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AN INTEGRATED APPROACH OF GRA COUPLED WITH PRINCIPAL COMPONENT ANALYSIS FOR FRICTION STIR WELDED AM20 MAGNESIUM ALLOY

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ABSTRACT

Magnesium alloys possess highly desirable properties and become increasingly popular in various practical applications due to their lightweight nature as a replacement for aluminum alloys. The purpose of this study is to optimize the process parameter to get the better mechanical properties of friction stir welded AM20 magnesium alloy using Taguchi Grey relational analysis (GRA) Coupled with Principal Component Analysis (PCA). The considered process parameters are plunging depth (PD), tool rotation speed (RPM), welding speed (WS), shoulder diameter (SD), and. The experiments were carried out by using Taguchi's L18 factorial design of experiment. The processes parameters were optimized and ranked the parameters based on the W-GRG. The responses are ultimate tensile strength (UTS), yield strength (YS), percentage of elongation (% E), compressive stress (CS), bending angle, average hardness at the nugget zone (NZ), thermo mechanical affected zone (TMAZ) and heat affected zone (HAZ). Case-1 is preferable when high values of quality parameters are desired, while Case-2 is more suitable when some parameters need to be low values. The optimal combination of parameters in case-1 is PD1, RPM3, WS3, and SD1, while the optimal combination of parameters in case-2 is PD1, RPM1, WS2, SD1. In both cases, the most influence response in is UTS, while the maximum influence of factor is SD. We suggest further research to be able to use confirmatory experiments so that we can find out how well the new setup is suggested.

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1. INTRODUCTION

Magnesium alloys possess highly desirable properties that make them an appealing option to replace aluminum and steel in mechanical and structural applications. These properties include exceptional stiffness-to-weight ratio, high damping capacity, the lowest density of all engineering metallic materials, and the ease of recycling [1]. As a result, magnesium alloys have become increasingly popular in various practical applications due to their lightweight nature as a replacement for aluminum alloys [2].

In conventional welding processes, magnesium alloy components suffer from several issues, including low strength, hot cracking, alloy segregation, partial melting zones, and porosity in the welded joint. As a result, the mechanical properties and corrosion resistance of the welded joints tend to decrease [3]-[5]. To address the aforementioned limitations, the friction stir welding (FSW) process can be employed as a viable option to weld magnesium alloys.

Friction stir welding (FSW) is a solid-state joining process that has become increasingly important in welding technology. Initially developed for welding aluminum alloys, FSW has been extended to a variety of materials such as magnesium, copper, steel, and composites. During FSW, the material being welded is heated and softened by the frictional heat generated between the surface of the plates or components being welded and the contact surface of a specialized rotating tool [6].

This research presents firstly the material and process parameters selection followed by the Taguchi GRA coupled with PCA. Then, we discuss and analyze the ANOVA and find which parameters contribute the most. At the final, we will suggest the optimum settings for better output.

2. RESEARCH METHODS

2.1 Taguchi Methods

The Taguchi method, also known as the Taguchi design of experiments, is a statistical technique developed by Genichi Taguchi in the 1950s. The method aims to optimize product and process design by identifying the optimal combination of design parameters that minimize the effects of variations in manufacturing and usage conditions [7].

According to Taguchi, quality is defined as minimum loss to society and can be measured by the consistency of performance. This consistency is achieved when the performance is close to the target with minimum variation. To enhance quality, Taguchi introduced a two-step optimization approach: first, identify the factor-level combination that reduces performance variability, and second, adjust the factor levels to bring the performance closer to the target. Taguchi's techniques for quality improvement include the use of orthogonal arrays to design experiments, the calculation of signal-to-noise ratios, and the use of parameter design and tolerance design to optimize product and process design [8]. The Taguchi method has been widely applied in various fields, including manufacturing, engineering, and management. Its key concepts and techniques have been shown to be effective in improving product quality, reducing costs, and enhancing customer satisfaction. Some examples of its applications include process optimization in manufacturing systems, machining operations, and drilling of composites [9] [10].

2.2 GRA with PCA

Grey relational analysis (GRA) is used for solving interrelationships among the multiple responses. In this approach, a grey relational grade is obtained for analyzing the relational degree of the multiple responses. Grey relational based approach has been used to solve multi-response problems in the Taguchi methods [11]. The GRA theory adopts the Grey theory, which is derived from the mixing of clear and unclear information. For example, Black is denoted as vague information, which is considered rudimentary information. In contrast, white signifies completely clear information. However, some information falls in between black and white, referred to as Grey, information that has some things that are clear and unclear or less perfect [12].

However, in real cases, this method doesn’t work at best, Hotelling and Pearson developed principal component analysis (PCA), which calculates prioritized weights for each quality response. Kumar et al. applied PCA in GRA to optimize the mechanical properties of silica fly ash composites [13]. PCA has been vastly applied in fields of EDM [14], weaving [15], welding [16] [17], etc.

