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COMPARISON OF APARCH-TYPE MODELS: DOES THE CONTINUOUS AND JUMP COMPONENTS OF REALIZED VOLATILITY IMPROVE THE FITTING?

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ABSTRACT

This study aims to extend an APARCH-X(1,1) model to the APARCH-CJ(1,1) by separating the exogenous variable X into two components: continuous and discontinuous (jump). The study was based on the application of models to 1-min intraday high-frequency data from the Tokyo Stock Price Index from 2004 to 2011, where its dependent variable is daily return and its exogenous variability is Realized Volatility. As a basic framework, the return errors follow a Normal distribution. An Adaptive Random Walk Metropolis (ARWM) method was constructed in the Markov Chain Monte Carlo algorithm to estimate model parameters so that the model fits the observed return time series. By visual inspection, the parameter trace plots showed good convergence of the Markov chains, indicating that the ARWM method is efficient in estimating the studied models. Based on the results of the Akaike Information Criterion for model fitting to data, this study found that APARCH-CJ(1,1) is inferior to APARCH(1,1).

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1. INTRODUCTION

The volatility of a financial asset plays an important role in the calculation and hedging of financial derivatives (such as options), as well as the measurement of risk in a financial or investment portfolio. The volatility phenomenon refers to how large an asset's prices change around the average price over a given period of time—this is a statistical measure of the commodity return margin [1]. The existence of volatility is closely related to risk in the market. In general, a sudden increase in volatility can trigger demand for higher premia on more risky assets and thereby lead to market losses [2].

There are several approaches to estimating financial market volatility, and the most popular is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model in [3]. This model has the ability to model non-constant volatility, where conditional volatilities depend not only on past returns but also on past conditional volatilities. A new version of the GARCH model that includes intraday volatility measurements later called the GARCH-X model, was introduced [4]. The model is formed from the standard GARCH model by adding an exogenous variable X to the volatility dynamic equation. The variable describes the intraday volatility of each trading day. An illustrative result using the Realized Volatility (RV) of the Deutsche mark –U.S. dollar exogenous variable shows a better fit than the GARCH model.

On the other hand, to overcome the GARCH weaknesses in capturing the asymmetric volatility phenomenon, study in [5] proposed a class of APARCH (Asymmetric Power Autoregressive Conditional Heteroskedasticity) model that was able to capture the phenomenon. Asymmetry refers to the fact that the asset market volatility tends to be higher during periods when the return is negative [6]. This study chose to focus on the APARCH model due to the fact that it can act in a manner similar to the other seven GARCH-type models [7], and many current applications have found it to be especially pertinent. Similar to the GARCH-X model, the standard APARCH model was extended to the APARCH-X in [8] to the APARCH-X. An empirical study of the APARCH-X(1,1) model fitting for 49 stocks proved that RV can help predict squared return.

Furthermore, to increase the strength in predicting volatility, studies in [9] decomposed the exogenous variable X into two parts, namely continuous (C) and discontinuous or jump (J). The application using high-frequency data from the HUSHEN 300 index in China, which are fitted to GARCH and EGARCH models, demonstrated that models with continuous and jump components are superior to the basic and X-type models.

Motivated by the above studies, this study aims to evaluate the goodness of fit of the APARCH-CJ model constructed from the APARCH-X model by decomposing X into C and J. In order to investigate this, the APARCH(1,1), APARCH-X(1,1), and APARCH-CJ(1,1) models are compared by using 1-minute intraday high-frequency data from the Tokyo Stock Price Index (TOPIX) in Japan. A study of [10] showed that APARCH models have better performances compared with the GARCH model for the daily TOPIX Sector Indexes, which suggests the existence of an asymmetric effect. To our knowledge, there are no results for the estimation of the APARCH-CJ model.

The performance of competing models is examined based on the estimates of model parameters using the Adaptive Random Walk Metropolis (ARWM) method introduced in [11]. This method is adopted in the Markov Chain Monte Carlo (MCMC) algorithm to sample the model parameters from their full conditional posterior distributions. Model comparison is conducted by calculating the Akaike Information Criterion (AIC).

2. RESEARCH METHODS

This section begins with the development of the APARCH-CJ model and realized measures. We then describe how to estimate the model parameters by using the ARWM method in the MCMC algorithm and how to select the best-fit model based on AIC. Finally, we explain dataset selection and describe some properties of the data.

