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PRICING OF CALL OPTIONS USING THE QUASI MONTE CARLO METHOD

Indah Oktaviani [1], Evy Sulistianingsih [2], Neva Satyahadewi [3*]

[1,2,3]*Statistics Study Program, Faculty of Mathematics and Natural Sciences, Tanjungpura University*
St. Prof. Dr. H. Hadari Nawawi, Pontianak, 78124, Indonesia

*Corresponding author's e-mail: *neva.satya@math.untan.ac.id*

ABSTRACT

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A call option is a type of option that grants the option holder the right to buy an asset at a specified price within a specified period of time. Determining the option price period of time within a certain period of time is the most important part of determining an investment strategy. Various methods can be employed to determine the prices of options, such as Quasi-Monte Carlo and Monte Carlo simulations. The purpose of this research is to determine the price of European-type call options using the Quasi-Monte Carlo method. The data used is daily stock closing price data on the Apple Inc. for the period October 1, 2021, to September 30, 2022. Apple Inc. stock options in this study were chosen because it is the largest technology company in the world in 2022. The steps taken in this study are to determine the parameters obtained from historical data such as the initial risk-free interest rate (r), stock price (S_0), volatility (σ), maturity time (T), and strike price (K). Next is to generate Halton's quasi-randomized sequence and simulate the stock price by substituting the parameters (S_t) by substituting the parameters. Then proceed to calculate the call option payoff and estimate the call option price by averaging the call option payoff values. The results showed that the call option price of the company Apple Inc. using the Quasi-Monte Carlo with Halton's quasi-randomized sequence on the 1000000th simulation with a standard error of 0,045 is $90,163. The call option price obtained can be used as a reference for investors in purchasing options to minimize losses from call option investments in that period.

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1. INTRODUCTION

In the world of business and finance, investment activities are no longer a new thing. Investment activities are a form of fund allocation to get future profits [1]. An alternative form of investment that is currently popular is derivative products. A derivative product refers to a type of financial instrument that heavily relies on the value of the underlying asset. The function of derivative products is to avoid investment risks and increase profits. One form of derivative product is an option. An option is a contractual agreement between the option buyer and the option seller that contains the right, not the obligation, for the option buyer to sell or buy a specific asset at a predetermined price within a specified time frame [2].

Investors can use options to prevent the impact of falling market prices on their portfolios and to speculate on stock price movements. However, a common problem with stock options trading is the losses that investors incur. One of the factors leading to losses is due to a lack of knowledge about strategies for pricing options. Therefore, determining the option price within a certain period of time is an important part of determining strategies for investing. The option price is the price that the option holder must pay to the option seller to obtain the option right [3]. Various approaches exist for determining option prices, namely Monte Carlo and Quasi-Monte Carlo simulations.

Previous research indicates that when utilizing the Quasi-Monte Carlo method, the valuation of option contracts demonstrates accelerated convergence and yields a smaller standard error compared to the Monte Carlo method [4]. Another research study, conducted by [5] has demonstrated that the utilization of Halton's quasi-randomized sequence within the Quasi-Monte Carlo method yields more accurate results because it produces a smaller error value, which means that the simulation in the Quasi-Monte Carlo method is closely the market's option price. In addition, recent research conducted by [6] which aims to determine the pricing of European-type stock options using the Quasi-Monte Carlo method with Halton's quasi-randomized sequence shows results in the form of the more simulations performed, the smaller the standard error value produced so that the option calculation results will be more convergent.

The Monte Carlo simulation method is a computational algorithm that requires repetition to assess risk by calculating the probability of the final result due to uncertainty involving random variables [7]. The Quasi-Monte Carlo method is an improved method of the Monte Carlo simulation because it makes the Quasi-Monte Carlo method effective for use in determining option prices. The Quasi-Monte Carlo method serves as an improvement to the Monte Carlo simulations, offering a more precise approach. The Quasi-Monte Carlo is a variety of the Monte Carlo simulation that utilizes quasi-randomized sequences as a substitute for the pseudo-random sequence [8]. A quasi-randomized order is a random number that has a certain pattern. The quasi-randomized sequence is divided into Halton, Van der Corput, Faure, and Sobol.

The most basic and most commonly used quasi-randomized sequences in the multidimensional form are Halton's quasi-randomized sequences [9]. This research uses Halton's quasi-random sequence in Quasi Monte Carlo simulation. Then calculate the standard error value of the call option price. This method will be used to determine the price of European-type call options on Apple Inc. for the period October 1, 2021 to September 30, 2022.