2.3 Experimental Work

The experiments use the Taguchi method and specific orthogonal array to maximize the number of process parameters included in the empirical matrix and their levels and minimize the number of experiments. The design of the orthogonal array is influenced by the number of factors and their degrees of freedom (dof) for each factor. **Table 1** shows the factors used and their levels.

Table 1. Input Parameters and Levels

No	Factors (Units)	Symbols	Level			Degree of Freedom
			Level 1	Level 2	Level 3	
1	Plunge Depth (mm)	A	0.12	0.21	-	1
2	Tool Rotational Speed (rev/min)	B	600	815	1100	2
3	Welding Speed (mm/min)	C	63	98	132	2
4	Shoulder Diameter (mm)	D	16	20	24	2

The factors and their levels that were taken into consideration in this study are shown in **Table 1**. There are three factors that have three levels and one factor with only two levels, so the mixed levels approach will be used in this study. The full factorial of this experiment is $2x3^3 = 54$ trials. Based on Taguchi method, this mixed level of experiment can be done by using L_{18} orthogonal array. The array has 4 columns with mixed degree of freedom based on its level. **Table 2** shows the L_{18} orthogonal array.

Table 2. L_{18} Orthogonal Array

Trial no.	A	B	C	D
1	0.12	600	63	24
2	0.12	600	98	20
3	0.12	600	132	16
4	0.12	815	63	24
5	0.12	815	98	20
6	0.12	815	132	16
7	0.12	1100	63	20
8	0.12	1100	98	16
9	0.12	1100	132	24
10	0.21	600	63	16
11	0.21	600	98	24
12	0.21	600	132	20
13	0.21	815	63	20
14	0.21	815	98	16
15	0.21	815	132	24
16	0.21	1100	63	16
17	0.21	1100	98	24
18	0.21	1100	132	20

This design is applied to generate multi-responses approach of weld quality parameters **[18]**. There are eight weld quality parameters, namely ultimate tensile strength (UTS), yield strength (YS), percentage of elongation (% E), compressive stress (CS), bending angle, average hardness at the nugget zone (NZ), thermo mechanical affected zone (TMAZ) and heat affected zone (HAZ) were measured after the experiment and are given in **Table 3**.

Table 3. Experimentally Measured Output Responses Corresponding to The Parameters Settings

Trial no.	UTS (MPa)	YS (MPa)	% E	CS (MPa)	Bending Angle (')	Avg. Hat NZ (HV)	Avg. Hat TMAZ (HV)	Avg. Hat HAZ (HV)
1	132.17	115.56	2.17	9.46	45	55.76	52.03	48.5
2	112.46	105.96	1.89	4.38	30	56.52	53.63	50.67
3	59.48	48.5	1.07	1.5	15	54.38	52.78	50.42
4	91.2	74.55	2.63	7.06	35	56.71	54.34	50.92
5	101.1	90.16	1.83	7.39	45	55.9	53.33	49.92
6	65.56	60.13	1.57	3.72	20	56.43	53.73	50.42
7	100.99	90.55	2.76	5.11	30	54.43	53.58	50.75
8	54.9	46.5	1.33	7.38	55	53.48	51.33	50.66
9	127.27	86.86	5.87	5.42	30	59.38	58.9	53.5
10	63.25	60.89	1.33	5.23	30	57.24	54.66	50.92
11	113.04	109.64	1.46	5.23	35	52	50.36	48.66
12	90.83	86.5	1.07	1.51	15	54.38	52.3	49.41
13	49.02	43.63	1.63	2.38	20	54.43	52	51
14	68.47	60.01	2	9.78	90	55.9	53.41	50.71
15	101.02	84.61	2.93	5.2	30	51.29	50.33	48.75
16	46.02	38.15	1.26	6.73	60	52.52	51.67	49.67
17	78.55	77.47	1.76	8.37	85	53.95	53.67	47.83
18	107.52	84.6	2.66	5.13	30	54.16	52.5	49.17

The study utilized Taguchi orthogonal arrays to generate a design matrix that encompasses the entire parametric space with a limited number of experiments. The experiments were done based on the Taguchi orthogonal array design, which is frequently used to optimize engineering problems [19]-[21]. However, the Taguchi method is a single-optimization process and cannot effectively handle the optimization of multiple responses, which is required for several processes [22].

