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MODELING STOCHASTIC ADVERSE EFFECTS OF CBN 2023 REDESIGNED NAIRA NOTES POLICY ON RURAL FARMERS IN NIGERIA

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ABSTRACT

The recent Central Bank of Nigeria (CBN) 2023 redesigned naira notes is of good benefits to strengthen the economy of the country by checking counterfeiting and hoarding of large volume of banknotes by the public. Despite all the efforts made by the CBN for citizens to enjoy the benefits of this implementation, most rural farmers were faced with adverse effects of uncertainties in the production and marketing of their agricultural produce due to lack of redesigned new naira notes in circulation. The adverse effects of these uncertainties are modeled as Advanced Stochastic Time-Delay Differential Equation (ASTDDE). The modeled equation is solved using Extended Second Derivative Block Backward Differentiation Formulae Method (ESDBBDFM) without the use of interpolation techniques in the evaluations of the delay term and noise term. In comparing the numerical results of this method with other existing methods in literature, the newly developed mathematical expressions for the evaluations of the delay term and the noise term in solving ASDDEs with the discrete schemes of ESDBBDFM gives better results for step number $k = 4$ than step numbers $k = 2$ and 3 by producing Least Minimum Absolute Random Error (LMARE) in a Lower Computational Processing Unit Time (LCPUT) faster than other existing methods that applied interpolation techniques in evaluations of the delay term and the noise term.

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1. INTRODUCTION

The Advanced Stochastic Time-Delay Differential Equations (ASTDDE) is a stochastic differential equation where the increment of the process depends not only on current state but also on the future uncertainties as the future part of the system being modeled which contains the random values of the noise term. A noise term is an uncertainty process on any family of random variables $\{X_t, t\epsilon T\}$ where X_t is, in practice, the observation at time t, and T is the time range involved which its applications can be seen in applied sciences, economics and engineering [1]. Uncertainty is defined as a probability process for collection of random variables on set of discrete time points controlled by probabilistic laws [2]. The emergence of the adoption and fueling of conspiracy theories among stakeholders, mostly the rural farmers in the production and marketing of their agricultural produce may result to adverse effect of uncertainties. The adverse effects of uncertainties such as low level of farm productivity, delay in supplying of farm produce to the market, lack of farm inputs in the market and its rise in price are capable of resulting to future delay and uncertainty noise in the financial market during business transactions among rural farmers as studied by [3]. Also, due to high rate of illiteracy, lack of awareness and lack of commercial banks in the rural areas, majority of the rural farmers prefer to hold the old currency instead of opening and depositing them to their bank accounts which may directly or indirectly result to these adverse effects of uncertainties as the old currency cannot be used for any market transactions [4]. These challenges are considered in this work and it is therefore modeled as Advanced Stochastic Time-Delay Differential Equation (ASTDDE) which are solved using Extended Second Derivative Block Backward Differentiation Formulae Method (ESDBBDFM) without the use of interpolation techniques for the evaluations of the delay term and noise term. The recent implementation of redesigned naira notes forced the public into cashless economy. In the quest of obtaining the numerical solution of ASTDDE, most scholars used Euler-Maruyama scheme to formulate continuous split-step schemes of ASTDDE on a continuous interval $t_0 \leq t \leq t_a$ for the numerical solutions and encountered some setbacks in the use of interpolation techniques in evaluating the delay term and noise term which affected the accuracy of their results as studied by ([5], [6], [7]). To tackle or reduce the challenges of these adverse effects of uncertainties e on the rural farmers, buyers ability and to overcome the setbacks encountered by the researchers in the use of interpolation techniques for evaluation of the delay term and noise term of ASTDDE, three new mathematical expressions developed by [8] for the evaluations of the delay term and noise term were applied to improve rural farmers' satisfactions and buyers ability which other researchers failed to address. The rest of the paper is organized as follows: Section 2 presents the Research Method. Result and Discussion are presented in section 3, while Section 4 concludes the work.

2. RESEARCH METHOD

2.1 Derivation of the Stochastic model and the Proposed Method

The general equation for Advanced Stochastic Time- Delay Differential Equation (ASTDDE) was developed and solved numerically with the help of interpolation techniques for the evaluations of the delay term and noise term [1].

The developed governing equation for ASTDDE takes into account the current state and the future uncertainties as the future part of the system being modeled and was expressed as;

$$dy(t) = \alpha(y(t), y(t+\tau), t)dt + \beta(y(t), y(t+\tau), t)d\Phi(t) fort > 0, \tau > 0$$
$$y(t) = \varphi(t), \text{ for } t > 0 \tag{1}$$

Adapting (1), the general modeled equation for this study is stated as:

$$dARF(t) = \alpha(ARF(t), ARF(t+\tau), t)dt + \beta(ARF(t), ARF(t+\tau), t)dSAECBNNRNP(t)$$
$$\text{for } t > 0, \tau > 0$$
$$ARF(t) = \varphi(t), \text{ for } t > 0 \tag{2}$$

where $\varphi(t)$ is the initial function, α, β are drift and stochastic coefficients, $ARF(t)$ represents the activities of the rural farmers in the production and marketing of their agricultural produce, t is the time delay in months , τ is called the delay, $(t+\tau)$ is called the future time-delay term and $ARF(t+\tau)$ is the solution of the future time-delay term for the activities of the rural farmers in the production and marketing of their agricultural produce on the drift part of **Equation (2)**. The Standard Brownian motion representing the stochastic adverse effects of CBN redesigned new naira note policy on rural farmers is denoted as $SAECBNNRNP(t)$ with its differential equivalence $dSAECBNNRNP(t)$ as the noise term together with the solution of the future time-delay term for the activities of the rural farmers in the production and marketing of their agricultural produce expressed as $ARF(t+\tau)dSAECBNNRNP(t)$ on the stochastic or diffusion part of **Equation (2)**. The drift part of the **Equation (2)** $dARF(t) = \alpha(ARF(t), ARF(t+\tau), t)dt$ is deterministic and takes care of the average time rate of the investment returns without any risk. The stochastic or diffusion part $dARF(t) = \beta(ARF(t), ARF(t+\tau), t)dSAECBNNRNP(t)$ is stochastic, which takes care of the random change or the stochastic adverse risk effects of CBN redesigned new naira note policy on rural farmers in the modeled **Equation (2)**.