2. RESEARCH METHODS

2.1 Options

Options as derivative products whose value is highly dependent on the underlying asset. Options can be divided into two types, namely based on the time of exercise and based on the rights owned by the holder. According to [10], options based on the time of exercise consist of two, namely:

a. An American-type

This option is a contract that grants the holder to exercise the option either at the expiration date or before expiration date.

b. A European-type

This option is a contract that grants the holder to exercise the option only at the expiration date.

While options are divided into two parts according to the type of rights granted to the option holder, namely:

a. A put option is an option type that provides the holder to sell an asset at a predetermined price within a specified time frame. The equation representing the payoff of a put option is as follows.

$$P_p = maks(K - S_T, 0) \tag{1}$$

b. A call option is an option type that provides the holder to buy an asset at a strike price within a specified time frame. The equation representing the payoff of a call option is as follows.

$$P_c = maks(S_T - K, 0), \tag{2}$$

with P_c represents the call option payoff, K represents the strike price, and S_T denotes the stock price at maturity **[11]**.

2.2 Stock Price Movement Model

One of the factors that affects option price trading is stock prices. Stock prices or underlying assets often occur due to demand and supply, which causes stock prices to fluctuate over time. The model for stock price movement follows Brownian motion, which assumes that the returns of stock prices follow a normal distribution. Brownian motion is a stochastic process characterized by relatively rapid changes occurring within a short timeframe **[12]**. If S_t expresses the stock price at time t, then the expectation of S_t can be assumed as μS_t with μ denotes the constant expected stock price return. Suppose σ is defined as the stock price volatility then the model for stock price movements becomes:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW \tag{3}$$

where dW is Brownian motion.

Furthermore, *Lemma Ito* is used to calculate the price and time functions in the stock price movement model. By applying *Lemma Ito* for $f = \ln(S_t)$, so the stock price movement model can be expressed in **Equation (4) [12]**:

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma W_T} \tag{4}$$

where S_t, σ, S_0, T and μ are the initial stock price at maturity, volatility, stock price, maturity time, and risk-free interest rate. Notation μ in some literature is also denoted as r **[13]**.

2.3 Normality Test

The Kolmogorov-Smirnov test is employed as a normality test to ascertain whether a specific data distribution adheres to a normal distribution. This test compares the empirical cumulative frequency (observed) with the cumulative frequency of the theoretical distribution **[14]**. The hypothesis utilized in the Kolmogorov-Smirnov test is as below.

H_0: data is normally distributed

H_1: data is not normally distributed

Furthermore, the largest standard deviation can be calculated using **Equation (5)**:

$$D = maximum|F_o(X) - S_n(X)|, \tag{5}$$

where $F_o(X)$ is the cumulative frequency distribution and $S_n(X)$ is the observed cumulative frequency distribution. If $D < D_{tabel}$ then H_0 is not rejected, which means that the data is normally distributed **[14]**.

2.4 Return, Variance, and Volatility

Return is the result obtained from a stock investment. The return value on stocks can be positive or negative **[15]**. If the stock return is positive, it means getting a profit (capital gain), while if the stock return is negative, it means experiencing a loss (capital loss). Stock price return can be calculated using **Equation (6)**.

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right), \tag{6}$$

with S_t expresses the stock price at time *t*, R_t expresses the stock price return at time *t*, and S_{t-1} expresses the stock price at time *t*-1 [6].

Variance is a measure of the magnitude of the spread of data that shows how much the random variable spreads to the average. The equation to calculate the variance value can be done using Equation (7).

$$S^2 = \frac{\sum_{t=1}^{n}(R_t - E(R))^2}{n-1}, \tag{7}$$

with S^2 denotes the variance, R_t expresses the stock return at time *t*, *n* denotes the number of stock price data, and $E(R)$ denotes the expected stock price return [6]. The equation for calculating stock return expectations is as follows:

$$E(R) = \frac{\sum_{t=1}^{n} R_t}{n-1} \tag{8}$$

Volatility is a measure of rapidly fluctuating stock price changes. The volatility that describes annual stock price movements in the past (historical data) can be calculated using Equation (9).

$$\sigma = \sqrt{n \times S^2}, \tag{9}$$

with *n* stating the number of days in a trading year and S^2 denotes the variance of stock price returns [6].

2.5 Quasi-Monte Carlo

The option pricing method known as the Monte Carlo simulation method was first introduced by Boyle in 1977. Monte Carlo simulation is a numerical analysis method that involves random number trial samples and requires repetition, which is generally done using a computer. Random numbers are used to describe random events at any time from random variables and sequentially follow any changes that occur in the simulation process [7].