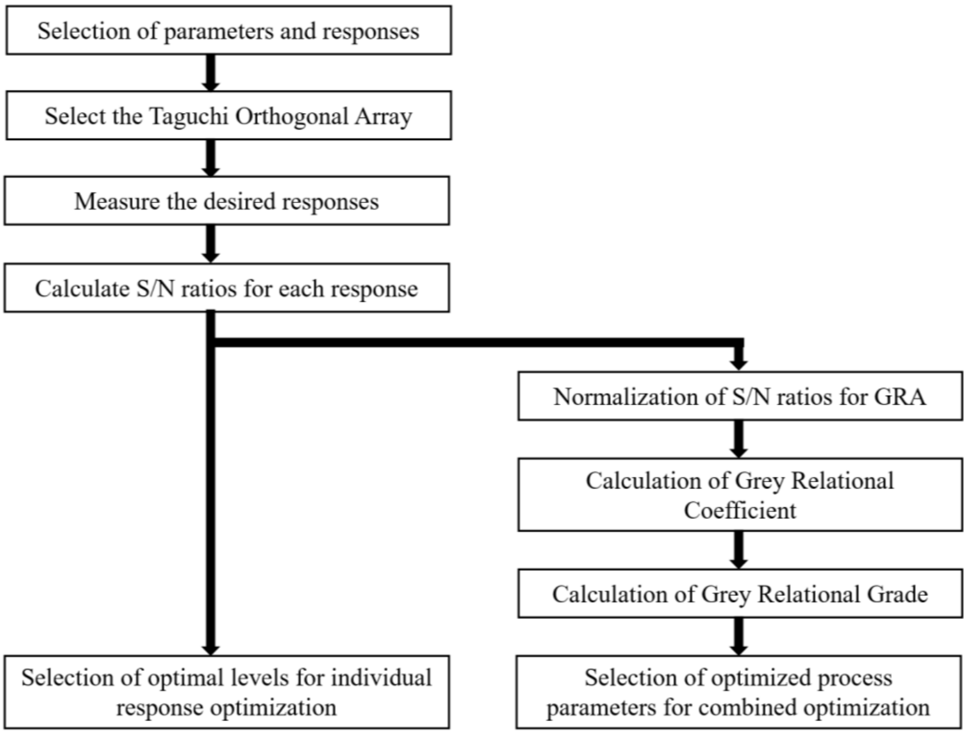

Figure 1. Concept Behind PCA-GRA

The larger the better S/N ratio as computed from **Equation (1)**:

$$S/_N\, ratio = (-10) \times log_{10}\left(\frac{1}{x}\right) \sum_{i=1}^{x} \frac{1}{y_{ij}^2} \quad (1)$$

The smaller the better S/N ratio as computed by **Equation (2)**:

$$S/_N\, ratio = (-10) \times log_{10}\left(\frac{1}{x}\right) \sum_{i=1}^{x} y_{ij}^2 \quad (2)$$

where x is number of replications and y_{ij} is measured observation.

Welding process has multiple responses and welding quality sturdily depends upon optimizing all responses simultaneously. Therefore, researchers frequently employ GRA coupled with PCA as weights for optimization of multiple responses simultaneously. These techniques are entirely different from traditional single response optimization. These are effective statistical methods and offer quite successful results in obtaining a combination of parameters for multiple response optimizations **[23]**. **Figure 1** depicts the concept of PCA-GRA.

As a result, a larger-the-better criterion has been chosen for these quality characteristics. The normalized results can be expressed as **Equation (3)**.

$$y_j^*(q) = \frac{y_j(q) - \min y_j(q)}{\max y_j(q) - \min y_j(q)} \quad (3)$$

Thus, the smaller-the-better is used, as represented in **Equation (4)**.

$$y_j^*(q) = \frac{\max y_j(q) - y_j(q)}{\max y_j(q) - \min y_j(q)} \quad (4)$$

where $y_j^*(q)$ are the generated grey relational values, while $max\ y_i(q)$ and $min\ y_i(q)$ are the largest and smallest values of $y_j(q)$ for q^{th} trial, respectively. $q = 8$ is the number of response variables. The eighteen observations of the experiments are comparability sequence $y_j(q)$, $j = 1,2, \dots ,18$,. The best normalized results should be equal to 1. Therefore, for achieving better performance, we expect larger value of normalized results.

Data normalization is followed by calculation of grey relational coefficients (GRC) that explains the relationship between desirable and real experimental normalized results. Expression of GRC $\xi_j(q)$ is determined, as follows in **Equation (5)**.

$$\xi\left(y_j^*(q), y_0^*(q)\right) = \frac{\Delta_{min}(q) + \varsigma\, \Delta_{max}(q)}{\Delta_{0j}(q) + \Delta_{max}(q)} \quad (5)$$

where $\Delta_{0i}(q) = |y_0^*(q) - y(q)|$ is deviation sequence, defined as absolute of difference between reference sequence $y_0^*(q)$ and comparability sequence $y_j^*(q)$. The identification or distinguishing coefficient (ζ), takes value as $\zeta \in [0, 1]$, which is generally and in this study were set as 0.5 **[24].** Grey relational grade (GRG) provides information about correlation strength between the experimental runs, which is computed by weighted mean of respective GRC's for all experimental. GRG value lies between 0 and 1, $\gamma \in [0, 1]$. Usually, an experimental run with larger GRG is considered the ideal case, which indicates the strength of correlation between corresponding experiments and the ideally normalized value. When equal weights are opted for all quality responses, **Equation (6)** is used for GRG calculation.