The discrete schemes of the Extended Second Derivative Block Backward Differentiation Formulae Method (ESDBBDFM) for step numbers $k = 2, 3$ and 4 are derived through matrix inversion techniques of the first and second characteristics continuous formulations of the D-matrix on the k-step multistep collocation method developed by **[9]** and presented as;

For $k = 2$ of (ESDBBDFM)

$$
\begin{aligned}
y_{n+1} &= \frac{101}{60}d^2v_{n+2} - \frac{17}{60}d^2u_{n+3} + \frac{29}{10}df_{n+1} - \frac{19}{10}df_{n+2} + y_n \\
y_{n+2} &= -\frac{3}{29}y_n + \frac{32}{29}y_{n+1} + \frac{26}{29}df_{n+2} - \frac{34}{87}d^2v_{n+2} + \frac{4}{87}d^2v_{n+3} \\
y_{n+3} &= -\frac{4}{29}y_n + \frac{33}{29}y_{n+1} + \frac{54}{29}df_{n+2} - \frac{1}{29}d^2v_{n+2} + \frac{5}{29}d^2v_{n+3}
\end{aligned} \tag{3}
$$

For $k = 3$ of (ESDBBDFM)

$$
\begin{aligned}
y_{n+1} &= \frac{463}{1282}d^2v_{n+3} - \frac{127}{2564}d^2u_{n+4} + \frac{5239}{5128}df_{n+1} - \frac{2081}{5128}df_{n+3} - \frac{137}{641}y_n + \frac{778}{641}y_{n+2} \\
y_{n+2} &= \frac{3552}{5975}d^2v_{n+3} - \frac{374}{5975}d^2u_{n+4} + \frac{10478}{5975}df_{n+2} - \frac{5168}{5975}df_{n+3} - \frac{133}{1195}y_n + \frac{1328}{1195} \\
y_{n+3} &= \frac{136}{5239}y_n - \frac{1161}{5239}y_{n+1} + \frac{6264}{5239}y_{n+2} + \frac{4350}{5239}df_{n+3} - \frac{1530}{5239}d^2v_{n+3} + \frac{108}{5239}d^2u_{n+4} \\
y_{n+4} &= \frac{136}{5239}y_n - \frac{1161}{5239}y_{n+1} + \frac{6264}{5239}y_{n+2} + \frac{4350}{5239}df_{n+3} - \frac{1530}{5239}d^2v_{n+3} + \frac{108}{5239}d^2u_{n+4}
\end{aligned} \tag{4}
$$

For $k = 4$ of (ESDBBDFM)

$$
\begin{aligned}
y_{n+1} &= \frac{578}{247}d^2v_{n+4} - \frac{118}{475}d^2u_{n+5} + \frac{218}{305}df_{n+1} + \frac{158}{305}df_{n+4} - \frac{561}{435}y_n + \frac{261}{415}y_{n+2} + \frac{327}{445}y_{n+3} \\
y_{n+2} &= \frac{530}{173}d^2v_{n+4} - \frac{488}{173}d^2u_{n+5} + \frac{218}{173}df_{n+2} + \frac{672}{173}df_{n+4} - \frac{301}{419}y_n + \frac{448}{591}y_{n+2} + \frac{780}{419}y_{n+3} \\
y_{n+3} &= \frac{224}{619}d^2v_{n+4} - \frac{162}{619}d^2u_{n+5} + \frac{854}{619}df_{n+3} - \frac{324}{619}df_{n+4} + \frac{176}{619}y_n + \frac{161}{619}y_{n+1} + \frac{894}{619}y_{n+3} \\
y_{n+4} &= -\frac{141}{109}y_n + \frac{122}{109}y_{n+1} - \frac{520}{109}y_{n+2} + \frac{198}{109}y_{n+3} + \frac{100}{109}df_{n+4} - \frac{358}{109}d^2v_{n+4} + \frac{178}{109}d^2u_{n+5} \\
y_{n+5} &= -\frac{356}{109}y_n + \frac{255}{109}y_{n+1} - \frac{190}{109}y_{n+2} + \frac{250}{109}y_{n+3} + \frac{220}{109}df_{n+4} + \frac{520}{109}d^2v_{n+4} + \frac{140}{109}d^2
\end{aligned} \tag{5}
$$

2.2 Analysis of Basic Properties of the Method

The order, error constant, consistency, zero stability, convergence and region of absolute stability of **Equation (3)**, **Equation (4)** and **Equation (5)** are analyzed using the conditions proposed by (**[10]**, **[11]**).

2.2.1 Order and Error Constant

As the proposed method is one of the families of Linear Multistep Method, the Linear Multistep Method is said to be of order p if $C_0 = C_1 = 0, \dots, C_p = 0$ and $C_{p+1} \neq 0$. C_{p+1} is the error constant.

The order and error constants for **Equation (3)** are analyzed and presented as follows;

$C_0 = C_1 = (0 \quad 0 \quad 0)^T$ and $C_2 = \begin{pmatrix} \frac{7}{5} & -\frac{10}{29} & \frac{6}{29} \end{pmatrix}^T$.

Therefore, **Equation (3)** has order $p = 1$ and error constant, $\frac{7}{5}, -\frac{10}{29}, \frac{6}{29}$.

Following the same approach to **Equation (4)**, we obtained

$C_0 = C_1 = (0 \quad 0 \quad 0 \quad 0)^T$ and $C_2 = \left(\frac{799}{2564} \quad \frac{3178}{5975} \quad -\frac{1422}{5239} \quad \frac{1788}{5239}\right)^T$.

Therefore, **Equation (4)** has order $p = 1$ and error constant, $\frac{799}{2564}, \frac{3178}{5975}, -\frac{1422}{5239}, \frac{1788}{5239}$.
Applying the same approach to **Equation (5)**, we obtained

$C_0 = C_1 = (0 \quad 0 \quad 0 \quad 0 \quad 0)^T$ and $C_2 = \left(-\frac{25604}{127035} \quad \frac{15074}{50791} \quad \frac{193662}{667619} \quad -\frac{34200}{149809} \quad -\frac{3611305}{299618}\right)^T$.

Therefore, **Equation (5)** has order $p = 1$ and error constant, $-\frac{25604}{127035}, \frac{15074}{50791}, \frac{193662}{667619}, -\frac{34200}{149809}, -\frac{3611305}{299618}$.

