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MIXED ESTIMATORS OF TRUNCATED SPLINE-EPANECHNIKOV KERNEL ON NONPARAMETRIC REGRESSION AND ITS APPLICATIONS

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ABSTRACT

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Research on innovations in the statistics and statistical computing program systems implemented in the health sector. The development of a mixed estimator model is an innovation of nonparametric regression analysis by combining two approaches in nonparametric regression, namely the truncated spline estimator and the Epanechnikov kernel. The urgency of this study is that there are often cases where there are different data patterns from each predictor variable. In addition, by using only one form of the estimator in estimating a multivariable regression curve, the result is that the estimator obtained will not match the data pattern. The research objective was to find a mixed estimator between the truncated spline and the Epanechnikov kernel and the estimator results were applied to Dengue Hemorrhagic Fever case data. The unit of observation is a province in Indonesia and This study relied on secondary data received from the Central Statistical Agency (BPS) and the Health Office. Based on the analysis results, it was found that the best model of nonparametric regression with a mixed estimator of the truncated spline and Epanechnikov Kernel is a model with 3 knots with a combination of variables. The coefficient of determination (R^2) is 98.11%. We can conclude that the mixed estimator tends to follow actual data and represents a nonparametric regression model with a mixed estimator that can predict the number of Dengue Hemorrhagic Fever Cases in Indonesia

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1. INTRODUCTION

The The development of methods in the field of nonparametric regression has evolved. One of the statistical methods is nonparametric regression, which seeks to establish the pattern of association between the predictor variable and the responder variable when the form of the function is unknown [1]. The fundamental idea is that the data seeks its own estimation of the regression curve, independent of the researcher's subjectivity [2]. This means that the nonparametric regression approach is very flexible and objective [3], [4]. There are many estimators in the nonparametric regression approach that researchers have developed, including splines [5]–[8], kernels [9]–[11], Fourier series [12]–[15], wavelets [16], [17], and local polynomials [18]–[20].

According to Budiantara et al. [21], the nonparametric regression models developed by researchers so far, if explored further, basically have significant and fundamental assumptions in the models. Each predictor variable in multi-predictor nonparametric regression modeling is considered to have the same pattern, so the researchers forced the use of only one form of the model estimator for all predictor variables [22]. Therefore, using only one form of the estimator in various states of different data relationship patterns will undoubtedly result in the resulting estimator not matching the data pattern. As a result, the estimation of the regression model is not good and produces significant errors. Therefore, to overcome this problem, several researchers have developed a nonparametric mixed regression curve estimator in which the appropriate curve estimator approximates each data pattern in the nonparametric regression model.

Several studies have examined and developed nonparametric mixed regression models, spline-kernel [23]–[25], spline-Fourier series [26]–[28], and kernel-Fourier series [10], [22], [29]. This nonparametric mixed regression curve estimator is expected to be an appropriate estimator that can estimate data patterns reasonably. This research is a continuation of previous research on nonparametric regression of the truncated spline. The next innovation in this study is nonparametric regression using mixed estimators, including truncated spline and Epanechnikov kernel with application based on computational programs using Dengue Hemorrhagic Fever data in Indonesia.

2. RESEARCH METHODS

2.1 Basic Theory of Mixed Estimators

Suppose there are paired data $(t_i, w_i, y_i)$where the pattern between the predictor variables (t_i, w_i) and response variable (y_i) following the nonparametric regression model in **Equation (1)**.

$$y_i = \eta(t_i, w_i) + \varepsilon_i \tag{1}$$

It is assumed that the shape of the regression curve is unknown and that it need only be continuous and differentiable. Random error ε_i is normally distributed with a mean of zero and $E\left(\varepsilon_i^2\right) = \sigma^2$. Then the regression curve $\eta(t_i, w_i)$ is assumed to be additive.

$$\eta(t_i, w_i) = f(t_i) + g(w_i) \tag{2}$$

$f(t_i)$ and $g(w_i)$being functions that are assumed to be smooth. The main problem that must be solved in modeling the nonparametric regression curve mixed estimator is to obtain the estimation form of the regression curve $\eta(t_i, w_i)$. In this study, the combination estimator that will be used is truncated spline to estimate the regression curve $f(t_i)$, and Epanechnikov Kernel estimates the regression curve $g(w_i)$.