2.1 Model construction

The APARCH model is an extension of the GARCH model constructed to allow asymmetric effects of positive and negative returns on conditional volatility. That is, negative returns tend to increase more in volatility than positive ones of the same magnitude. Consider the return $R_t = \varepsilon_t$ at time t, where ε_t is independent identically distributed (i.i.d) Normal with zero-mean and nonconstant conditional variance σ_t^2. The return refers to the difference in the natural logarithm of the asset price. Specifically, the APARCH (1,1) model may be defined as follows:

$$\sigma_t^\delta = \omega + \alpha(|R_{t-1}| - \gamma R_{t-1})^\delta + \beta\sigma_{t-1}^\delta. \tag{1}$$

Usual restrictions on parameters are that

$$\omega, \alpha, \beta, \delta > 0 \text{ and } -1 < \gamma < 1.$$

Asymmetry in the above model is captured by the term $\gamma \neq 0$. When $\gamma > 0$ ($\gamma < 0$), the negative (resp. positive) returns lead to higher volatility than positive (resp. negative) ones.

Notice that the APARCH(1,1) model nests several other volatility models as follows [12], [13]:

- Set $\delta = 2$, $\beta = 0$, and $\gamma = 0$ to obtain ARCH(1) model.
- Set $\delta = 2$ and $\gamma = 0$ to obtain a GARCH(1,1) model.
- Set $\delta = 1$ and $\gamma = 0$ to obtain Taylor–Schwert GARCH.
- Set $\delta = 2$ to obtain the Glosten–Jagannathan–Runkle GARCH model.
- Set $\delta = 1$ to obtain Threshold GARCH.
- Set $\beta = 0$ and $\gamma = 0$ to obtain Non-linear ARCH.
- Set $\delta \to 0$ to obtain Log-GARCH.

Recently, the APARCH model was extended in [8] to be APARCH-X, where X refers to an exogenous variable that is added in the volatility process. An APARCH-X(1,1) model with 1 lagged value of an exogenous variable assumes that:

$$\sigma_t^\delta = \omega + \alpha(|R_{t-1}| - \gamma R_{t-1})^\delta + \beta\sigma_{t-1}^\delta + \lambda X_{t-1}^\delta, \tag{2}$$

where $\lambda > 0$ to ensure the positivity of model since X_{t-1} is strictly positive. In general practice, many studies take RV as an exogenous variable because it is a considerably more accurate volatility estimator than the squared return. RV is defined as the square root of the sum of squared intraday returns and can be expressed as

$$RV_t^2 = \sum_{i=1}^{M} R_{t,i}^2, \tag{3}$$

where i is the fraction of the regular trading session, $R_{t,i}$ is the return of the i-th 1-minute interval (in our case) of the t-th day, and M is the number of observations for each trading day. The RV with 1-minute returns is then called 1-min RV.

Similar to study in [9], the APARCH-CJ(1,1) is constructed from Equation (2) by decomposing X_{t-1} into two parts: C_{t-1} as a continuous part and J_{t-1} as a jump part. Here is the APARCH-CJ(1,1) model:

$$\sigma_t^\delta = \omega + \alpha(|R_{t-1}| - \gamma R_{t-1})^\delta + \beta\sigma_{t-1}^\delta + \lambda_1 C_{t-1}^\delta + \lambda_2 J_{t-1}^\delta. \tag{4}$$

The estimators for continuous and jump parts, respectively, are defined as

$$C_t = I_{[Z_t \leq \phi_\alpha]} RV_t + I_{[Z_t > \phi_\alpha]} MedRV_t \tag{5}$$

$$J_t = I_{[Z_t > \phi_\alpha]}(RV_t - MedRV_t) \tag{6}$$

where

I	: Indicator function
ϕ_α	: α-quantile of the standard Normal distribution function
$MedRV$	: Median RV
Z	: Test statistic

Z_t is expressed as follows:

$$Z_t = \frac{(RV_t - MedRV_t)RV_t^{-1}}{\sqrt{\frac{1}{M}\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\max\left(1, \frac{MedRTQ_t}{MedRV_t}\right)}}, \tag{7}$$

Where,

$$MedRV_t = \frac{\pi}{6 - 4\sqrt{3} + \pi}\left(\frac{M}{M-2}\right) \times \sum_{i=2}^{M-1} Med\left(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|\right)^2 \tag{8}$$

$$MedRTQ_t = \frac{3\pi M}{9\pi + 72 - 52\sqrt{3}}\left(\frac{M}{M-2}\right) \times \sum_{i=2}^{M-1} Med\left(|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|\right)^4 \tag{9}$$

Based on previous research, this study chose 99% quantile.