$$\gamma_j\left(y_0^*, y_j^*\right) = \frac{1}{n} \sum_{q=1}^{n} \xi\left(y_j^*(q), y_0^*(q)\right) \quad (6)$$

In some applied application, weights of quality characteristics are different likewise weights obtained from Principal Component Analysis (PCA). In such cases, **Equation (6)** is modified as **Equation (7)** by applying a weight **[25]**.

$$\gamma_j\left(y_0^*, y_j^*\right) = \frac{1}{n} \sum_{q=1}^{n} w_q\, \xi\left(y_j^*(q), y_0^*(q)\right) \quad (7)$$

where $\gamma_j\left(y_0^*, y_j^*\right)$ is GRG for j^{th} experimental run, n is number of quality response, w_q is weight of q^{th} quality response and $\sum_{q=1}^{n} w_q = 1$.

PCA is a powerful multivariate statistical technique for multi-objective optimization [26] that reduces the complexity, correlation, vagueness, and dimensions of information by simplifying and combining numerous allied arrays into few uncorrelated arrays and principal components. PCA employs linear permutation for conserving unique information to maximum extent [27]. Thus, it converts multi-response optimization to single response optimization without compromising original information [28]. It starts by setting a structure of linear combinations arrays of multi-responses. The GRC's computed for response variables is employed to form a matrix, presented as Equation (8).

$$y = \begin{bmatrix} y_1(1) & y_1(2) & \cdots & y_1(k) \\ y_2(1) & y_2(2) & \cdots & y_2(k) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ y_j(1) & y_j(2) & \cdots & y_j(k) \end{bmatrix} \tag{8}$$

where $y_p(q)$ is GRC of each quality responses, $p = 1, 2, \dots, j$, experiments and $q = 1, 2, \dots, k$, quality responses. In this research, $j = 18$ and $k = 8$. Thereafter, the coefficient correlation matrix can be produced by the following Equation (9)

$$R_{jl} = \left(\frac{Cov(y_p(q), y_p(l))}{\sigma_{yp}(q) * \sigma_{yp}(l)}\right);\ q = 1,2, \dots, k;\ l = 1,2, \dots, k \tag{9}$$

where $Cov(y_p(q), y_p(l))$ is covariance of sequences $y_p(q)$ and $y_p(l)$. $\sigma_{yp}(q)$ is standard deviation of sequence $y_p(q)$ and $\sigma_{yp}(l)$ is standard deviation of sequence $y_p(l)$. The eigen values and eigen vectors are computed from R_{jl} array as per Equation (10)

$$(R - \lambda_k l_j) V_{pk} = 0 \tag{10}$$

Thereafter, eigenvalues (λ_k) and eigenvectors (V_{pk}) of square matrix R are used to determine the uncorrelated principal components (PC's) by using Equation (11)

$$Z_{jk} = \sum_{i=1}^{n} \Upsilon_j(p) \times V_{pk} \tag{11}$$

where Z_{jk} corresponds to k^{th} principal component. Eigenvalues and principal components are arranged in descending order with respect to explained variance. Therefore, first eigenvalue associated with first PC accounts for largest variance contribution. Eigenvalues corresponding to eigenvectors are presented in the Table 4 for case 1 and Table 5 for case 2.

Table 4. Principal Component Analysis for Case 1

Component	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8
Eigen Value	0.0944	0.0652	0.0565	0.0063	0.0044	0.0015	0.0003	0.0002
Variation (%)	0.413	0.285	0.247	0.027	0.019	0.007	0.001	0.001
Cumulative (%)	0.413	0.698	0.945	0.972	0.991	0.998	0.999	1.00
Eigen Vector	0.475	-0.38	-0.299	-0.151	0.227	-0.227	0.5	0.405
	0.257	-0.478	-0.474	0.009	-0.446	0.332	-0.336	-0.244
	0.442	0.073	0.18	-0.526	0.32	-0.096	-0.115	-0.604
	-0.033	-0.615	0.385	0.398	0.492	0.055	-0.261	-0.028
	-0.181	-0.408	0.564	-0.417	-0.482	0.027	0.273	0.038
	0.418	0.096	0.237	0.6	-0.347	-0.261	0.312	-0.337
	0.412	0.128	0.259	-0.069	-0.216	-0.292	-0.6	0.5
	0.367	0.228	0.253	0.046	0.076	0.82	0.155	0.212

Table 5. Principal Component Analysis for Case 2

Component	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8
Eigen Value	0.0811	0.0663	0.0635	0.0214	0.0086	0.0037	0.0017	0.0007
Variation (%)	0.328	0.268	0.257	0.087	0.035	0.015	0.007	0.003
Cumulative (%)	0.328	0.597	0.854	0.941	0.975	0.99	0.997	1
Eigen Vector	0.639	-0.05	0.181	-0.011	0.035	-0.003	-0.729	-0.152
	0.555	-0.257	0.141	-0.388	0.025	-0.397	0.505	0.205
	-0.409	-0.191	-0.037	-0.789	0.154	-0.078	-0.359	0.1314
	0.281	-0.018	-0.64	-0.052	0.463	0.466	0.054	0.272
	0.009	0.001	-0.711	0.071	-0.214	-0.609	-0.128	-0.238
	-0.139	-0.582	0.021	0.41	-0.042	-0.165	-0.216	0.631
	-0.093	-0.622	0.047	0.138	0.441	0.018	0.119	-0.611
	0.092	-0.411	-0.167	-0.177	-0.72	0.47	0.063	-0.142