i. Regression Curve Estimator for the Truncated Spline

Suppose there are paired data $(t_{1i}, t_{2i}, \dots, t_{qi}, y_i)$, which assumes the pattern between predictor and response variables follows the nonparametric regression model.

$$y_i = f(t_{1i}, t_{2i}, \dots, t_{qi}) + \varepsilon_i \tag{3}$$

The regression curve of $f(t_{1i}, t_{2i}, \dots, t_{qi})$ is unknown, and it is only assumed that the curve is smooth because it is continuous and differentiable. The random error ε_i is normally distributed with zero means and $E\left(\varepsilon_i^2\right) = \sigma^2$. Furthermore, the regression curve $f(t_{1i}, t_{2i}, \dots, t_{qi})$ is assumed to be additive.

$$f\left(t_{1i}, t_{2i}, \dots, t_{qi}\right) = f_1(t_{1i}) + f_2(t_{2i}) + \cdots + f_q\left(t_{qi}\right) \tag{4}$$

$$= \sum_{p=1}^{q} f_p(t_{pi})$$

Then the form of the response relationship pattern (y_i) with each predictor variable (t_i) is assumed to vary at certain sub-intervals. In theory, $f_p(t_{pi})$ is a predictor variable component that is approximated by the truncated spline regression curve of degree 1 (linear) with a total of q predictor variables. The regression curve $f_p(t_{pi})$ is assumed to be contained in a degree m spline space with ξ being knot points $K_1, K_2, \ldots, K_r$. The components of the degree 1 (linear) regression curve of multivariable truncated spline are written in **Equation (5)**.

$$f_p(t_{pi}) = \delta_{0p} + \sum_{j=1}^{m}\sum_{p=1}^{q} \delta_{jp} t_p^j + \sum_{k=1}^{r}\sum_{p=1}^{q} \xi_{(m+k)} \left(t_p - K_{kp}\right)_+^m \tag{5}$$

The multivariable regression of the truncated spline model can be presented in matrix form.

$$\begin{bmatrix} f_{p1} \\ f_{p2} \\ \vdots \\ f_{pn} \end{bmatrix} = \begin{bmatrix} 1 & t_{11} & t_{21} \cdots & t_{q1} \\ 1 & t_{12} & t_{22} \cdots & t_{q2} \\ \vdots & \vdots & \vdots \ \ddots & \cdots \\ 1 & t_{1n} & t_{2n} \cdots & t_{qn} \end{bmatrix} \begin{bmatrix} \delta_0 \\ \delta_{11} \\ \vdots \\ \delta_{1q} \end{bmatrix} + \begin{bmatrix} (t_{11} - K_{11}) & \ldots & (t_{11} - K_{r1}) \\ (t_{12} - K_{11}) & \ldots & (t_{12} - K_{r1}) \\ \vdots & \ddots & \vdots \\ (t_{1n} - K_{11}) & \ldots & (t_{1n} - K_{r1}) \end{bmatrix} \begin{bmatrix} \xi_{21} \\ \xi_{31} \\ \vdots \\ \delta_{1q} \end{bmatrix} + \cdots$$
$$+ \begin{bmatrix} (t_{q1} - K_{1q}) & \ldots & (t_{q1} - K_{rq}) \\ (t_{q2} - K_{1q}) & \ldots & (t_{q2} - K_{rq}) \\ \vdots & \ddots & \vdots \\ (t_{qn} - K_{1q}) & \ldots & (t_{qn} - K_{rq}) \end{bmatrix} \begin{bmatrix} \xi_{2q} \\ \xi_{3q} \\ \vdots \\ \delta_{rq} \end{bmatrix} \tag{6}$$

The matrix form in **Equation (6)** can be presented in the state in **Equation (7)**.

$$\boldsymbol{f_p(t_{pi})} = [\boldsymbol{T_0} \quad \boldsymbol{T_1(K_1)} \quad \ldots \quad \boldsymbol{T_q(K_q)}] \begin{bmatrix} \boldsymbol{\delta} \\ \boldsymbol{\xi_1} \\ \vdots \\ \boldsymbol{\xi_q} \end{bmatrix} \tag{7}$$

