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A FRACTIONAL DIFFERENTIAL EQUATION MODEL FOR THE SPREAD OF POTATO LEAF ROLL VIRUS (PLRV) ON POTATOES

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ABSTRACT

Potatoes infected with the PLRV virus will experience a decrease in production up to 90%. In this paper, The PLRV distribution fractional differential equation model with potato and vector populations is reformulated by adding one new parameter, namely the rate of vector death due to predators. The model is divided into susceptible and infected classes. The PLRV dispersion model was developed and converted to a fractional order form for $0<\sigma\leq 1$*. Next, the invariant region, positive solutions, basic reproduction number, equilibrium point, and stability were determined. Based on the stability analysis, it is shown that the stability of the disease-free equilibrium point is locally stable and globally stable if the basic reproduction number* $(R_0)<1$*, and the stability of the endemic equilibrium point is globally stable if the basic reproduction number* $(R_0)>1$*. Numerical solutions were also carried out to determine the effect of several parameters on the PLRV distribution model on potatoes. The numerical solution results show that the elimination rate of infected potatoes and the infection rate of potatoes have a significant role in controlling the spread of PLRV in potatoes.*

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1. INTRODUCTION

In Indonesia, potatoes are one of the essential commodities of vegetables. Indonesia is the largest country that produces potatoes in South East Asia [1]. Potatoes are susceptible to several diseases, but Potato Leaf Roll Virus (PLRV) is the dominant one worldwide [2]. Potatoes infected with the PLRV virus will experience a decrease in production up to 90%. The potato plant can be infected with the PLRV in two courses: primary and secondary infections. A primary infection is caused by the virus-carrying aphids (vector) during the growing season, while a secondary infection occurs when contaminated bulbs are planted [3].

Controlling the spread of PLRV involves various measures, including using virus-free seed potatoes, implementing strict aphid control strategies, and promoting good agricultural practices. Mathematical models can be utilized to understand the dynamics of PLRV spread better and assess the effectiveness of different control measures. Early detection and prompt action are crucial in managing and minimizing the impact of the Potato Leaf Roll Virus on potato crops.

Fractional differential equations can be used to understand the dynamics of real-life situations. Using fractional differential equations can provide better and more detailed information in approaching a problem. The fractional differential equation involves the derivative of an unknown function with fractional orders. For example, Ahmad et al. used fractional differential equations for the reaction-diffusion model [16], and Singh et al. applied them to obtain a hyperbolic-type solution for a particular equation [17]. Fractional differential equations form a mathematical model that can describe the propagation of PLRV. Mapinda et al. have used a typical differential equations system to model the spread of Banana Xanthomas Wilt bacteria (BXW) on bananas. Studies have shown that pruning bananas infected with BXW and sterile farming tools are necessary strategies to control the distribution of BXW [4]. Shah et al. applied a fractional differential equation to study the spread of pests in tea plants. The findings suggest that selecting predators as the enemy of problems has reduced the spread of pests in tea plants [5]. Ali et al. described regression modeling strategies to predict PLRV disease [18]. Furthermore, Bonyah formulated a potato disease model in a fractional-order derivative [19].

Tilahun et al. have also researched the mathematical model of PLRV deployment using differential fractional equations and stability [15]. This article reviewed the research by adding another new parameter: the rate of vector death because predators are viewed as an enemy to the vector. In this article, predators can help control the spread of PLRV in potatoes because predators are considered enemies of the vectors. Also, several studies have shown that leaf predators can help control vector populations and indirectly reduce the spread of diseases caused by these vectors.

2. RESEARCH METHODS

The research method contains sources of data and data analysis to show how to control the spread of PLRV on Potatoes.

2.1 Sources of Data

This paper uses data from published articles from Tilahun et al. [15]. The data has been collected in Table 2 to perform numerical simulations to investigate the effect of some parameters on disease control and support the theoretical analysis.

2.2 Data Analysis

This paper has developed a system of fractional differential equations of the PLRV model using fractional derivative Caputo to the ordinary differential equations. The PLRV model is analyzed qualitatively to determine the conditions of the PLRV model, and numerical simulation is performed by using MATLAB and presenting the results as a graph.

3. RESULTS AND DISCUSSION

3.1 PLRV Model using Fractional Differential Equation

The fractional differential equation model of PLRV spread considers the populations of potatoes and vectors, each population consisting of susceptible and infected subclasses. The fractional differential equation model of PLRV spread has four compartments: $S_p(t)$ representing the number of susceptible potato populations at time t, $I_p(t)$ representing the infected potato population at time t, $S_v(t)$ representing the susceptible vector population at time t, and $I_v(t)$ representing the infected vector population at time t.

The plot and parameters of the model for the spread of PLRV in potatoes can be seen in **Figure 1** and **Table 1** below, respectively.