2.2.2 Consistency

A Linear Multistep Method is said to be consistent if the order $p \geq 1$. Since the order of our proposed method ESDBBDFM as analyzed using the discrete schemes **Equation (3)**, **Equation (4)** and **Equation (5)** is $p \geq 1$, therefore the method is consistent**.**

2.2.3 Zero Stability Analysis

A Linear Multistep Method is said to be zero stable if no roots $e_i, i = 1,2,3, \ldots, n$ of the first characteristic polynomial $M(e)$ expressed as $M(e) = det\left(eA_m^{(n)} - A_n^{(n)}\right)$ is greater than 1 which satisfies $|e_i| \leq 1$ and the roots $|e_i|$ is simple or distinct where $A_m^{(n)}$and $A_n^{(n)}$ are the matrices of the first characteristic polynomial obtained from **Equation (3)**, **(4)** and **(5).**

The zero stability for **Equation (3)** is determined as follows:

$$M(e) = det\left(eA_2^{(1)} - A_1^{(1)}\right) = \left|eA_2^{(1)} - A_1^{(1)}\right| = 0. \quad (6)$$

The following are obtained using Maple (18) software,

$$M(e) = e^3 + e^2 \Rightarrow e^3 + e^2 = 0$$
$$\Rightarrow e_1 = -1, e_2 = 0, e_3 = 0.$$

Since $|e_i| < 1, i = 1,2,3$, **Equation (3)** is zero stable.

By the same procedure for **Equation (4):**

$$M(e) = det\left(eA_2^{(2)} - A_1^{(2)}\right) = \left|eA_2^{(2)} - A_1^{(2)}\right| = 0. \quad (7)$$

Using Maple (18) software, we obtain:

$$M(e) = -\frac{267189}{765995}e^4 - \frac{267189}{765995}e^3 \Rightarrow -\frac{267189}{765995}e^4 - \frac{267189}{765995}e^3 = 0$$
$$\Rightarrow e_1 = -1, e_2 = 0, e_3 = 0, e_4 = 0.$$

Since$|e_i| < 1, i = 1,2,3,4$, **Equation (4)** is zero stable.

Following the same procedure for **Equation (5)**:

$$M(e) = det\left(eA_2^{(3)} - A_1^{(3)}\right) = \left|eA_2^{(3)} - A_1^{(3)}\right| = 0. \quad (8)$$

Using Maple (18) software, we obtain:

$$M(e) = \frac{26572199993504}{95725210403667}e^5 + \frac{26572199993504}{95725210403667}e^4 \Rightarrow \frac{26572199993504}{95725210403667}e^5 + \frac{26572199993504}{95725210403667}e^4 = 0$$
$$\Rightarrow e_1 = -1, e_2 = 0, e_3 = 0, e_4 = 0, e_5 = 0.$$

Since$|e_i| < 1, i = 1, 2, 3,4,5$, **Equation (5)** is zero stable.

2.2.4 Convergence

The necessary and sufficient condition for a linear multistep method to be convergent is that it must be consistent and zero stable. Since the discrete schemes **Equation (3)**, **Equation (4)** and **Equation (5)** of the proposed method are all consistent and zero stable, therefore the method is convergent.

2.2.5 Region of Absolute Stability

The P- and Q- regions of absolute stability of the numerical methods for discrete schemes **Equation (3)**, **Equation (4)** and **Equation (5)** are plotted using Maple 18 and MATLAB software and are presented in **Figure 1** to **Figure 4** below:

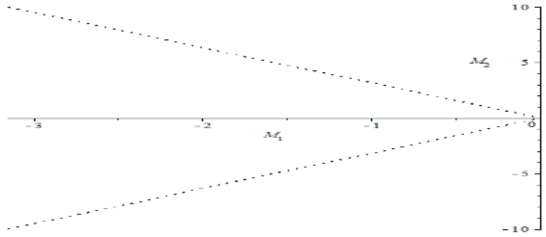

Figure 1. ***P*-stability in Equation (3)**

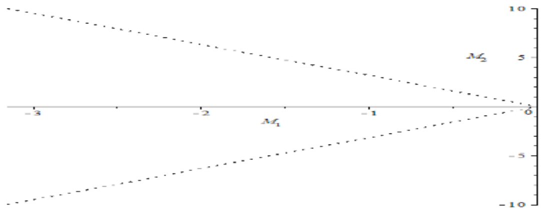

Figure 2. ***P*-stability in Equation (4)**

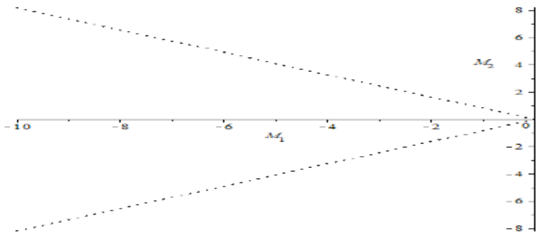

Figure 3. ***P*-stability in Equation (5)**

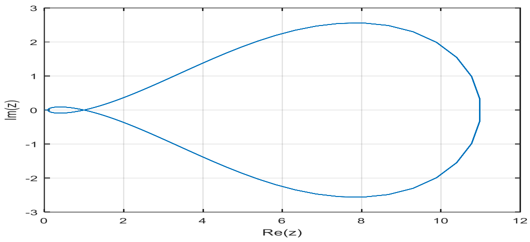

Figure 4. ***Q*-stability in Equation (3)**

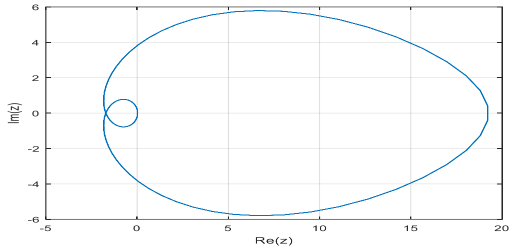

Figure 5. ***Q*-stability in Equation (4)**

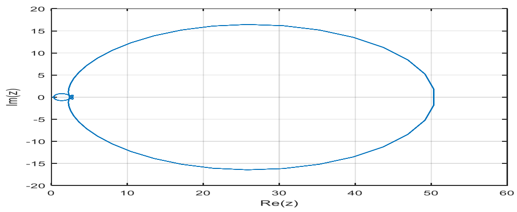

Figure 6. ***Q*-stability in Equation (5)**

The P-stability regions in **Figure 1** to **Figure 3** lie inside the open-ended region while the Q-stability regions in **Figure 4** to **Figure 6** lie inside the enclosed region. Therefore, the region of absolute stability of our proposed method is satisfied.