Therefore, we can summarize **Equation (7)** in **Equation (8)**.

$$\boldsymbol{f_p(t_{pi})} = \boldsymbol{T}(\xi)\boldsymbol{\beta} \tag{8}$$

ii. Regression Curve Estimator for the Epanechnikov Kernel

For example, there are paired data w_i dan y_i, that follow a nonparametric regression model, where y_i is the response variable, w_i is the predictor variable. The relationship w_i and y_i can be modeled functionally according to **Equation (9)**.

$$y_i = g(w_i) + \varepsilon_i \tag{9}$$

The regression curve of $g(w_i)$ is unknown and will be approximated by the estimated regression curve of **Equation (10)**.

$$g_\tau(w) = n^{-1} \sum_{i=1}^{n} \left[\frac{P_\tau(w - w_i)}{n^{-1} \sum_{i=1}^{n} P_\tau(w - w_i)} \right] y_i = n^{-1} \left[\sum_{i=1}^{n} G_{\tau i}(w) \right] y_i \tag{10}$$

Where:

$P_\tau(w - w_i) = \frac{1}{\tau} P\left(\frac{w - w_i}{\tau}\right)$.

The Kernel function used is the Epanechnikov Kernel in **Equation (11)**.

$$P(z) = \frac{3}{4}(1 - z^2); I_{[-1,1]}(z) \tag{11}$$

Kernel Estimator is highly dependent on Kernel functionality and bandwidth. It can be written in matrix form **Equation (12)**.

$$\begin{bmatrix} \hat{g}_\tau(w_1) \\ \hat{g}_\tau(w_2) \\ \vdots \\ \hat{g}_\tau(w_n) \end{bmatrix} = \begin{bmatrix} n^{-1} \sum_{i=1}^{n} G_{\tau i}(w_1)\, y_i \\ n^{-1} \sum_{i=1}^{n} G_{\tau i}(w_2)\, y_i \\ \vdots \\ n^{-1} \sum_{i=1}^{n} G_{\tau i}(w_n)\, y_i \end{bmatrix} \tag{12}$$

The Epanechnikov Kernel Estimator can be written according to **Equation (13)**.

$$\boldsymbol{g}_\tau(w) = \boldsymbol{G}(\tau)\boldsymbol{y} \tag{13}$$

Vector $\boldsymbol{g}_\tau(w)$ has size $(n \times 1)$, vector $\boldsymbol{y}$ has size $(n \times 1)$, and matrix $\boldsymbol{G}(\tau)$ has size $(n \times n)$.

2.2 Mixed Estimators of Truncated Spline and Epanechnikov Kernel

Based on **Equation (8)** and **Equation (13)** and the shape of each component, the estimator combination that will be used is a truncated spline to estimate the regression curve $f(t_i)$ and Epanechnikov Kernel to estimate the regression curve $g(w_i)$, presented in matrix form in **Equation (14)**.

$$\boldsymbol{y} = \boldsymbol{T}(\xi)\boldsymbol{\beta} + \boldsymbol{G}(\tau)\boldsymbol{y} + \boldsymbol{\varepsilon} \tag{14}$$

Parameter estimation of $\boldsymbol{\beta}$ can be obtained using the Least Squares method. The error can be written in **Equation (15)**.

$$\begin{aligned} \boldsymbol{\varepsilon} &= \boldsymbol{y} - [\boldsymbol{T}(\xi)\boldsymbol{\beta} + \boldsymbol{G}(\tau)\boldsymbol{y}] \\ &= [\boldsymbol{I} - \boldsymbol{G}(\tau)]\boldsymbol{y} - \boldsymbol{T}(\xi)\boldsymbol{\beta} \end{aligned} \tag{15}$$

The sum of squared errors in **Equation (16)**.