3. RESULTS AND DISCUSSION

3.1 Implementation of Taguchi GRA with PCA

In the present investigation, Taguchi GRA has been applied for selection of optimal parameter settings. All the eight output responses presented in **Table 3** were standardized using **Equations (3)** or **Equation (4)**. Two optimization scenarios were considered: In Case-1, all quality parameters were assumed to be "higher the better," which means that the purpose of Case-1 was to maximize all quality parameters. In Case-2, UTS, YS, CS, and bending angle were considered "higher the better" while percentage elongation and average hardness values at NZ, TMAZ, and HAZ were considered "lower the better" with the goal of maximizing UTS, YS, CS, and bending angle and minimizing percentage elongation and hardness values at the same time. The choice between Case-1 or Case-2 depends on the user's preference or specific application requirements. Case-1 is preferable when high values of quality parameters are desired, while Case-2 is more suitable when the tensile property needs to be high and the hardness needs to be low.

3.2 Case-1: All the Output Responses are taken as “Higher the Better”

For Case-1, all the outputs responses are taken as “higher the better”. It is preferable when high values of quality parameters are desired. Hence, for this case, it uses **Equations (3)** to do normalization and the results are shown in **Table 6**. After getting the normalized data, GRCs are calculated using **Equations (5)**, and the results of the GRCs are obtained in **Table 7**.

Then, the results of the GRCs will be subjected to PCA analysis, which will later be used to calculate the weighting when calculating the GRG. The component used is the first component that shown in **Table 4**. The weight is the squared result of the eigenvector PC1, which will later calculate the GRG value using **Equations (7)** and the results can be seen also in **Table 7** along with the rank order. From the results of the GRG weighting, it was found that the value of GRG was around from 0 to 1 with results trial no. 9, 1, and 2 sequentially are the three best rank trials.

After getting W-GRG of each trial, then it can be calculated the Average W-GRG of each parameter, including plunging depth (PD), tool rotation speed (RPM), welding speed (WS), and shoulder diameter (SD). From that Average W-GRG, the rank and optimum parameters can be found. **Table 8** shows the optimum parameters level (PD1, RPM3, WS3 and SD1) namely plunging depth 0.12 mm, tool rotation speed 1100 rev/min, welding speed 132 mm/min, and shoulder diameter 16 mm.

Table 6. Calculated Normalized for Case 1

Trial no.	UTS (MPa)	YS (MPa)	% E	CS (MPa)	Bending Angle (')	Avg. Hat NZ (HV)	Avg. Hat TMAZ (HV)	Avg. Hat HAZ (HV)
1	1.000	1.000	0.229	0.961	0.400	0.553	0.198	0.118
2	0.771	0.876	0.171	0.348	0.200	0.646	0.385	0.501
3	0.156	0.134	0.000	0.000	0.000	0.382	0.286	0.457
4	0.524	0.470	0.325	0.671	0.267	0.670	0.468	0.545
5	0.639	0.672	0.158	0.711	0.400	0.570	0.350	0.369
6	0.227	0.284	0.104	0.268	0.067	0.635	0.397	0.457
7	0.638	0.677	0.352	0.436	0.200	0.388	0.379	0.515
8	0.103	0.108	0.054	0.710	0.533	0.271	0.117	0.499
9	0.943	0.629	1.000	0.473	0.200	1.000	1.000	1.000
10	0.200	0.294	0.054	0.450	0.200	0.735	0.505	0.545
11	0.778	0.924	0.081	0.450	0.267	0.088	0.004	0.146
12	0.520	0.625	0.000	0.001	0.000	0.382	0.230	0.279
13	0.035	0.071	0.117	0.106	0.067	0.388	0.195	0.559
14	0.261	0.282	0.194	1.000	1.000	0.570	0.359	0.508
15	0.638	0.600	0.388	0.447	0.200	0.000	0.000	0.162
16	0.000	0.000	0.040	0.632	0.600	0.152	0.156	0.325
17	0.378	0.508	0.144	0.830	0.933	0.329	0.390	0.000
18	0.714	0.600	0.331	0.438	0.200	0.355	0.253	0.236