2.2 Bayesian Inference

The most extensively used sampling technique for Bayesian inference is the MCMC algorithm. When attempting to estimate the posterior distribution of parameters, the MCMC algorithm offers an adaptable and effective method [14]. The MCMC algorithm generally consists of two different steps: Monte Carlo and Markov Chain [15]. First, the Monte Carlo approach is to draw a large number of random samples that form a Markov chain with the desired equilibrium distribution. Second, the Monte Carlo approach calculate the statistical parameters (such as mean, standard deviation, and confidence interval) from the random draws.

Several MCMC methods are available to construct a Markov chain. This study chose the ARWM method of [11], which is recommended in [16]–[18] due to its effectiveness in estimating GARCH-type models. To draw samples for the unknown parameter θ at the n-th iteration:

1. A candidate for the new sample is created by formula:

$$\theta^n = \theta^{(n-1)} + \sqrt{\Delta^{(n)}} z^{(n)}, z^{(n)} \sim N(0,1), \tag{10}$$

   where $\Delta^{(n)} > 0$ is the step size for the proposed move.

2. The new candidate is accepted if $\frac{p(\theta^{(n)}|Y)}{p(\theta^{(n-1)}|Y)} > U(0,1)$, where U denotes the uniform distribution and $p(\theta|Y)$ denotes the posterior probability distribution of the parameter θ conditional on the available data Y. The posterior distribution can be calculated using Bayes' rule: $p(\theta|Y) = L\,(Y|\theta) \times p(\theta)$, where L represents the likelihood function and $p(\theta)$ represents the prior distribution for the parameter θ. In our case, given the observation $Y = \{R_1, R_2, \dots, R_T\}$, the general form of log-likelihood function for the model with a parameter vector $\boldsymbol{\theta}$ can be expressed as

$$\ln L(Y|\boldsymbol{\theta}) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{R_t^2}{\sigma_t^2}, \tag{11}$$

   where the σ_t's are defined by Equations (1–2) and (4). In particular, we use the same prior for all parameters: a truncated Normal with a mean of 0 and a variance of 10.

3. For a proposal acceptance frequency of m, the step size changes adaptively as follows:

$$\Delta^{(n)} = \Delta^{(n-1)} + \frac{\frac{m}{n} - 0{,}44}{n^{0{,}6}}, \tag{12}$$

For the in-sample analysis, the best fitting model is selected based on AIC, a function that penalizes the adjusted model's quality in line with the number of estimated parameters. The primary benefit is that the competing models do not need to be nested. The equation for the AIC statistic is

$$AIC = -2\ln L + 2k, \tag{13}$$

in which L represents the maximum value of the log-likelihood function of the model and k is the number of estimated parameters. The model with the lowest AIC value will be chosen as the best.

2.3 Data selection

It is not easy to evaluate volatility models. Since the volatility process is latent, it cannot be directly seen. Fortunately, the availability of high-frequency financial data is growing, enabling us to obtain more accurate measures of volatility. However, because the data is not free to the public, as an empirical example, this study applies a model to the TOPIX data used in [19]. TOPIX (Tokyo Stock Price Index) is one of the primary indices of the TSE (Tokyo Stock Exchange). Considering its total market value, TSE is one of the four biggest stock exchanges in the world [20].

The daily data used for model estimation spans the years 2004 through 2011, with a total of 1963 trading days. TOPIX is recorded every 1 minute of every trading day. For each trading day, we have roughly 270 observations. Figure 1 displays the time series plots of daily absolute returns, continuous variation, and jump variation of the index. The highest RV levels were attained on 14 October 2008 and 15 March 2011, when returns were around 12.87% and –9.95%, respectively. The financial crisis in October 2008 was due to the collapse of Lehman Brothers. On 15 March 2011, jump variation was at its highest level, which refers to the market crash after Japan's Fukushima disaster on 11 March 2011. The intraday price changes for 15 March 2011 are more drastic than on the previous day, resulting in a higher jump variation. This emphasizes how crucial it is to account for volatility jumps in order to adequately reflect the dynamics of extreme events.