The foundation of the Monte Carlo method lies in the concept that an integral in the form of $\int_0^1 f(x)dx$ can be represented as the expected value of a function's value, given a uniform distribution over the interval [0,1], i.e.:

$$E[f(x)] = \int_{[0,1]} f(x)dx \tag{10}$$

This result can also be applied to calculate multidimensional integral forms, so Equation (10) can be written as follows.

$$E[f(x)] = \int_{C^d} f(x)dx \tag{11}$$

As a result,

$$E[f(x)] \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i) \tag{12}$$

where $x_i = (x_1, x_2, \dots, x_d)$ taken from a uniform distribution on $[0,1]^d$ that is normally distributed [16].

Over time, in 1995, Paskov and Traub introduced the Quasi-Monte Carlo method as an approach to pricing options. Quasi-Monte Carlo shares similarities with Monte Carlo simulation, but it uses quasi-randomized sequences as a replacement of pseudo-random sequences. Quasi-randomized sequences can generate representative samples of the probability distribution simulated in a problem. These sequences, often referred to as low discrepancy sequences are used because they provide more accurate results that can improve the performance of Monte Carlo simulation [8]. The key distinction between Quasi-Monte Carlo and Monte Carlo simulation is that when the data is generated repeatedly, the sequence generated by quasi-randomized will be the same because they already have their patterns, while the sequence from the Monte Carlo simulation will produce different values for each repetition [17].

The Quasi-Monte Carlo method uses a quasi-randomized numbers x_n to approximate the function f as the average of the functions evaluated on the set of points $x_1, x_2, \dots, x_n$.

$$\int_{C^d} f(x)dx \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i) \tag{13}$$

where C is a hypercube with dimension d, $C^d = [0,1] \times ... \times [0,1]$, so that each x_i is a vector of d elements. In the Monte Carlo simulation, x_i is a pseudo-random sequence, while in the Quasi-Monte Carlo method, x_i is a quasi-randomized sequence [16].

The Quasi-Monte Carlo method requires a uniform quasi-randomized sequence so that the results given are more accurate. Several quasi-randomized sequences can be used, including Halton, Faure, Van der Corput, and Sobol.

2.6 Haltons's Quasi-Randomized Sequence

Halton's quasi-randomized sequence is the most basic low discrepancy sequence in multidimensional form. Halton's quasi-randomized sequence is a d-dimensional sequence on the unit hypercube $[0,1]^d$ [5]. For example, given an integer $n \geq 1$, Halton's quasi-randomized sequence can be written as follows:

$$n = \sum_{i=0}^{I} a_i(n) b^i \tag{14}$$

where n is the *1st*, *2nd*, ..., *Mth* terms of the sequence, $a_i(n)$ is a constant in the sequence i that is non-negative to satisfy the value of n, b is a base of prime numbers, and I is the smallest integer from $\left|\frac{\ln(n)}{\ln(b)}\right|$ [13].

Furthermore, the value of $a_i(n)$ that has been obtained will be entered into **Equation (15)**.

$$\phi_b(n) = \sum_{i=0}^{I} \frac{a_i(n)}{b^{i+1}} \tag{15}$$

The d-dimensional Halton's quasi-random sequence can be written as follows:

$$H_b(n) = (\phi_b(1), \phi_b(2), ..., \phi_b(M)) \tag{16}$$

The first dimension of Halton's quasi-randomized sequence is Van der Corput's quasi-randomized sequence of base 2 and the second dimension of Halton's quasi-randomized sequence is Van der Corput's quasi-randomized sequence of base 3. The d-dimension of Halton's quasi-randomized sequence is Van der Corput's quasi-randomized sequence using the d-th prime as the base [18].

For example, the following quasi-random Halton series of dimension one, dimension two, and dimension three for the first term to the sixth term are given in **Table 1** [11].

Table 1. Example of Halton's Quasi-Random Sequence

n	Dimension = 1 (Base 2)	Dimension = 2 (Base 3)	Dimension = 3 (Base 5)
1	1/2	1/3	1/5
2	1/4	2/3	2/5
3	3/4	1/9	3/5
4	1/8	4/9	4/5
5	5/8	7/9	1/25
6	3/8	2/9	6/25

The steps taken to determine the European-type call options price using the Quasi-Monte Carlo method with Halton's quasi-randomized sequence are as follows:

a. Calculating the stock price return value (R_t) using **Equation (6)**.

b. Normality test on stock price returns.

c. Calculating the value of stock price volatility (σ) using **Equation (9)**.

d. Determining parameters obtained from historical data in the form of risk-free interest rate (r), initial stock price (S_0), strike price (K), maturity time (T), and volatility (σ).

e. Generating quasi-random Halton numbers and simulating stock prices (S_t) by substituting the parameters.

f. Determine the payoff value of the call option using **Equation (2)**.

g. Estimate the call option price by averaging the call option payoff values.