Table 7. Calculated GRC and W-GRG for Case 1

Trial no.	UTS (MPa)	YS (MPa)	% E	CS (MPa)	Bending Angle (')	Avg. Hat NZ (HV)	Avg. Hat TMAZ (HV)	Avg. Hat HAZ (HV)	W-GRG	Rank
1	1.000	1.000	0.393	0.928	0.455	0.528	0.384	0.362	0.591	2
2	0.686	0.801	0.376	0.434	0.385	0.586	0.448	0.500	0.540	3
3	0.372	0.366	0.333	0.333	0.333	0.447	0.412	0.479	0.397	16
4	0.513	0.486	0.426	0.604	0.405	0.602	0.484	0.524	0.503	4
5	0.581	0.604	0.373	0.634	0.455	0.538	0.435	0.442	0.487	6
6	0.393	0.411	0.358	0.406	0.349	0.578	0.453	0.479	0.440	11
7	0.580	0.607	0.436	0.470	0.385	0.450	0.446	0.508	0.492	5
8	0.358	0.359	0.346	0.633	0.517	0.407	0.361	0.500	0.389	17
9	0.898	0.574	1.000	0.487	0.385	1.000	1.000	1.000	0.928	1
10	0.385	0.415	0.346	0.476	0.385	0.654	0.503	0.524	0.465	8
11	0.692	0.867	0.352	0.476	0.405	0.354	0.334	0.369	0.465	9
12	0.510	0.571	0.333	0.334	0.333	0.447	0.394	0.409	0.429	14
13	0.341	0.350	0.361	0.359	0.349	0.450	0.383	0.531	0.398	15
14	0.403	0.411	0.383	1.000	1.000	0.538	0.438	0.504	0.463	10
15	0.580	0.556	0.449	0.475	0.385	0.333	0.333	0.374	0.434	12
16	0.333	0.333	0.342	0.576	0.556	0.371	0.372	0.425	0.368	18
17	0.445	0.504	0.369	0.746	0.882	0.427	0.450	0.333	0.431	13
18	0.636	0.556	0.428	0.471	0.385	0.437	0.401	0.396	0.475	7

Table 8. **Response Table of Average W-GRG for Case 1**

Parameters	Levels			Delta	Rank
	1	2	3		
PD	0.5296*	0.4364	-	0.0932	2
RPM	0.4811	0.4539	0.5139*	0.0600	3
WS	0.4693	0.4625	0.5171*	0.0547	4
SD	0.5585*	0.4700	0.4205	0.1380	1

* Optimal level of parameters (PD1, RPM3, WS3, SD1)

Table 9. **ANOVA for individual W-GRG Responses Case 1**

Source	DF	Adj Sum of squares	Adj Mean Squares	F-Value	P-Value	Percentage of contribution	Rank
PD	1	0.0391	0.0391	2.76	0.128	14.995*	2
RPM	2	0.0108	0.0054	0.38	0.692	4.1410	3
WS	2	0.0106	0.0053	0.38	0.696	4.0760	4
SD	2	0.0587	0.0294	2.07	0.177	22.493*	1
Residual error	10	0.1417	0.0142			54.295	
Total	17	0.2610					

The ANOVA analysis in **Table 9** shows the performance of statistical significance and the percentage contribution to each parameter. Unfortunately, in this case, all of the parameters are not significant, with the percentage contribution to each parameter sequentially starting from the largest SD with 22.49%, PD with 14.99%, RPM with 4.14% and WS with 4.08%. Then, the variance contribution of each response can be seen in **Table 10**.

Table 10. **Variance contribution of response variables for first PC**

Response Variable	Contribution
UTS (MPa)	0.226
YS (MPa)	0.066
% E	0.195
CS (MPa)	0.001
Bending Angle (')	0.033
Avg. Hat NZ (HV)	0.175
Avg. Hat TMAZ (HV)	0.170
Avg. Hat HAZ (HV)	0.135

The ANOVA analysis also need to be checked of residual assumption. The assumption that required to be satisfy are normal distribution of residual. In this case, the residual normal distribution test using Anderson-Darling Test in **Figure 2** with the result that this residual doesn't satisfy the normal distribution. In this analysis also calculate the variance contribution of response variables for first PC shown at **Table 10** which is conclude that UTS is the highest value.

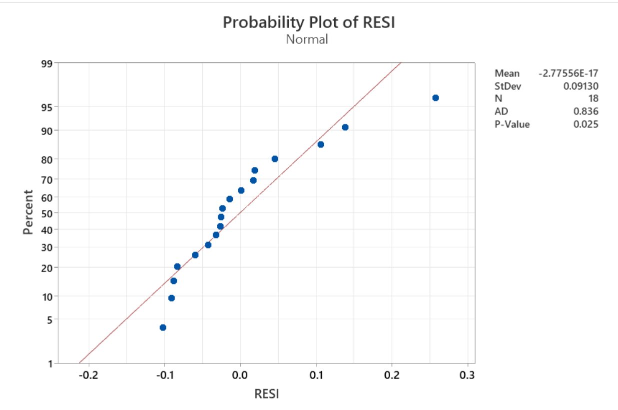

Figure 2. **Normal Distribution Test for ANOVA Case-1**

3.2 Case-2: Some of Output Responses Taken as "Higher the better" and "Lower the Better"

For Case-2, some parameters UTS, YS, CS, and bending angle were considered "higher the better" while percentage elongation and average hardness values at NZ, TMAZ, and HAZ were considered "lower the better". This case is more suitable when the tensile property needs to be high and the hardness needs to be low.