2.3 Evaluations of the Delay Term $(t+\tau)$ and the Noise Term $dSAECBNNRNP(t)$

The delay term $(t+\tau)$and the noise term $dSAECBNNRNP(t)$ are derived and evaluated using newly developed mathematical expressions developed by **[8]** different from the interpolation techniques used by other researchers as cited in the literature for better and faster evaluations, computations, performances and accurate results. The newly developed mathematical expressions for the evaluations of the delay term and the noise term are incorporated into some examples of the Advanced Stochastic Time- Delay Differential Equation (ASTDDE) with the derived discrete schemes of **Equation (2)**, **Equation (3)** and **Equation (4)** before its numerical experiments at constant step size $d=0.01$ to obtain the numerical solutions of $dARF(t)$ with the help of Maple 18 software.

Researchers (**[12]**, **[13]**, **[14]**) applied the formula developed by **[15]** for the evaluation of the delay term of first order delay differential equations and discovered that it gives lesser accurate results, it takes more time to compute and cannot be adequately use in solving different classes of DDEs. In the sequel, one states;

Theorem 1. *Let the current state and future part of the drift and stochastic coefficients of* **Equation (2)** *be represented as* α *and* β, *then the corresponding value of the functions* $ARF(t+\tau)$*and* $ARF(t+\tau)dSAECBNNRNP(t)$ *with an accurate formula for the evaluation of the delay term* $(t+\tau)$ *is given as:*

$$ARF(t_n+\tau)=\frac{n}{c}\big((cq+(n+a+g-1)d)\big), c\neq 0. \quad (9)$$

where $a\in(-k,k), k$ *is a step number* $g=\frac{\tau}{d}\in\mathbb{Z}, \tau=gd, \tau$ *is the delay term,* $n=0,1,2,\dots,N-1$ *and* N *is the number of solutions in the giving interval which is implemented to approximate the delay term* $(t+\tau)$*at*

the point $t = t_n + \tau$ *using the values of* ψ_{n+a} *at* $t_n + \tau > 0$ *whenever* $t_n + \tau > 0$ *where* $\psi_{n+a}(t)$ *is the approximation to* $ARF(t_n + \tau)$.
Proof. **Theorem 1** expressed in **Equation (9)** has been proved in **[8]**.

Theorem 2. *Let* $SAECBNNRNP(t)$ *be a stochastic normalized Brownian Motion Process for hyperbolic equivalence of Euler's exponential function with the mean* μ*and the volatility* σ *given as* $N(0,1)$. *Then the discrete noise term* $dSAECBNNRNP(t)$ *is given as:*

$$dSAECBNNRNP(t) = \frac{1}{\sqrt{2\pi}}\left(e^{\frac{-3t^2}{2}} - te^{\frac{-t^2-2}{2}}\right) for\ 0 \le t \le 12 \qquad (10)$$

Proof. **Theorem 2** expressed in **Equation (10)** has been proved in **[8]**.

Theorem 3. *Let theorem 3.2 exists, then the modified discrete noise term,* $dSAECBNNRNP(t)$, *using Iterative Adomian Decomposition Method (IADM) is given as:*

$$dSAECBNNRNP(t) = \frac{V_0}{\sqrt{2\pi}} + \sum_{h=1}^{\infty}\sqrt{\frac{2}{\pi}}\frac{V_h(e^t + e^{-t})}{2}, for\ 0 \le t \le 12 \qquad (11)$$

Proof. **Theorem 3** expressed in **Equation (11)** has been proved in **[8]**.

2.4 Numerical Implementation and Computations

In this section, following the algorithm below, the three newly developed mathematical expressions **Equation (9)**, **Equation (10)** and **Equation (11)** for the evaluations delay term and the noise term and the discrete schemes **Equation (3)**, **Equation (4)** and **Equation (5)** of the proposed method shall be incorporated into the ASTDDE below before its numerical evaluation at constant step size d = 0.01 using Maple 18 software to obtain the approximate solutions of $dARF(t)$:

i. *Eval* $(t + \tau)$ *and* $dSAECBNNRNP(t)$
ii. *Input Discrete Schemes*
iii. *Incorporate (i) and (ii) into ASTDDE*
iv. *Obtain the Linear Algebra*
v. *Compute the sequence*
vi. *Eval (i) to (iv) to obtain the approximate solutions of* $dARF(t)$

Theorem 2 and **Theorem 3** are used separately with **Theorem 1** for numerical evaluations of the modeled equation.

Example 1

$$dARF(t) = 1000\big(ARF(t) + 997e^{-3}ARF(t+1) + (1000 + 997e^{-3})\big)dt + \big(ARF(t) + 997e^{-3}ARF(t+1) + (1000 + 997e^{-3})\big)\ dSAECBNNRNP(t), 0 < t \le 12$$

$ARF(t) = 1 + e^{-3t}, t > 0$, *Exact Solution* $ARF(t) = 1 + e^{-3t}$

Example 2

$$dARF(t) = \big(ARF(t + 1 + e^{-t}) + sin(t + 1 + e^{-t}) + cos(t)\big)dt + \big(ARF(t + 1 + e^{-t}) + sin(t + 1 + e^{-t}) + cos(t)\big)dSAECBNNRNP(t), 0 > t \le 12$$

$ARF(t) = sin(t)\ , t > 0$, *Exact Solution* $ARF(t) = sin\ (t)$

The interpretation of the two examples above relating to the main problem are modeled and explained in **Equation (2).** The constant parameters represent the constant coefficients of the drift and stochastic or diffusion part of the modeled equation for this study as expressed in **Equation (2).**

3. RESULTS AND DISCUSSION

The above examples were solved using the three newly developed mathematical expressions **Equation (9)**, **Equation (11)** and **Equation (13)** and the discrete schemes **Equation (3)**, **Equation (4)** and **Equation (5)** of the proposed method and the numerical results and absolute random errors computed and presented in **Table 1** to **Table 4**:

