(Q_n)$ that satisfies the lower bound,
3. Calculating the weight of each side of the alternate triangular snake $A(T_n)$ and double alternate quadrilateral snake $DA(Q_n)$ so that all edges have different weight,
4. Looking for general pattern res alternate triangular snake graph $A(T_n)$ and double alternate quadrilateral $DA(Q_n)$.

3. RESULTS AND DISCUSSION

In this section describes the results about reflexive edge strength of alternate triangular snake $A(T_n)$ and double alternate quadrilateral snake $DA(Q_n)$.

3.1 Alternate Triangular Snake $A(T_n)$

An alternate triangular snake $A(T_n)$ has been obtained from a path $u_1, u_2, \ldots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i . That is every alternate edge of a path is replaced by C_3 [15]. Alternate triangular snake $A\ (T_n)$ consist of the set vertices $V(A(T_n)) = \{u_i: 1 \le i \le n,\} \cup \{v_i: 1 \le i \le \lceil\frac{n-1}{2}\rceil\}$ and the set edges $E(A(T_n)) = \{u_iu_{i+1}: 1 \le i \le n-1\} \cup \{u_{2i-1}v_i: 1 \le i \le \lfloor\frac{n}{2}\rfloor\} \cup \{u_{2i}v_i: 1 \le i \le \lfloor\frac{n}{2}\rfloor\}$. Thus, alternate triangular snake $A(T_n)$ has $|V(A(T_n))| = \frac{3n-1}{2}$ and $|E(A(T_n))| = 2n-2$ if n odd and $|V(A(T_n))| = \frac{3n}{2}$ and $|E(A(T_n))| = 2n-1$ if n even.

The reflexive edge strength of alternate triangular snake $A(T_n)$ can be got through **Theorem 1**.

Theorem 1. *For every positive integer $n \ge 3$, the reflexive edge strength of alternate triangular snake $A(T_n)$ is*

a. $for\ n\ even$

$$res\ (A(T_n)) = \begin{cases} \left\lceil\frac{2n-1}{3}\right\rceil, & 2n-1 \not\equiv 2,3\ (mod\ 6), \\ \left\lceil\frac{2n-1}{3}\right\rceil + 1, & 2n-1 \equiv 2,3\ (mod\ 6). \end{cases}$$

b. $for\ n\ odd$

$$res\ (A(T_n)) = \begin{cases} \left\lceil\frac{2n-2}{3}\right\rceil, & 2n-2 \not\equiv 2,3\ (mod\ 6), \\ \left\lceil\frac{2n-2}{3}\right\rceil + 1, & 2n-2 \equiv 2,3\ (mod\ 6). \end{cases}$$

Proof.

a) First, we prove the lower bound of $res\ (A(T_n))$. Because alternate triangular snake $A(T_n)$ with n even has $2n-1$ edges, then by **Lemma 1** we get.

$$res\ (A(T_n)) \ge \begin{cases} \left\lceil\frac{2n-1}{3}\right\rceil, & \text{if } 2n-1 \not\equiv 2,3\ (\text{mod } 6), \\ \left\lceil\frac{2n-1}{3}\right\rceil + 1, & \text{if } 2n-1 \equiv 2,3\ (\text{mod } 6). \end{cases}$$

b) Second, we prove the lower bound of $res\ (A(T_n))$. Because alternate triangular snake $A(T_n)$ with n odd has $2n-2$ edges, then by **Lemma 1** we get.

$$res\ (A(T_n)) \ge \begin{cases} \left\lceil\frac{2n-2}{3}\right\rceil, & 2n-2 \not\equiv 2,3\ (\text{mod } 6), \\ \left\lceil\frac{2n-2}{3}\right\rceil + 1, & 2n-2 \equiv 2,3\ (\text{mod } 6). \end{cases}$$

The lower bound $res\ (A(T_n))$ is shown. The lower bound in **Theorem 1** is the same as the lower bound $res\ (A(T_n))$ shown by Ryan *et al* [3].