$$Q(\boldsymbol{\beta}|\xi,\tau) = \|[\boldsymbol{I} - \boldsymbol{G}(\tau)]\boldsymbol{y}\|^2 - 2\boldsymbol{\beta}^T\boldsymbol{T}(\xi)^T[\boldsymbol{I} - \boldsymbol{G}(\tau)]\boldsymbol{y} + \boldsymbol{\beta}^T\boldsymbol{T}(\xi)^T\boldsymbol{T}(\xi)\boldsymbol{\beta} \tag{16}$$

To obtain an estimate of $\boldsymbol{\beta}$, partial derivatives are used.

$$\frac{\partial Q(\boldsymbol{\beta}|\xi,\tau)}{\partial \boldsymbol{\beta}} = -2\boldsymbol{T}(\xi)^T[\boldsymbol{I} - \boldsymbol{G}(\tau)]\boldsymbol{y} + \boldsymbol{2}\boldsymbol{T}(\xi)^T\boldsymbol{T}(\xi)\boldsymbol{\beta} \tag{17}$$

The estimator for $\hat{\boldsymbol{\beta}}$:

$$\hat{\boldsymbol{\beta}} = [\boldsymbol{T}(\xi)^T\boldsymbol{T}(\xi)]^{-1}\boldsymbol{T}(\xi)^T[\boldsymbol{I} - \boldsymbol{G}(\tau)]\boldsymbol{y} \tag{18}$$

We can summarize **Equation (18)** in **Equation (19)**.

$$\hat{\boldsymbol{\beta}} = \boldsymbol{R}(\xi,\tau)\boldsymbol{y} \tag{19}$$

In **Equation (8)**, we derive the estimation of the truncated spline regression curve:

$$\hat{\boldsymbol{f}}_{\boldsymbol{p}}(\boldsymbol{t}_{\boldsymbol{pi}}) = \boldsymbol{T}(\xi)\hat{\boldsymbol{\beta}} \tag{20}$$

Hence:

$$\hat{\boldsymbol{f}}_{\boldsymbol{p}}(\boldsymbol{t}_{\boldsymbol{pi}}) = \boldsymbol{T}(\xi)([\boldsymbol{T}(\xi)^T\boldsymbol{T}(\xi)]^{-1}\boldsymbol{T}(\xi)^T[\boldsymbol{I} - \boldsymbol{G}(\tau)]\boldsymbol{y}) \tag{21}$$

$$\hat{\boldsymbol{f}}_{\boldsymbol{p}}(\boldsymbol{t}_{\boldsymbol{pi}}) = \boldsymbol{A}(\xi,\tau)\,\boldsymbol{y} \tag{22}$$

With $\boldsymbol{A}(\xi,\tau) = \boldsymbol{T}(\xi)([\boldsymbol{T}(\xi)^T\boldsymbol{T}(\xi)]^{-1}\boldsymbol{T}(\xi)^T[\boldsymbol{I}-\boldsymbol{G}(\tau)])$.

Based on **Equation (21)** and **(22)**, the mixed estimator forms of the truncated spline and Epanechnikov Kernel are as follows:

$$\hat{\eta}(t_i,w_i) = \hat{f}(t_i) + \hat{g}(w_i)$$
$$\hat{\eta}(t_i,w_i) = \boldsymbol{A}(\xi,\tau)\boldsymbol{y} + \boldsymbol{G}(\tau)\boldsymbol{y} \tag{23}$$
$$\hat{\eta}(t_i,w_i) = \boldsymbol{S}(\xi,\tau)\boldsymbol{y}$$

Matrix $\boldsymbol{S}(\boldsymbol{\xi},\boldsymbol{\tau})$ is strongly dependent on $\boldsymbol{A}(\boldsymbol{\xi},\boldsymbol{\tau})$ which is part of the truncated spline estimator with the knot point $\boldsymbol{\xi} = (\boldsymbol{K_1},\boldsymbol{K_2},\dots,\boldsymbol{K_r})^{\boldsymbol{T}}$. $\boldsymbol{G}(\boldsymbol{\tau})$ is a component of the Epanechnikov Kernel estimator and has bandwidth (τ) as a smoothing parameter.