3.1 Numerical Solutions with its Graphical Presentations

Table 1. Numerical Solution of Example 1 with the incorporation of Theorem 1 and Theorem 2 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	Exact Solution	$k=2$ Numerical Solution	$k=3$ Numerical Solution	$k=4$ Numerical Solution
1	0.970445534	0.970442037	0.97026926	0.970419306
2	0.960789439	0.960794693	0.960784392	0.959542
3	0.951229425	0.951225995	0.951233939	0.951219778
4	0.941764534	0.941769684	0.941534196	0.941764353
5	0.93239382	0.932390458	0.932388597	0.932403009
6	0.923116346	0.923121395	0.923120399	0.92323405
7	0.913931185	0.913927889	0.913687709	0.913885631
8	0.904837418	0.904842366	0.904832237	0.904800959
9	0.895834135	0.895830906	0.895837958	0.895806679
10	0.886920437	0.886925287	0.886677455	0.88564472

CPU time of ESDBBDFM for $k=2$ is 0.2s, $k=3$ is 0.04 and $k=4$ is 0.01s

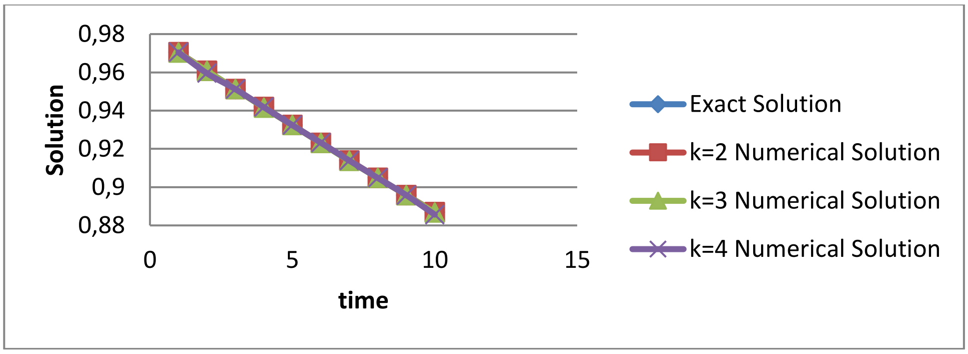

Figure 7: Graphical Presentation of Table 1

Table 2. Numerical Solution of Example 2 with the incorporation of Theorem 1 and Theorem 2 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	Exact Solution	$k=2$ Numerical Solution	$k=3$ Numerical Solution	$k=4$ Numerical Solution
1	1.913931185	1.913911862	1.913164307	1.9136689
2	1.886920437	1.886949516	1.886876817	1.87607662
3	1.860707976	1.860689767	1.860742997	1.860623911
4	1.835270211	1.835297601	1.834311469	1.83526246
5	1.810584246	1.810567098	1.810542954	1.810650552
6	1.786627861	1.786653657	1.786658441	1.78845191
7	1.763379494	1.763363344	1.762416543	1.762989522
8	1.740818221	1.740842513	1.740780001	1.740495934
9	1.718923733	1.718908523	1.718951201	1.718667129
10	1.697676326	1.697699203	1.696767095	1.68722167

CPU time of ESDBBDFM for $k=2$ is 0.3s, $k=3$ is 0.05 and $k=4$ is 0.02s

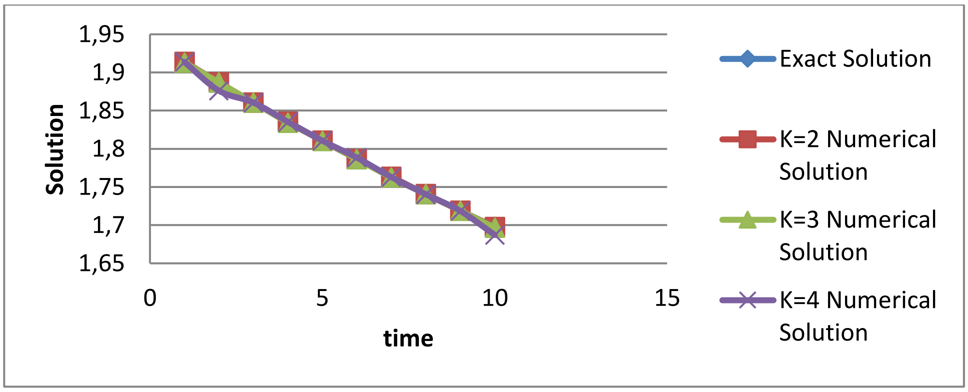

Figure 8: Graphical Presentation of Table 2

Table 3. Numerical Solution of Example 1 with the incorporation of Theorem 1 and Theorem 3 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	Exact Solution	k = 2 Numerical Solution	k = 3 Numerical Solution	k = 4 Numerical Solution
1	0.472366553	0.430738175	0.513468959	0.430008728
2	0.367879441	0.317481402	0.378140722	0.337148612
3	0.286504797	0.249693367	0.319853832	0.245681157
4	0.22313016	0.184835774	0.247047534	0.17348597
5	0.173773943	0.13602625	0.194290129	0.135862998
6	0.135335283	0.106865461	0.143138782	0.104988665
7	0.105399225	0.078972253	0.121092862	0.082167968
8	0.082084999	0.058014885	0.093554596	0.059694048
9	0.063927861	0.045520285	0.073593603	0.04201957
10	0.049787068	0.033572236	0.05423944	0.032832814