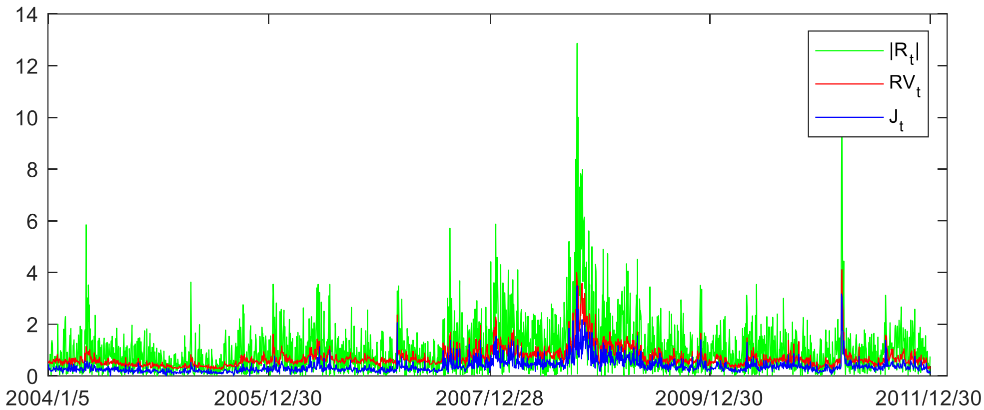

Figure 1. Time series plots of the percentage daily absolute return, realized volatility, and jump variation in volatility. The underlying asset is the TOPIX, with time periods ranging from January 2004 to December 2014

3. RESULTS AND DISCUSSION

This section presents a practical application of the proposed model using a real-world dataset. Firstly, the convergence of the estimation is visually analyzed. Secondly, the key parameters are analyzed for their significance. Finally, a comparison of competing models is performed to find out the model that provides a better fit.

3.1 Diagnosis of Convergence of a Method

A total of 6000 iterations were conducted in the MCMC algorithm, with an initial 1000 discarded in the burn-in period. The burn-in period is required to reduce the possibility of non-convergence caused by the effect of the parameter's initial value. There is no established method for determining the length of the necessary burn-in since the convergence rates of different methods on different target distributions may vary significantly [21]. One approach to help indicate whether the burn-in period has run long enough is to evaluate the trace plot. The trace plot provides a visual way to display the realizations of the Markov chain at each iteration. Furthermore, with the help of plots, it is possible to see how the Markov chain travels through the state space and how well it mixes [22]. Trace plots with flat bits (where the chain stays in the same state space for too long) or too many consecutive steps indicate slow convergence. Meanwhile, visible trends or changes in the trace plot's spread indicate that stationarity has not been reached yet. It is sometimes said that a good trace plot should resemble a hairy caterpillar.