Next step, we have shown an upper bound on the labeling of alternate triangular snake $A(T_n)$. Then create the k-labeling f with $k = \left\lceil\frac{2n-1}{3}\right\rceil$ for n even , $2n-1 \not\equiv 2,3\ (mod\ 6)$, $\left\lceil\frac{2n-1}{3}\right\rceil + 1$ for n even, $2n-1 \equiv 2,3\ (\text{mod } 6)$, $\left\lceil\frac{2n-2}{3}\right\rceil$ for n odd, $2n-2 \not\equiv 2,3\ (mod\ 6)$, and $\left\lceil\frac{2n-2}{3}\right\rceil + 1$ for n odd, $2n-2 \equiv 2,3\ (\text{mod } 6)$.

$$f(u_i) = \begin{cases} 0, i = 1,2, \\ 2\left\lceil \frac{i}{3} \right\rceil, i \equiv 2 \ (\mathrm{mod}\ 3), 1 \le i \le n, \\ 2\left\lfloor \frac{i}{3} \right\rfloor, i \equiv 0,1 \ (\mathrm{mod}\ 3), 1 \le i \le n. \end{cases}$$

$$f(v_i) = \begin{cases} 0, i = 1, \\ 2, i = 2, \\ 2\left\lceil \frac{2i}{3} \right\rceil, i \equiv 0,1 \ (\mathrm{mod}\ 3), 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil, \\ 2\left\lfloor \frac{2i}{3} \right\rfloor, i \equiv 2 \ (\mathrm{mod}\ 3), 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil. \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 2, i \le 5, \\ 2\left\lfloor \frac{2i}{6} \right\rfloor, 6 \le i \le n-1. \end{cases}$$

$$f(v_i u_{2i-1}) = \begin{cases} 1, i = 1,2,3\,, \\ i - \left\lceil \frac{i-3}{3} \right\rceil, i \equiv 4 \text{ and } 6, \\ i + \left\lfloor \frac{i-5}{3} \right\rfloor, i \equiv 1,2 \ (\mathrm{mod}\ 3), 5 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \\ i + \left\lfloor \frac{i-5}{5} \right\rfloor, i \equiv 0 \ (\mathrm{mod}\ 3), 9 \le i \le \left\lfloor \frac{n}{2} \right\rfloor. \end{cases}$$

$$f(v_i u_{2i}) = \begin{cases} 3, i \le 4, \\ \left\lfloor \frac{4i-3}{3} \right\rfloor, i \equiv 0 \ (\mathrm{mod}\ 3), 6 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \\ \left\lfloor \frac{4i-7}{3} \right\rfloor, i \equiv 1 \ (\mathrm{mod}\ 3), 7 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \\ \left\lfloor \frac{4i+3}{3} \right\rfloor, i \equiv 2 \ (\mathrm{mod}\ 3), 5 \le i \le \left\lfloor \frac{n}{2} \right\rfloor. \end{cases}$$

Proving the lower and upper bounds, found that the maximum labels of the vertex and the edges $\left\lceil \frac{2n-1}{3} \right\rceil$ for n even , $2n-1 \not\equiv 2,3 \ (\mathrm{mod}\ 6)$, $\left\lceil \frac{2n-1}{3} \right\rceil + 1$ for n even, $2n-1 \equiv 2,3 \ (\mathrm{mod}\ 6)$, $\left\lceil \frac{2n-2}{3} \right\rceil$ for n odd, $2n-2 \not\equiv 2,3 \ (\mathrm{mod}\ 6)$, and $\left\lceil \frac{2n-2}{3} \right\rceil + 1$ for n odd, $2n-2 \equiv 2,3 \ (\mathrm{mod}\ 6)$. In this case, the edge weight is expressed as,

$$w_t(u_i u_{i+1}) = 2i, 1 \le i \le n-1.$$
$$w_t(v_i u_{2i-1}) = 4i-3, 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor.$$
$$w_t(v_i u_{2i}) = 4i-1, 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor.$$

It can be seen from the weight of an alternate triangular snake $A(T_n)$ weight is different . Therefore, f requires that the component on an edge irregular reflexive k-labeling. The trial in the theorem is complete. ■

Example 1. The example of an edge irregular reflexive-4 labeling for alternate triangular snake graph $A(T_5)$ is shown in **Figure 1**. The red color shows edge weight, the blue color shows edge label, and the black color show vertex label.