Table 11. Calculated Normalized for Case 2

Trial no.	UTS (MPa)	YS (MPa)	% E	CS (MPa)	Bending Angle (')	Avg. Hat NZ (HV)	Avg. Hat TMAZ (HV)	Avg. Hat HAZ (HV)
1	1.000	1.000	0.771	0.961	0.400	0.447	0.909	1.345
2	0.771	0.876	0.829	0.348	0.200	0.354	0.711	1.077
3	0.156	0.134	1.000	0.000	0.000	0.618	0.816	1.108
4	0.524	0.470	0.675	0.672	0.267	0.330	0.623	1.046
5	0.639	0.672	0.842	0.711	0.400	0.430	0.748	1.169
6	0.227	0.284	0.896	0.268	0.067	0.365	0.698	1.108
7	0.638	0.677	0.648	0.436	0.200	0.612	0.717	1.067
8	0.103	0.108	0.946	0.710	0.533	0.729	0.995	1.078
9	0.943	0.629	0.000	0.473	0.200	0.000	0.059	0.727
10	0.200	0.294	0.946	0.450	0.200	0.265	0.583	1.046
11	0.778	0.924	0.919	0.450	0.267	0.912	1.115	1.325
12	0.520	0.625	1.000	0.001	0.000	0.618	0.875	1.232
13	0.035	0.071	0.883	0.106	0.067	0.612	0.912	1.036
14	0.261	0.282	0.806	1.000	1.000	0.430	0.738	1.072
15	0.638	0.600	0.613	0.447	0.200	1.000	1.119	1.314
16	0.000	0.000	0.960	0.632	0.600	0.848	0.953	1.200
17	0.378	0.508	0.856	0.830	0.933	0.671	0.706	1.428
18	0.714	0.600	0.669	0.438	0.200	0.645	0.850	1.262

Table 12. Calculated GRC and W-GRG for Case-2

Trial no.	UTS (MPa)	YS (MPa)	% E	CS (MPa)	Bending Angle (')	Avg. Hat NZ (HV)	Avg. Hat TMAZ (HV)	Avg. Hat HAZ (HV)	W-GRG	Rank
1	1.000	1.000	0.686	0.928	0.455	0.475	0.704	0.858	0.927	1
2	0.686	0.801	0.745	0.434	0.385	0.436	0.551	0.588	0.704	3
3	0.372	0.366	1.000	0.333	0.333	0.567	0.623	0.610	0.480	15
4	0.513	0.486	0.606	0.604	0.405	0.427	0.502	0.567	0.525	11
5	0.581	0.604	0.759	0.634	0.455	0.467	0.574	0.659	0.620	5
6	0.393	0.411	0.828	0.406	0.349	0.440	0.543	0.610	0.476	16
7	0.580	0.607	0.587	0.470	0.385	0.563	0.554	0.581	0.580	8
8	0.358	0.359	0.902	0.633	0.517	0.649	0.802	0.588	0.482	14
9	0.898	0.574	0.333	0.487	0.385	0.333	0.321	0.416	0.650	4
10	0.385	0.415	0.902	0.476	0.385	0.405	0.483	0.567	0.490	13
11	0.692	0.867	0.860	0.476	0.405	0.851	0.993	0.830	0.764	2
12	0.510	0.571	1.000	0.334	0.333	0.567	0.672	0.719	0.601	6
13	0.341	0.350	0.811	0.359	0.349	0.563	0.708	0.561	0.433	18
14	0.403	0.411	0.721	1.000	1.000	0.467	0.568	0.584	0.510	12
15	0.580	0.556	0.563	0.475	0.385	1.000	1.000	0.815	0.575	9
16	0.333	0.333	0.927	0.576	0.556	0.767	0.751	0.687	0.466	17
17	0.445	0.504	0.777	0.746	0.882	0.603	0.548	1.000	0.551	10
18	0.636	0.556	0.602	0.471	0.385	0.585	0.651	0.751	0.592	7

Hence, for this case, it needs to do normalization by using **Equations (3)** and the results are shown in **Table 11**. After getting the normalized data, then GRCs are calculated using **Equations (5)** and the results of the GRCs are obtained in **Table 12**. Then, the results of the GRCs will be subjected to PCA analysis which will later be used to calculate the weight when calculating the GRG. The component that used is the first component that shown in **Table 5**. The weight is the squared result of the eigen vector PC1, which will later calculate the GRG value using **Equations (7)** and the results can be seen also in **Table 12** along with the rank order. From the results of the weighted GRG, it was found that the value of GRG was around from 0 to 1 with results trial no. 1, 11, and 2 sequentially are the three best rank trials.