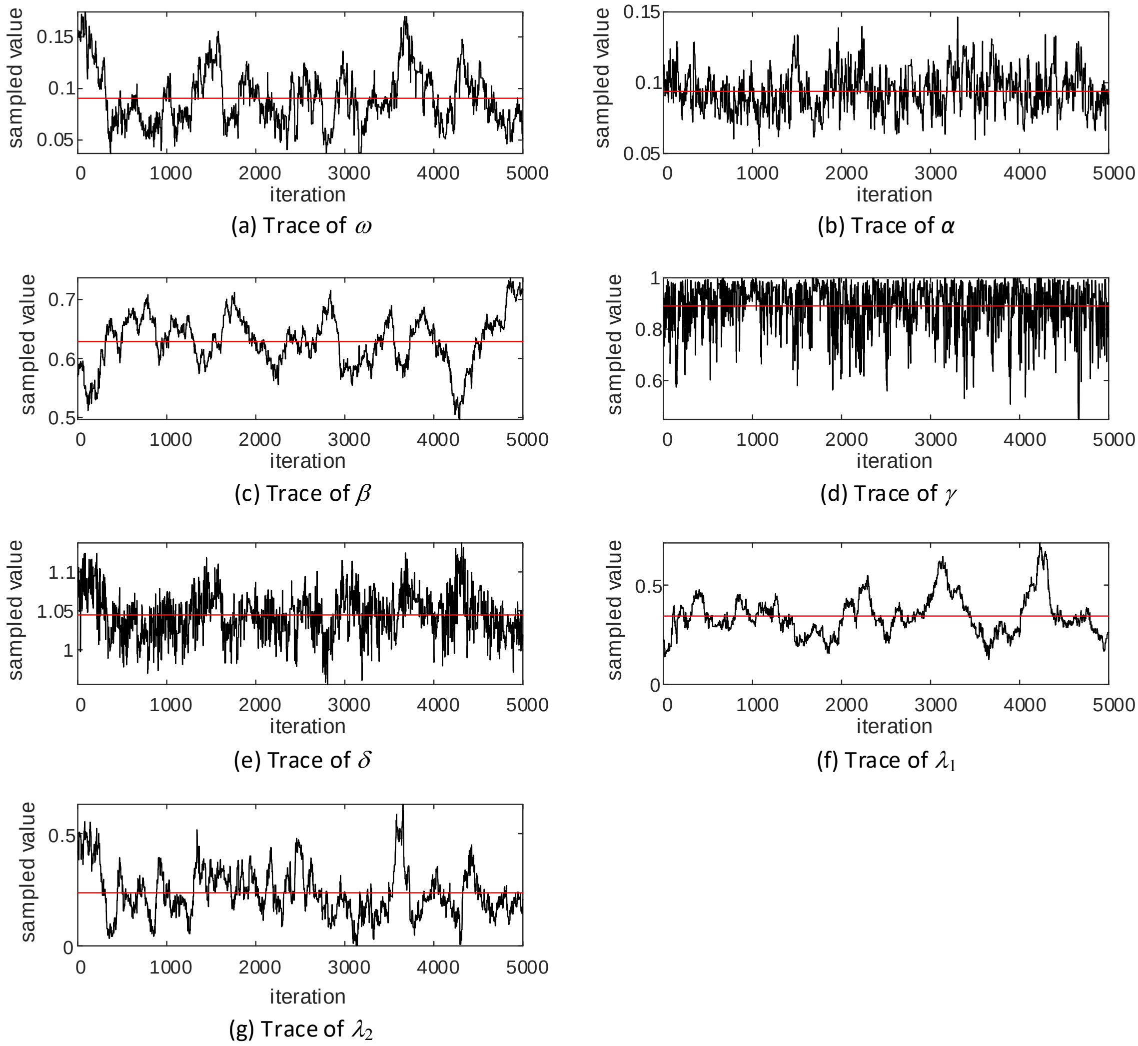

Figure 2. Trace plots of the ARWM chains for the APARCH-CJ(1,1) model: (a) estimates of ω, (b) estimates of α, (c) estimates of β, (d) estimates of γ, (e) estimates of δ, (f) estimates of λ_1, and (g) estimates of λ_2.

The results of running the ARWM sampler once in estimating the APARCH-CJ(1,1) model are shown in Figure 2, using the remaining iterations after burn-in. The 5000 draws were generated for all parameters with the same initial value $\boldsymbol{\theta}^{(0)} = (0.5, 0.1, 0.5, 0.1, 1, 0.1, 0.1)$. Looking at the trace plots, a burn-in period of 1000 seems sufficient since the earlier draws close to the sample mean (red line). The sampled values are centered around the sample mean. The absence of frequent flat bits in the trace plots indicates good enough mixing of the chain. Although the chain converges rather slowly for the β and λ_1 parameters, the ARWM method can be said to be successful in exploring the posterior distribution of the parameters in the APARCH-CJ(1,1) model. This result supports the findings of the study in [17], [23], [24] in the context of the GARCH model. Therefore, these samples can be used for Monte Carlo purposes, namely estimating the marginal posterior distribution of each parameter and its related characteristics (mean, standard deviation, probability interval, etc.).

3.2 Parameter Estimation and Analysis

Table 1 lists the estimates for three competing APARCH(1,1)-type models that were found after the burn-in period was eliminated. We report the posterior mean, posterior standard deviation, and the lower bound and upper bound for the 95% highest posterior distribution (HPD) interval. The HPD intervals were estimated by using the Chen–Shao approach in [24], [25].

First, we see the flexibility of the APARCH specification. The result shows the presence of a structure in the TGARCH model, indicated by the HPD intervals of power (δ) that include 1. Meanwhile, the HPD intervals of asymmetry coefficient (γ) exclude 0, which suggests an asymmetrical effect on volatility for the

TOPIX data. In particular, we found that the estimates of γ are positive, known as the "leverage effect" by definition [26]. These findings confirm the conclusions in [12], [27], [28] that underlined that the Threshold GARCH (TGARCH)-type model has the potential to be the best fit in its class.