CPU time of ESDBBDFM for $k = 2$ is 0.4s, $k = 3$ is 0.06 and $k = 4$ is 0.03s

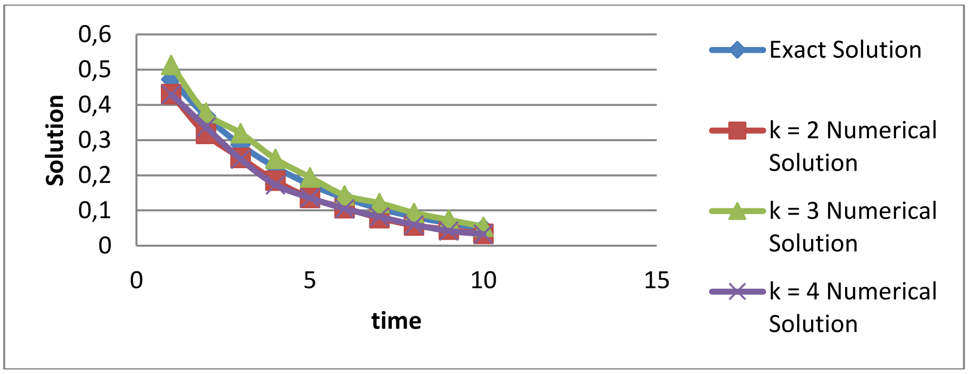

Figure 9: Graphical Presentation of Table 3

Table 4. Numerical Solution of Example 2 with the incorporation of Theorem 1 and Theorem 3 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	Exact Solution	k = 2 Numerical Solution	k = 3 Numerical Solution	k = 4 Numerical Solution
1	1.0299955	1.030262832	1.02972058	1.030445892
2	1.039989334	1.040235898	1.039786271	1.040425843
3	1.049979169	1.050373612	1.049483505	1.050497676
4	1.059964006	1.060337301	1.059492497	1.06096945
5	1.069942847	1.070466624	1.069454703	1.070919453
6	1.079914694	1.080416952	1.079501049	1.080903846
7	1.089878549	1.090533832	1.089162343	1.090853196
8	1.099833417	1.100466818	1.099142187	1.100893732
8	1.109778301	1.110567209	1.109069872	1.111345291
10	1.119712207	1.120478879	1.119080799	1.121249191

CPU time of ESDBBDFM for $k = 2$ is 0.5s, $k = 3$ is 0.07s and $k = 4$ is 0.02s

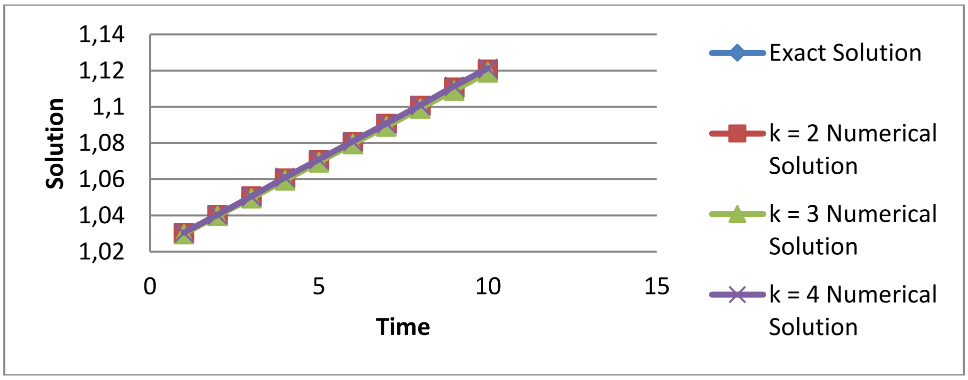

Figure 10: Graphical Presentation of Table 4

3.2 Absolute Random Errors

The absolute random errors of the obtained numerical solutions are computed by determining absolute differences of the exact solutions and the numerical solutions.

Table 5. Absolute Random Errors of Example 1 with the incorporation of Theorem 1 and Theorem 2 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	$k = 2$ Absolute Random Error	$k = 3$ Absolute Random Error	$k = 4$ Absolute Random Error
1	3.49705E-06	0.000176274	2.62275E-05
2	5.25355E-06	5.04695E-06	0.001247439
3	3.4298E-06	4.514E-06	9.647E-06
4	5.15002E-06	0.000230338	1.80484E-07
5	3.36171E-06	5.22331E-06	9.18939E-06
6	5.04821E-06	4.05271E-06	0.000117704
7	3.29587E-06	0.000243476	4.55541E-05
8	4.94806E-06	5.18114E-06	3.64592E-05
9	3.2297E-06	3.8225E-06	2.74561E-05
10	4.85018E-06	0.000242982	0.001275717

CPU time of ESDBBDFM for $k = 2$ is 0.2s, $k = 3$ is 0.04 and $k = 4$ is 0.01s

Table 6. Absolute Random Errors of Example 2 with the incorporation of Theorem 1 and Theorem 2 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	$k = 2$ Absolute Random Error	$k = 3$ Absolute Random Error	$k = 4$ Absolute Random Error
1	1.93233E-05	0.000766878	0.000262285
2	2.90793E-05	4.36197E-05	0.010843817
3	1.82094E-05	3.50206E-05	8.40654E-05
4	2.73896E-05	0.000958742	7.75141E-06
5	1.7148E-05	4.1292E-05	6.6306E-05
6	2.57959E-05	3.05799E-05	0.001824049
7	1.61503E-05	0.000962951	0.000389972
8	2.42923E-05	3.82197E-05	0.000322287
9	1.52104E-05	2.74676E-05	0.000256604
10	2.28769E-05	0.000909231	0.010454656

CPU time of ESDBBDFM for $k = 2$ is 0.3s, $k = 3$ is 0.05 and $k = 4$ is 0.02s

Table 7. Absolute Random Errors of Example 1 with the incorporation of Theorem 1 and Theorem 3 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.

t	$k = 2$ Absolute Random Error	$k = 3$ Absolute Random Error	$k = 4$ Absolute Random Error
1	0.041628378	0.041102406	0.042357825
2	0.050398039	0.010261281	0.030730829
3	0.03681143	0.033349035	0.04082364
4	0.038294386	0.023917374	0.04964419
5	0.037747693	0.020516186	0.037910946
6	0.028469822	0.007803499	0.030346618
7	0.026426971	0.015693637	0.023231256
8	0.024070114	0.011469597	0.02239095
9	0.018407576	0.009665742	0.021908292
10	0.016214833	0.004452371	0.016954254

CPU time of ESDBBDFM for $k = 2$ is 0.4s, $k = 3$ is 0.06 and $k = 4$ is 0.03s

Table 8. **Absolute Random Errors of Example 2 with the incorporation of Theorem 1 and Theorem 3 using the ESDBBDFM for Step Numbers k = 2, 3 & 4.**

t	$k = 2$ Absolute Random Error	$k = 3$ Absolute Random Error	$k = 4$ Absolute Random Error
1	0.000267332	0.00027492	0.000450392
2	0.000246564	0.000203063	0.000436509
3	0.000394443	0.000495664	0.000518507
4	0.000373295	0.000471509	0.001005444
5	0.000523777	0.000188144	0.000976606
6	0.000502258	0.000413645	0.000989152
7	0.000655283	0.000716206	0.000974647
8	0.000633401	0.00069123	0.001060315
9	0.000788908	0.000708429	0.00156699
10	0.0000766672	0.000631408	0.001536984