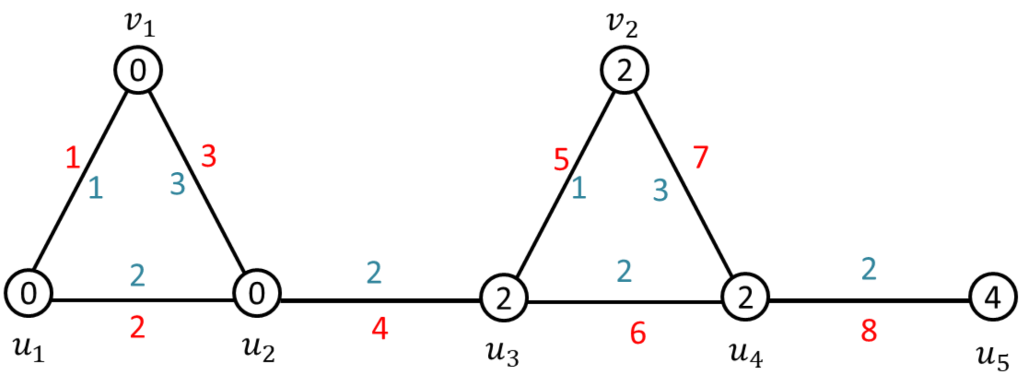

Figure 1. Edge irregular reflexive 4-labeling of alternate triangular snake $A(T_5)$

3.2 Double Alternate Quadrilateral Snake $DA(Q_n)$

A double alternate quadrilateral snake $DA\ (Q_n)$ consists of two alternate quadrilateral snakes that have a common path. Which is obtained from a path $u_1, u_2, \dots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices $v_i\ , x_i$ and $w_i\ , y_i$ respectively and adding the edges $v_i w_i$ and $x_i y_i$ [15].

A double alternate quadrilateral snake $DA\ (Q_n)$ consist of set vertices $V\big(DA(Q_n)\big) = \Big\{v_i: 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\Big\} \cup \{w_i: 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil \cup \{u_i: 1 \le i \le n \cup \{v_i': 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\} \cup \{w_i': 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\}$ and set edges $E\big(DA(Q_n)\big) = \{v_i w_i: 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil \cup \Big\{u_i v_i: 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\Big\} \cup \Big\{w_i u_{2i}: 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\Big\} \cup \{u_i u_{i+1}: 1 \le i \le n-1\} \cup \Big\{u_i v_i': 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\Big\} \cup \Big\{v_i' w_i': 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\Big\} \cup \Big\{w_i' u_{2i}: 1 \le i \le \left\lceil\frac{n-1}{2}\right\rceil\Big\}$.

Thus, double alternate quadrilateral snake $DA\ (Q_n)$ has $\big|V\big(DA(Q_n)\big)\big| = 2n+1$ and $|E(DA(Q_n))| = 4n-4$ if n odd; $|V(DA(Q_n)| = 2n+4$ and $|E(DA(Q_n))| = 4n-1$ if n even. **Theorem 2** shows the reflexive edge strength of double alternate quadrilateral snake $DA\ (Q_n)$.

Theorem 2. *For every positive integer $n \ge 3$, the reflexive edge strength of double alternate quadrilateral snake $DA(Q_n\ )$ is*

a. *for n odd*

$$res\ \big(DA\ (Q_n)\big) = \begin{cases} \left\lceil\dfrac{4n-4}{3}\right\rceil, & 4n-4 \not\equiv 2,3\ (mod\ 6), \\ \left\lceil\dfrac{4n-4}{3}\right\rceil + 1, & 4n-4 \equiv 2,3\ (mod\ 6). \end{cases}$$

b. *for n even*

$$res\ \big(DA\ (Q_n)\big) = \begin{cases} \left\lceil\dfrac{4n-1}{3}\right\rceil, & 4n-1 \not\equiv 2,3\ (mod\ 6), \\ \left\lceil\dfrac{4n-1}{3}\right\rceil + 1, & 4n-1 \equiv 2,3\ (mod\ 6). \end{cases}$$

Proof.

a) First, we prove the lower bound of $res\ \big(DA\ (Q_n)\big)$. Because double alternate quadrilateral snake $DA\ (Q_n)$ with n odd has $4n-4$ edges, then by **Lemma 1** we get.

$$res\ \big(DA\ (Q_n)\big) \ge \begin{cases} \left\lceil\dfrac{4n-4}{3}\right\rceil, & 4n-4 \not\equiv 2,3\ (\text{mod } 6), \\ \left\lceil\dfrac{4n-4}{3}\right\rceil + 1, & 4n-4 \equiv 2,3\ (\text{mod } 6). \end{cases}$$

b) Second, we prove the lower bound of $res\ \big(DA\ (Q_n)\big)$. Because double alternate quadrilateral snake $DA\ (Q_n)$ with n even has $4n-1$ edges, then by **Lemma 1** we get.