2.3 Smoothing Parameters Selection

The Generalized Cross-Validation (GCV) method developed by Wahba was used for the process of selecting the smoothing parameters. The modified GCV method formula for the mixed estimator form is shown in **Equation (24)**.

$$GCV(\xi_{opt},\tau_{opt}) = \left(\frac{n^{-1}\sum_{i=1}^{n}(y_i-\hat{y}_i)}{\left(n^{-1}trace(\boldsymbol{I}-[\boldsymbol{A}(\xi,\tau)\boldsymbol{y}+\boldsymbol{G}(\tau)])\right)^2}\right) \tag{24}$$

The GCV method in **Equation (24)** gives equal weight to each observation. The minimum GCV value provides optimal knot points and bandwidth in the mixed estimator modeling process.

2.4 Methodology

This study relied on secondary data received from the Central Statistical Agency (BPS) and the Health Office. In this study, the unit of observation is Indonesia's 34 provinces. Nonparametric Regression with Mixed Estimators of the Truncated Spline and the Epanechnikov Kernel was utilized in this study. The research variables are as follows:

Table 1. Research Variable

Variable	Notation	Details
Response	Y	Number of cases of dengue hemorrhagic fever
	X_1	Percentage of health services
Predictor	X_2	Percentage of public places that meet health requirements
	X_3	Percentage of the poor population
	X_4	Percentage of use of proper sanitation in households

3. RESULTS AND DISCUSSION

This part will go over data exploration with descriptive statistics and spatial mapping. Using a mixed estimator truncated spline and an Epanechnikov Kernel to model DHF cases in Indonesia.

a. Descriptive Statistics

Table 2 shows that each province in Indonesia has different characteristics for all variables.

Table 2. Descriptive Statistics

Variable	Minimum	Maximum	Mean	Median
Y	77	22613	3185.38	1760.5
X_1	2.61	8.54	5.16	4.95
X_2	0	94.6	49.98	53.95

Variable	Minimum	Maximum	Mean	Median
X_3	40.31	96.96	79.81	79.89
X_4	62.47	99.84	85.40	87.95

A complete visualization is shown in spatial mapping for the response variable.

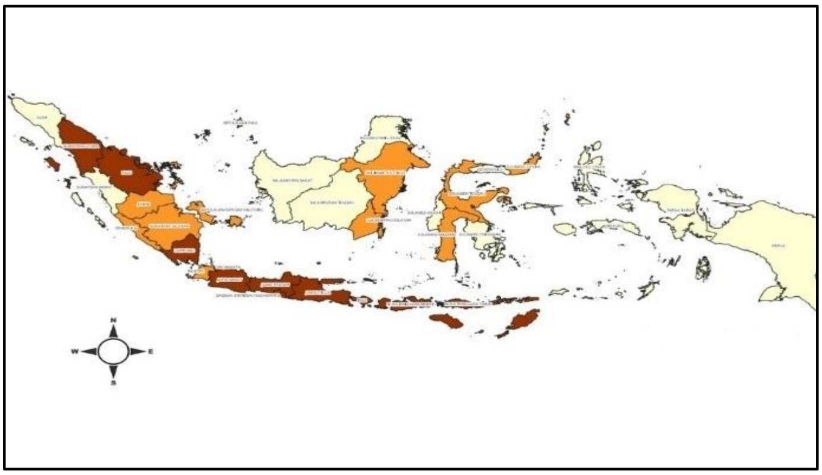

Figure 1. Spatial mapping of the number of cases of DHF in Indonesia

Based on **Figure 1** shows that on Java Island, the incidence of DHF is very high. This is because some areas are dark brown, with DHF incidents ranging from 2910 to 22613 cases. Spatial mapping then shows the other variables.

b. DHF Case Modeling with Mixed Estimators

Modeling begins with a scatter plot, which shows the relationship pattern between the response variable and each predictor variable. **Figure 2** depicts the scatter plot's results.