CPU time of ESDBBDFM for $k = 2$ is 0.5s, $k = 3$ is 0.07s and $k = 4$ is 0.02s

3.3 Graphical Presentation of Absolute Errors

Using R and R – studio softwares, the graphs of Absolute Random Error Results of ESDBBDFM for Examples1 and 2 above in **Table 1** to **Table 4** are plotted and presented as;

3.3.1 Graphical Presentations of the Absolute Random Errors for ESDBBDFM after the Incorporations of the Theorem 1 and Theorem 2 for Examples 1 and 2

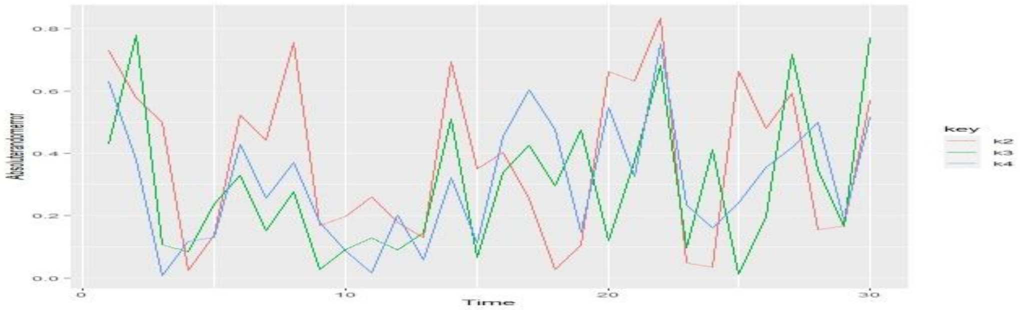

Figure 11: **Absolute Random Error Results for Example 1 using ESDBBDFM (as seen in the colors) against Time of future delay in days. The colorful lines represent the behavior or performance of the method for step numbers $k = 2, 3$ and 4 with different Absolute Random Errors.**

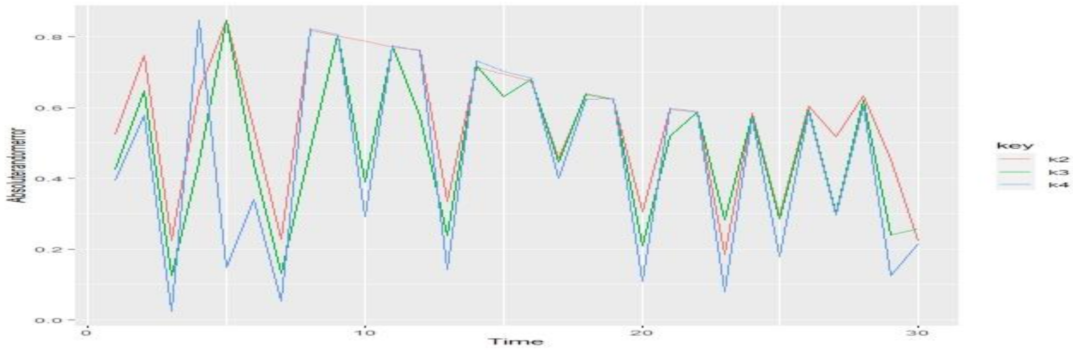

Figure 12: **Absolute Random Error Results for Example 2 using ESDBBDFM (as seen in the colors) against Time of future delay in days. The colorful lines represent the behavior or performance of the method for step numbers $k = 2, 3$ and 4 with different Absolute Random Errors.**

3.3.2 Graphical Presentations of the Absolute Random Errors for Implicit ESDBBDFM after the Incorporations of the Theorem 1 and Theorem 3 for Examples 1 and 2

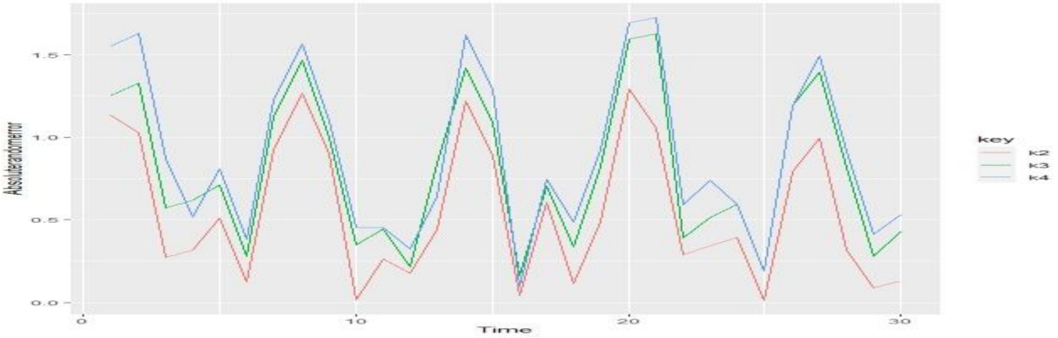

Figure 13: **Absolute Random Error Results for Example 1 using ESDBBDFM (as seen in the colors) against Time of future delay in days. The colorful lines represent the behavior or performance of the method for step numbers $k = 2, 3$ and 4 with different Absolute Random Errors.**

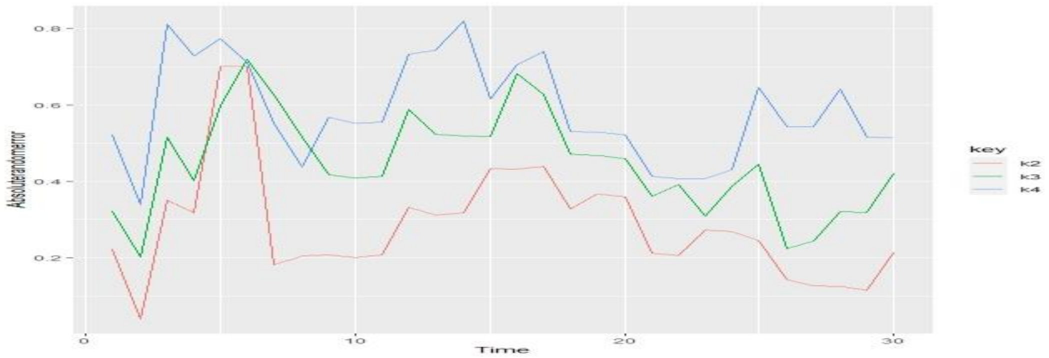

Figure 14: **Absolute Random Error Results for Example 2 using ESDBBDFM (as seen in the colors) against Time of future delay in days. The colorful lines represent the behavior or performance of the method for step numbers $k = 2, 3$ and 4 with different Absolute Random Errors.**