$$res\ \big(DA\ (Q_n)\big) \ge \begin{cases} \left\lceil\dfrac{4n-1}{3}\right\rceil, & 4n-1 \not\equiv 2,3\ (\text{mod } 6), \\ \left\lceil\dfrac{4n-1}{3}\right\rceil + 1, & 4n-1 \equiv 2,3\ (\text{mod } 6). \end{cases}$$

The lower bound $res\left(DA\left(Q_n\right)\right)$ is shown. The lower bound in **Theorem 2** is the same as the lower bound $res\ (DA\ (Q_n)$ shown by Ryan *et al.* [3].

Next step,we have shown an upper bound on the labeling of edge irregular reflexive labelling in double alternate quadrilateral snake $DA\ (Q_n)$. Let us create the k-labeling f with $k = \left\lceil \frac{4n-1}{3} \right\rceil$ for n even, $4n - 1 \not\equiv 2{,}3 \pmod 6$, $\left\lceil \frac{4n-1}{3} \right\rceil + 1$ for n even, $4n - 1 \equiv 2,3 \pmod 6$, $\left\lceil \frac{4n-4}{3} \right\rceil$ for n odd, $4n - 4 \not\equiv 2{,}3 \pmod 6$, and $\left\lceil \frac{4n-4}{3} \right\rceil + 1$ for n odd, $4n - 4 \equiv 2,3 \pmod 6$.

$$f(v_i) = f(w_i) = \begin{cases} 2\left\lceil \frac{4i-4}{3} \right\rceil, i \equiv 0 \pmod 3 \text{ and } i = 2, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil, \\ 2\left\lfloor \frac{4i-4}{3} \right\rfloor, i \equiv 1{,}2 \pmod 3 \text{ and } i \ne 2, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil. \end{cases}$$

$$f(v_i') = f(w_i') = 2\left\lfloor \frac{4i}{3} \right\rfloor, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil.$$

$$f(u_i) = \begin{cases} 2\left\lfloor \frac{2i-1}{3} \right\rfloor, i \equiv 1 \pmod 6, 1 \le i \le n, \\ 2\left\lceil \frac{2i-1}{3} \right\rceil, i \not\equiv 1 \pmod 6, 1 \le i \le n. \end{cases}$$

$$f(v_i w_i) = \begin{cases} 2, i = 2, \\ 2\left\lfloor \frac{4i-1}{3} \right\rfloor, i \equiv 0{,}1 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil, \\ 2\left\lceil \frac{4i-1}{3} \right\rceil, i \equiv 2 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil. \end{cases}$$

$$f(u_i u_{i+1}) = \begin{cases} 2\left\lceil \frac{4i-1}{6} \right\rceil, i \equiv 0{,}1 \pmod 6, 1 \le i \le n-1, \\ 2\left\lfloor \frac{4i-1}{6} \right\rfloor, i \not\equiv 0{,}1 \pmod 6, 1 \le i \le n-1. \end{cases}$$

$$f(u_i v_i) = f(w_i u_{2i}) = \begin{cases} 1, i = 2, \\ \left\lfloor \frac{8i-7}{3} \right\rfloor, i \equiv 0 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil, \\ \left\lceil \frac{8i-7}{3} \right\rceil, i \equiv 1{,}2 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil. \end{cases}$$

$$f(v_i' w_i') = \begin{cases} 2\left\lfloor \frac{4i-1}{3} \right\rfloor, i \equiv 0{,}1 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil, \\ 2\left\lceil \frac{4i-1}{3} \right\rceil, i \equiv 2 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil. \end{cases}$$

$$f(u_i v_i') = f(w_i' u_{2i}) = \begin{cases} 2\left\lfloor \frac{4i}{3} \right\rfloor + 1, i \not\equiv 0 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil, \\ 2\left\lfloor \frac{4i-1}{3} \right\rfloor + 1, i \equiv 0 \pmod 3, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil. \end{cases}$$