3. RESULTS AND DISCUSSION

The data utilized in this research is the daily closing stock prices of Apple Inc. (AAPL) obtained through *https://finance.yahoo.com*. The analyzed data is from October 1, 2021, to September 30, 2022, totaling 252. The first step is calculating the stock price return using **Equation (6)**. Based on the data used, 251 stock price returns were obtained.

3.1 Normality Test of Stock Price Return

The stock price return obtained is then tested for normality assumptions first. The normality test is used to identify whether stock returns are normally distributed or not using Kolmogorov-Smirnov (KS). The Kolmogorov-Smirnov test hypothesis used is as follows.

H_0 : the returns of Apple Inc.'s stock price are normally distributed

H_1 : the returns of Apple Inc.'s stock price are not normally distributed

Using *R software*, the results are shown in **Table 2**.

Table 2. Normality Test of Stock Price Return

Characteristics	Value
Sample Observation	251
Kolmogorov-Smirnov Z	0.054
p-value	0.453

In this research, the significance level or α value is 5%. Based on **Table 2**, p-value more than the value of α, so H_0 is not rejected. This means that the stock price return of Apple Inc. is normally distributed.

3.2 Volatility

After conducting the normality test, the subsequent stage involves computing the expected return, variance, and volatility obtained based on the daily stock return value using **Equation (7)**, **Equation (8)**, and **Equation (9)**.

From the return that has been obtained, investors also need an expected return to get an idea of the benefits of investing in Apple inc. shares. The expected return is the rate of return expected by investors from an investment by averaging the return value. The expected return in this research is -0,0001. To find out how far the data spread from the expected return value, it is necessary to know the variance value. Higher values of variance indicate more risk in investment returns, while lower values of variance indicate that returns tend to be more stable. The value of variance in this research is 0,0004. Volatility describes the annual movement of stock prices in the past. The higher the stock price volatility value, the higher the option price. The volatility value in this research is 0,320. The volatility value obtained is the standard deviation value of the annual stock price returns of Apple Inc. for the period October 1, 2021, to September 30, 2022.

3.3 European-Type Call Options Price Calculation

Some of the parameters required to calculate the European-type call options price using Quasi-Monte Carlo are as follows.

1. Risk-free interest rate (r)

   The risk-free interest rate used is the reference interest rate published by the US central bank of 4% or 0.04. The higher the interest rate, the higher the option price.

2. Initial stock price (S_0)

   The price of the stock underlying the option at the time of calculation. The initial stock price in this period is \$138,20.

3. Volatility (σ)

   A measure of how much the price of the underlying stock fluctuates. Higher volatility tends to increase the price of the option. Annual stock price volatility is 0,320.

4. Maturity time (T)

   The time remaining until the option expires. The longer the time remaining, the higher the value of the option. The maturity time used is one year.

5. Strike price (K)

   The pre-determined price for the purchase of shares under the option. The strike price used is \$50.

Next, Halton's quasi-randomized sequence is generated, and the stock price is simulated using **Equation (4)** by substituting the predetermined parameters. Then proceed to determine the payoff value of the call option using **Equation (2)**. The next step is to estimate the price of the European-type call options by averaging the call option payoff values. The European-type call options price using Quasi-Monte Carlo with Halton's quasi-randomized sequence is given in **Table 3**.

Table 3. Call Option Price Using Quasi-Monte Carlo with Halton's Quasi-Randomized Sequence

M	**Call Option Price (\$)**	**Standard Error**
10	79.477	10.512
100	87.783	4.231
1000	89.791	1.411
10000	90.104	0.453
100000	90.155	0.144
1000000	90.163	0.045

The results of the call option pricing using the Quasi-Monte Carlo simulation method are summarized in **Table 3**. As the number of simulations (M) performed increases, a convergence trend is observed, which is indicated by a decrease in the standard error. The European-type call option price for Apple Inc. stock determined at the 1,000,000th simulation is \$90,163. This convergence shows that the calculated option price stabilizes with a smaller and smaller standard error, reaching 0.045 in the last simulation. This means that if the call option is purchased at a price greater than the Quasi-Monte Carlo calculated price, then investors should consider it before buying because the call option is overpriced. However, if the call option is purchased at a price smaller than Quasi-Monte Carlo calculated price, the investors are recommended to buy because the call option is underpriced.

4. CONCLUSIONS

Based on the discussion and analysis previously described, this research concludes that the price of European-type call options on Apple Inc. using the Quasi-Monte Carlo method with Halton's quasi-randomized sequence of \$90,163 at the 1000000th simulations with a standard error of 0,045. The purchase option price obtained is expected to be a reference for investors in buying options to minimize losses or gain profits from investing in purchase options.

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