3.4 Comparison of Results Based on Minimum Absolute Random Errors (MAREs)

In order to determine the accuracy, efficiency and advantage of our method ESDBBDFM, we compared the Minimum Absolute Random Errors (MAREs) of our method with other existing methods below [9], [15]:

Table 9. **Comparison between the MAREs of ESDBBDFM after the incorporations of the evaluated Theorem 1, Theorem 2 and Theorem 3 for k = 2, 3 and 4 at constant step size d = 0.01 for Example 1 with [9], [15]**

Numerical Method	COMPARED MAREs with [9], [15], [29].
ESDBBDFM MARE for $k = 2$by theorems 1 and 2	3.2297E-06
ESDBBDFM MARE for $k = 3$by theorems 1 and 2	4.05271E-06
ESDBBDFM MARE for $k = 4$by theorems 1 and 2	1.80484E-07
ESDBBDFM MARE for $k = 2$by theorems 1 and 3	0.016954254
ESDBBDFM MARE for $k = 3$by theorems 1 and 3	0.016954254
ESDBBDFM MARE for $k = 4$by theorems 1 and 3	0.016214833
CSSEMM MARE for $k = 2$ [9]	4.76E-01
CSSEMM MARE for $k = 3$ [9]	9.17E-01
CSSEMM MARE for $k = 4$ [9]	1.62E-01
EMM MARE for $k = 2$ [15]	1.84E+00
EMM MARE for $k = 3$ [15]	2.47E-01
EMM MARE for $k = 4$ [15]	9.73E-01
BSM MARE for $k = 2$ [29]	7.04E-01
BSM MARE for $k = 3$ [29]	7.05E-01
BSM MARE for $k = 4$ [29]	7.06E-01

Table 10. Comparison between the MAREs of ESDBBDFM after the incorporations of the evaluated Theorem 1, Theorem 2 and Theorem 3 for k = 2, 3 and 4 at constant step size d = 0.01 for Example 2 with [9], [15]

Numerical Method	COMPARED MAREs with [9], [15], [29].
ESDBBDFM MARE for $k = 2$ by theorems 1 and 2	1.52104E-05
ESDBBDFM MARE for $k = 3$ by theorems 1 and 2	3.05799E-05
ESDBBDFM MARE for $k = 4$ by theorems 1 and 2	7.75141E-06
ESDBBDFM MARE for $k = 2$ by theorems 1 and 3	0.0000766672
ESDBBDFM MARE for $k = 3$ by theorems 1 and 3	0.000188144
ESDBBDFM MARE for $k = 4$ by theorems 1 and 3	0.000436509
CSSEMM MARE for $k = 2$ [9]	4.76E-01
CSSEMM MARE for $k = 3$ [9]	9.17E-01
CSSEMM MARE for $k = 4$ [9]	1.62E-01
EMM MARE for $k = 2$ [15]	1.84E+00
EMM MARE for $k = 3$ [15]	2.47E-01
EMM MARE for $k = 4$ [15]	9.73E-01
BSM MARE for $k = 2$ [29]	7.04E-01
BSM MARE for $k = 3$ [29]	7.05E-01
BSM MARE for $k = 4$ [29]	7.06E-01

3.5 Summary of Result Discussion

The results obtained after the numerical implementation of the method in solving the modeled Advanced Stochastic Time-Delay Differential Equation (ASTDDE) as presented in the from Table 1 to Table 4 and the absolute random errors in Table 5 to Table 8 with their respective graphical representations displayed in Figure 7 to Table 14 above revealed the economic chaos of uncertainties and instabilities in the production and marketing of agricultural produce by the rural farmers due to due to lack of redesigned naira notes in circulation which result to low patronage by the buyers, wastage of farm produce, discouragements in producing more which causes low supply of farm produce to the markets and reduction in buyers ability. To reduce the adverse effects of uncertainties caused by the CBN 2023 implementation of the redesigned new naira notes on its citizen mostly the rural farmers as expressed by this study, the government of Nigeria and the CBN should have a long-term implementation plan regarding its financial policies. Creation of awareness and establishment of commercials banks in the rural areas will easy circulation of redesigned naira notes for business transactions and discourage the situation of using money to buy money. Relating our findings to other existing results in the literature, comparisons of the Minimum Absolute Errors (MAREs) of our results were carried out which ascertained the advantage of our method over other existing methods in [9], [15] in terms of efficiency and accuracy. The proposed method performed better than existing methods in literature by producing the Least Minimum Absolute Errors as presented in Table 9 and Table 10 for numerical solution of ASTDDE.

4. CONCLUSION

This study has expressed and proffers solutions to the adverse effects of uncertainties suffered by the Nigerian rural farmers as a result of the CBN 2023 implementation of the redesigned naira notes for business transactions. The study has also demonstrated that Extended Second Derivative Block Backward Differentiation Formulae Method (ESDBBDFM) for step numbers k = 2, 3 and 4 is suitable for solving some Advanced Stochastic Time-Delay Differential Equation (ASTDDE) numerically without the use of interpolation techniques in evaluating the delay term and the noise term. As observed from Table 1 to Table 4, the numerical results of the discrete schemes of higher step number $k = 4$ of ESDBBDFM performed slightly better and faster than the lower step numbers $k = 2$ and 3 by producing the Least Minimum Absolute Random Error (LMARE) in Table 5 to Table 8. In comparing the numerical results of this method with other existing methods in literature as shown in **Table 5** to **Table 8**, the newly developed mathematical expressions for models **Theorem 1** and **Theorem 2** used for the evaluations of the delay term and the noise term in solving some ASTDDEs with the discrete schemes of ESDBBDFM gives better and accurate results by producing Least Minimum Absolute Random Error (LMARE) in a Lower Computational Processing Unit Time (LCPUT) as presented in **Table 9** to **Table 10** faster than the **Theorem 1** and **Theorem 3** and other existing methods that applied interpolation techniques in evaluations of the delay term and the noise term. The lower the Absolute Random Error (ARE) the lower the uncertainties and the lower the uncertainties the better the business transactions in the production and marketing of agricultural produce by the rural farmers which also strengthens buyers' ability. Further research should be carried-out for step numbers $k = 5,6,7,\ldots$ on the numerical solutions of ASTDDE using ESDBBDFM with some applications in finance.

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