By proving the lower and upper bounds, found that the maximum labels for the vertices and edges $\left\lceil \frac{4n-1}{3} \right\rceil$ for n even, $4n - 1 \not\equiv 2{,}3 \pmod 6$, $\left\lceil \frac{4n-1}{3} \right\rceil + 1$ for n even, $4n - 1 \equiv 2,3 \pmod 6$, $\left\lceil \frac{4n-4}{3} \right\rceil$ for n odd, $4n - 4 \not\equiv 2{,}3 \pmod 6$, and $\left\lceil \frac{4n-4}{3} \right\rceil + 1$ for n odd, $4n - 4 \equiv 2,3 \pmod 6$. In this case, the edge weight is expressed as,

$$w_t(v_i w_i) = 2\,(4i-3), 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil.$$

$$w_t(v_i u_i) = 8i - 7, 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil.$$

$$w_t(w_i u_{2i}) = 8i - 5, 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil.$$
$$w_t(u_i u_{i+1}) = 4i, \quad 1 \leq i \leq n-1.$$
$$w_t(u_i v_i') = 8i - 3, 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil.$$
$$w_t(w_i' u_{2i}) = 8i - 1, 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil.$$
$$w_t(w_i' v_i') = 8i - 2, 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil.$$

It can be seen from the weight of double alternate quadrilateral snake $DA\ (Q_n)$ have different weight. Therefore, f requires that the item on an edge irregular reflexive k-labeling. The trial in theorem is completed.

■

Example 2, The example of an edge irregular reflexive 6-labeling of double alternate quadrilateral snake $DA\ (Q_5)$ is shown in **Figure 2**. The red color shows edge weight, the blue color shows edge label, and the black color shows vertex label.

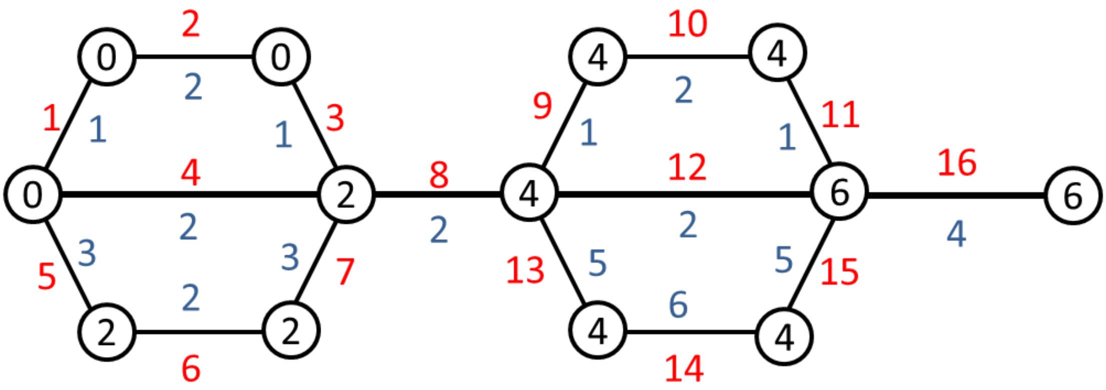

Figure 2. Edge irregular reflexive 6-labeling of double alternate quadrilateral snake $DA\ (Q_5)$

4. CONCLUSIONS

Based on these descriptions, conclusions are obtained

1. Reflexive edge strength of alternate triangular snake $A(T_n)$ $\left\lceil \frac{2n-1}{3} \right\rceil$ for n even, $2n-1 \not\equiv 2,3 \pmod 6$, $\left\lceil \frac{2n-1}{3} \right\rceil + 1$ for n even, $2n-1 \equiv 2,3 \pmod 6$, $\left\lceil \frac{2n-2}{3} \right\rceil$ for n odd, $2n-2 \not\equiv 2,3 \pmod 6$, and $\left\lceil \frac{2n-2}{3} \right\rceil + 1$ for n odd, $2n-2 \equiv 2,3 \pmod 6$.
2. Then, the reflexive edge strength of double alternate quadrilateral snake $DA\ (Q_n)$ $\left\lceil \frac{4n-1}{3} \right\rceil$ for n even, $4n-1 \not\equiv 2,3 \pmod 6$, $\left\lceil \frac{4n-1}{3} \right\rceil + 1$ for n even, $4n-1 \equiv 2,3 \pmod 6$, $\left\lceil \frac{4n-4}{3} \right\rceil$ for n odd $4n-4 \not\equiv 2,3 \pmod 6$, and $\left\lceil \frac{4n-4}{3} \right\rceil + 1$ for n odd, $4n-4 \equiv 2,3 \pmod 6$.

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