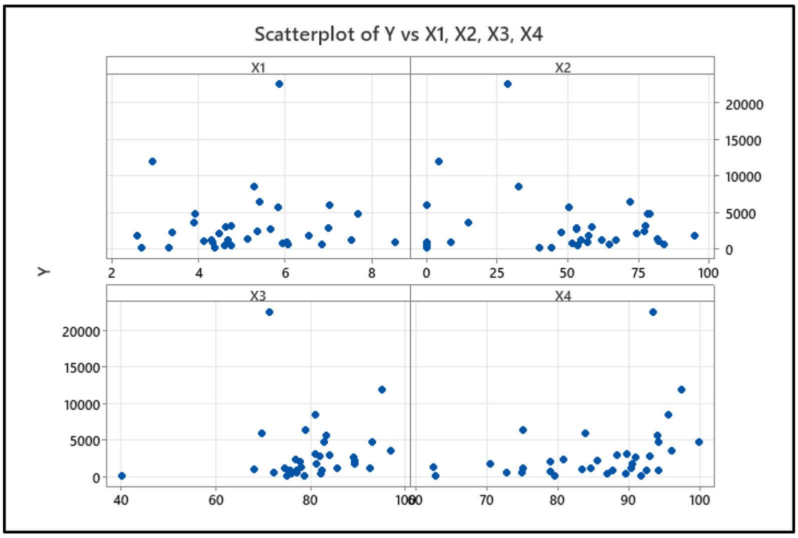

Figure 2. Scatterplot of predictor and response variables

Each predictor of the response variable tends to have a random pattern. Therefore, each predictor with the response variable is modeled with every possible estimator based on a combination of estimators.

Table 3. Predictor Combinations Based on the Estimators

Truncated Spline	Kernel
X_1	X_2, X_3, X_4
X_2	X_1, X_3, X_4
X_3	X_1, X_2, X_4
X_4	X_1, X_2, X_3

Truncated Spline	Kernel
X_1, X_2	$X_3,\ X_4$
X_1, X_3	$X_2,\ X_4$
X_1, X_4	$X_2,\ X_3$
$X_2,\ X_3$	$X_1,\ X_4$
X_2, X_4	$X_1,\ X_3$
X_3, X_4	$X_1,\ X_2$
$X_1, X_2,\ X_3$	X_4
$X_1, X_2,\ X_4$	X_3
$X_1, X_3,\ X_4$	X_2
$X_2, X_3,\ X_4$	X_1

Based on the results of the analysis, the best combination of estimators is the variables X_1, X_3, X_4 modeled with the truncated spline and variable X_2 with Epanechnikov Kernel.

Table 4. Optimal Knot Points and Bandwidth

Knot	Spline			Kernel	GCV
	X_1	X_3	X_4	X_2	
1	7.92	90.99	95.90	0.55	649433.4
2	4.17 7.60	55.22 88.02	72.30 93.94	0.55	392289.8
3	6.67 7.29 7.60	79.07 85.03 88.02	88.04 91.97 93.94	0.55	299738.2

Based on **Table 4** shows the minimum GCV value of 299738.2 with an optimal bandwidth of 0.55. The visualization of the estimated regression curve is based on the knot points in **Figure 3**.