Table 13. Response table of average W-GRG for Case-2

Parameters	Levels			Delta	Rank
	1	2	3		
PD	0.6049*	0.5534	-	0.0514	3
RPM	0.6608*	0.5231	0.5536	0.1378	2
WS	0.5702	0.6061*	0.5623	0.0428	4
SD	0.6653*	0.5883	0.4840	0.1813	1

* Optimal level of parameters (PD1, RPM1, WS2, SD1)

After getting W-GRG of each trial, then it can be calculated the Average W-GRG of each parameters including plunging depth (PD), tool rotation speed (RPM), welding speed (WS), and shoulder diameter (SD). From that Average W-GRG, the rank and optimum parameters can be found. **Table 13** shows the optimum parameters level (PD1, RPM1, WS2 and SD1) namely plunging depth 0.12 mm, tool rotation speed 600 rev/min, welding speed 98 mm/min, and shoulder diameter 16 mm.

Table 14. ANOVA For Individual W-GRG Responses for Case-2

Source	DF	Adj Sum of squares	Adj Mean Squares	F-Value	P-Value	Percentage of contribution	Rank
PD	1	0.0119	0.0119	1.53	0.245	4.6169	3
RPM	2	0.0628	0.0314	4.02	0.052	24.3158*	2
WS	2	0.0062	0.0031	0.40	0.682	2.4025	4
SD	2	0.0993	0.0497	6.36	0.017	38.4374*	1
Residual error	10	0.0781	0.0078			30.2278	
Total	17	0.2584					

The ANOVA analysis in **Table 14** shows the performance of statistical significance and the percentage contribution to each parameter. In this case all of the parameters are not significance with the percentage contribution to each parameter sequentially starting from the largest SD with 38.44%, RPM with 24.32%, PD with 4.62% and WS with 2.40%. The variance contribution of each response can be seen in **Table 15**.

Table 15. Variance Contribution of Response Variables for First PC

Response Variable	Contribution
UTS (MPa)	0.408
YS (MPa)	0.308
% E	0.167
CS (MPa)	0.079
Bending Angle (')	0.000
Avg. Hat NZ (HV)	0.019
Avg. Hat TMAZ (HV)	0.009
Avg. Hat HAZ (HV)	0.008

The ANOVA analysis also need to be checked of residual assumption. The assumption that required to be satisfy are normal distribution of residual. In this case, the residual normal distribution test using Anderson-Darling Test in **Figure 3** with the result that this residual does satisfy the normal distribution. In this analysis also calculate the variance contribution of response variables for first PC shown at Table 15 which is conclude that UTS is the highest value.

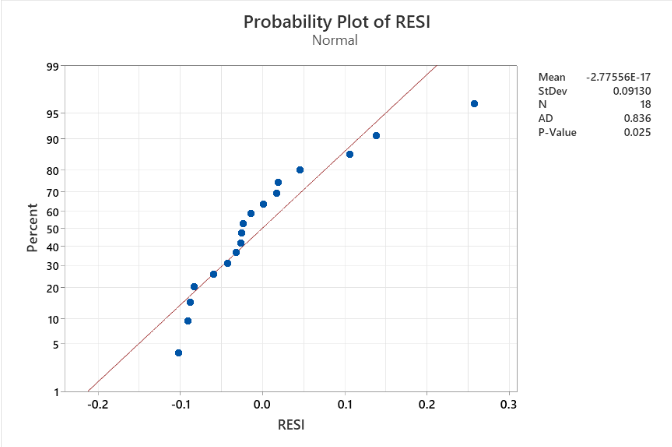

Figure 3. Normal Distribution Test for ANOVA Case-2

4. CONCLUSIONS

This paper tries to solve some parametric optimization of the quality response of Friction Stir Welded AM20 Magnesium Alloy. Initially, 18 experiments were designed and conducted as per Taguchi L_{18} Orthogonal Array, followed by the application of GRA with the PCA approach to extract optimal solutions from multi-objective optimization problems. PCA is used for weighting of quality responses that affect GRGs. By applying it to 2 cases, the following research results are obtained. The optimal combination of parameters in case-1 is Plunge Depth 0.12 mm, Tool Rotational Speed 1100 rev/min, Welding Speed 132 mm/min, and Shoulder Diameter 16 mm. While the optimal combination of parameters in case-2 is Plunge Depth 0.12 mm, Tool Rotational Speed 600 rev/min, Welding Speed 98 mm/min, and Shoulder Diameter 24 mm. In both cases, the maximum influence of factor is Shoulder Diameter, while the most influence response is Ultimate Tensile Strength. The result of W-GRA methods can used for solving multi-response optimization problem in the FSW process. Due to limited resources, this research has not been able to use a confirmatory experiment to find out how well the suggested setup is compared to the initial setup. We suggest further research to be able to use confirmatory experiments so that we can find out how well the new setup that has been suggested. Future work on this may concentrate on finite element analysis, with a focus on other parameters, tests, and statistical techniques.

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