Second, the analysis for the exogenous parameters, λ_1 and λ_1, follows the interpretation in [9]. The positive value for the coefficient of RV_{t-1} in the volatility equation of the APARCH-X model shows that TOPIX's volatility exhibits strong persistence, and the volatility of the previous period may be used as a predictor for the volatility of the present period. Considering the APARCH-CJ model, the positive coefficients of C_{t-1} and J_{t-1} indicate that, on TOPIX data, the lagged continuous and discontinuous jump path variations contain relatively more information for predicting the present volatility.

Table 1. The posterior mean, posterior standard deviation (SD), and lower bound (LB) and upper bound (UB) of the 95% HPD interval of the parameters in APARCH(1,1)-type models fitted by ARWM on daily returns of TOPIX

Stat.	Parameter						
	ω	α	β	γ	δ	λ_1	λ_2
APARCH							
Mean	0.0824	0.1161	0.8381	0.6683	1.019	-	-
SD	0.0161	0.0136	0.0153	0.1040	0.014	-	-
LB	0.0529	0.0922	0.8100	0.4687	0.991	-	-
UB	0.1133	0.1459	0.8672	0.8786	1.047	-	-
APARCH-X							
Mean	0.0908	0.0923	0.6273	0.9024	1.0573	0.2947	-
SD	0.0225	0.0125	0.0657	0.0814	0.0428	0.0733	-
LB	0.0481	0.0666	0.5116	0.7466	0.9826	0.1849	-
UB	0.1344	0.1160	0.7383	0.9999	1.1505	0.4252	-
APARCH-CJ							
Mean	0.0856	0.0909	0.6034	0.9080	1.059	0.3779	0.2645
SD	0.0218	0.0128	0.0451	0.0784	0.039	0.0899	0.0954
LB	0.0458	0.0653	0.5256	0.7511	0.978	0.1757	0.0784
UB	0.1468	0.1139	0.6999	0.9996	1.133	0.5459	0.4430

3.3 Model Comparison and Evaluation

In order to investigate whether the decomposition of realized volatility into continuous sample path and discontinuous jump components have better performance in fitting TOPIX time series, an assessment was carried out based on the AIC value. Table 2 reports the calculated values of AIC for symmetric GARCH-type and APARCH-type models. In the context of the GARCH model, it is clear that GARCH-CJ presents the lowest AIC, indicating that this model can be selected as the best model. This result shows that the decomposition of realized volatility into its continuous and jump components can improve volatility modeling, which is consistent with the study in [9]. Contrasting with that result, when the AIC values for the APARCH-X and APARCH-CJ are compared, the fitting for APARCH-X performs significantly better. This finding shows that the more advanced model does not necessarily provide a better fit.

Table 2. Comparison of the GARCH-type and APARCH-type models.

Stat.	Model					
	GARCH	GARCH-X	GARCH-CJ	APARCH	APARCH-X	APARCH-CJ
Log-likelihood	–3181.05 (0.12)	–3167.25 (0.16)	–3165.03 (0.12)	–3147.41 (0.42)	–3123.29 (0.63)	–3123.04 (0.29)
AIC	6368.10 (0.23)	6342.49 (0.32)	6340.06 (0.24)	6304.73 (0.83)	6258.64 (1.25)	6260.08 (0.58)

4. CONCLUSIONS

This study constructed the APARCH-CJ model on the basis of the APARCH-X model to investigate whether the volatility model can be better measured by decomposing the realized volatility into continuous and discontinuous jump components. In order to investigate the performance of the model, an empirical study was carried out using the 1-minute high-frequency data of TOPIX in Japan, which started in January 2004 and continued until December 2011. The considered models were estimated using the ARWM method in the MCMC algorithm, and their fitting performance was evaluated using AIC.

According to the results and discussion, the conclusions are as follows. On the comparison of the fitting performance of the GARCH-type and APARCH-type models, AIC reveals that both models have different performances. Consistent with the study in **[9]**, the GARCH-CJ model has a better fitting of the future volatility than the other two types of models. In contrast, the fitting of the proposed APARCH-CJ model is inferior to the APARCH-X model. It means the application of the APARCH-CJ model does not fit in measuring the volatility of the TOPIX dataset when 1-minute RV is decomposed into continuous and jump components.

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