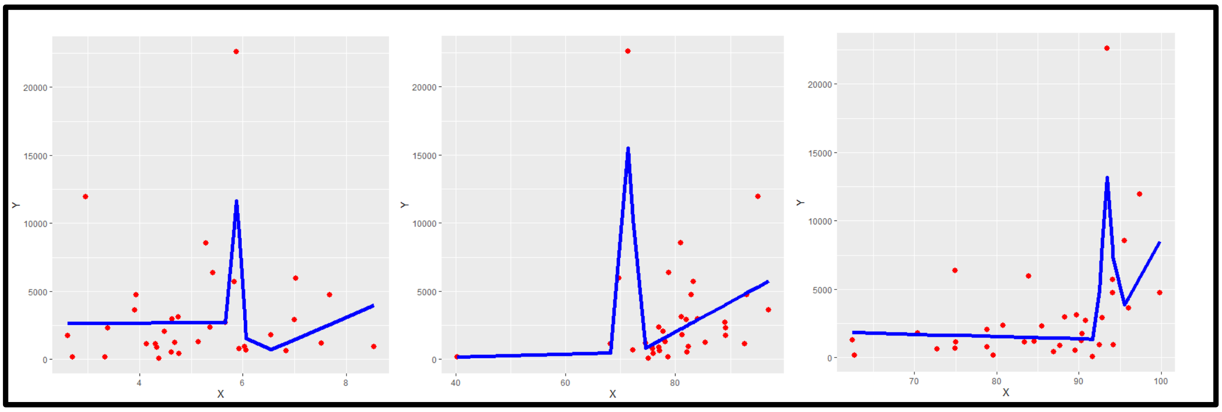

Figure 3. Illustration Of The Estimated Regression Curve Based On The Knot Point

The best nonparametric regression model with a mixed estimator is a model with 3 knots with a combination of variables. Variables X_1, X_3, X_4 modeled with the truncated spline and variable X_2 with Epanechnikov Kernel.

Table 5. Parameter Estimation Results

Variable	Parameter Estimation
Constant	$\hat{\delta}_0 = 350.24$
X_1	$\hat{\delta}_{11} = 85.04$ $\hat{\xi}_{21} = 5801.51$ $\hat{\xi}_{22} = -19573.93$ $\hat{\xi}_{23} = 14928.24$
X_3	$\hat{\delta}_{12} = -23.54$ $\hat{\xi}_{31} = 115.14$ $\hat{\xi}_{32} = 96.28$ $\hat{\xi}_{33} = -273.47$

Variable	Parameter Estimation
X_4	$\hat{\delta}_{13} = 10.42$ $\hat{\xi}_{41} = -354.42$ $\hat{\xi}_{42} = 882.77$ $\hat{\xi}_{43} = -582.81$
X_2	$\tau = 0.55$

The coefficient of determination (R^2) of the model formed is 98.11%. Visualization of the y and $\hat{y}$ from best model is shown in **Figure 4**.

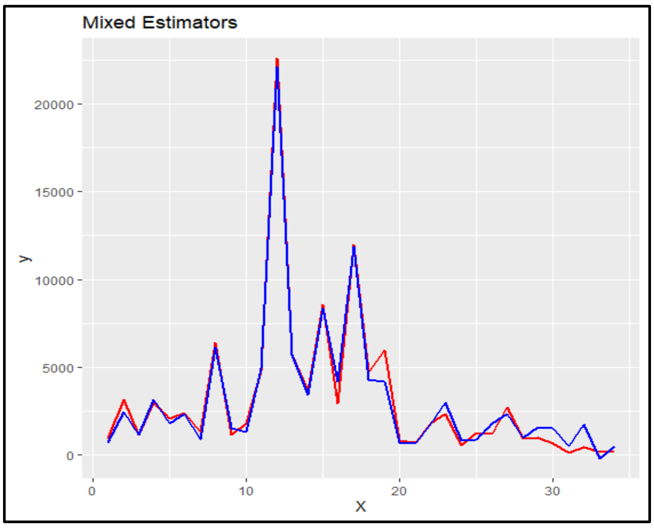

Figure 4. Graph of comparison of actual data with prediction data

The predictions follow actual data and represent that a nonparametric regression model with a mixed estimator of truncated spline and Epanechnikov kernel can be used to predict the number of Dengue Hemorrhagic Fever Cases in Indonesia. Based on the results of the study, it was found that the Percentage of health services, Percentage of public places that meet health requirements, Percentage of the poor population, and Percentage of use of proper sanitation in households affect the Number of cases of dengue hemorrhagic fever.

4. CONCLUSIONS

Based on the results of the analysis, it was found that the best model of nonparametric regression with a mixed estimator of the truncated spline and Epanechnikov Kernel is a model with 3 knots with a combination of variables, that is, variable X_1 (Percentage of Health Services), X_3 (Percentage of Poor Population), X_4 (Percentage of Use of Proper Sanitation in Households) modeled using the truncated spline and variable X_2 (Percentage of public places that meet health requirements) modeled using the Epanechnikov Kernel. The coefficient of determination (R^2) is 98,11%; hence we concluded that the mixed estimator tends to follow actual data and represents a nonparametric regression model with a mixed estimator of the truncated spline and Epanechnikov kernel can be used to predict the number of Dengue Hemorrhagic Fever Cases in Indonesia.

Declaration of competing interest

"The authors declare that they have no any known financial or non-financial competing interests in any material discussed in this paper."

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