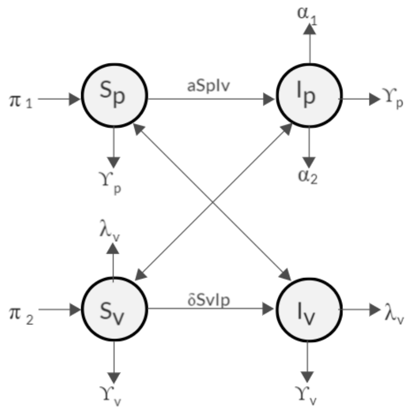

Figure 1. Flow diagram of PLRV model

Table 1. Parameters Description of the PLRV model

Parameters	Description	Parameters	Description
S_p	Population of susceptible potato	γ_v	Natural death rate of vector
S_v	Population of susceptible vector	π_1	Replanting rate of potato
I_p	Population of infected potato	π_2	Recruit rate of vector
I_v	Population of infected vector	a	Infection rate of potato
α_1	Virus induced death rate	δ	Infection rate of vector
α_2	Elimination rate of infected potato	λ_v	Death of vector by predators
γ_p	Natural death rate of potato		

From **Figure 1** and **Table 1**, system of differential equation are generated.

$$\begin{cases} \frac{dS_p}{dt} = \pi_1 - aS_pI_v - \Upsilon_pS_p, \\ \frac{dI_p}{dt} = aS_pI_v - (\alpha_2 + \alpha_2 + \Upsilon_p)I_p, \\ \frac{dS_v}{dt} = \pi_2 - \delta S_vI_p - (\Upsilon_v + \lambda_v)S_v, \\ \frac{dI_v}{dt} = \delta S_vI_p - (\Upsilon_v + \lambda_v)I_v, \end{cases} \quad (1)$$

with initial value $\boldsymbol{S_p(0) = S_{p_0} \geq 0, I_p(0) = I_{p_0} \geq 0, S_v(0) = S_{v_0} \geq 0, I_v(0) = I_{v_0} \geq 0}$ and $\boldsymbol{\alpha_1, \alpha_2, \gamma_p, \gamma_v, \pi_1, \pi_2, \delta, a, \lambda_v > 0}$.

Furthermore, system **Equation (1)** is converted into a fractional differential equation system for the order $0 < \sigma < 1$ and $n = 1$ using a fractional Caputo derivative [6]. For example, covert the first equation dS_p on system **Equation (1)** into a fractional derivative.

$$D_c^\sigma = \frac{1}{\Gamma(1-\sigma)} \int_0^t (t-u)^{n-\sigma-1} f^{(n)} du,$$
$$D_c^\sigma = \frac{1}{\Gamma(1-\sigma)} \int_0^t (t-u)^{-\sigma} \left(\frac{dS_p}{dt}\right) du = D^\sigma S_p. \quad (2)$$

Moreover, $D^\sigma I_p, D^\sigma S_v,$ and $D^\sigma I_v$ are obtained similarly, so the following fractional differential equation was collected.

$$\begin{cases} D^{\sigma}S_p = \pi_1 - aS_pI_v - \Upsilon_pS_p, \\ D^{\sigma}I_p = aS_pI_v - (\alpha_2 + \alpha_2 + \Upsilon_p)I_p, \\ D^{\sigma}S_v = \pi_2 - \delta S_vI_p - (\Upsilon_v + \lambda_v)S_v, \\ D^{\sigma}I_v = \delta S_vI_p - (\Upsilon_v + \lambda_v)I_v, \end{cases} \tag{3}$$

with order $\mathbf{0 < \sigma < 1}$, initial value $\boldsymbol{S_{p_0} \geq 0, I_{p_0} \geq 0, S_{v_0} \geq 0, I_{v_0} \geq 0}$ and $\boldsymbol{\alpha_1, \alpha_2, \gamma_p, \gamma_v, \pi_1, \pi_2, \delta, a, \lambda_v > 0}$.

3.2 Invariant Region

The following theorem is the feasible solution that satisfies all the constraints and conditions imposed by the given system.

Theorem 1. *The feasible solution set* $\{S_p, I_p, S_v, I_v\}$ *of the system equation of the model is* $\Omega = \left\{(S_p, I_p, S_v, I_v) \in \mathbb{R}_+^4 : \left(0 \leq N_p(t) \leq \frac{\pi_1}{\gamma_p}\right) \cup \left(0 \leq N_v(t) \leq \frac{\pi_2}{(\gamma_v + \lambda_v)}\right);\ N_p = S_p + I_p, N_v = S_v + I_v\right\}$.

Proof. Differentiate N_p and N_v with respect to time by considering fractional order

$$D^{\sigma}N_p = D^{\sigma}(S_p + I_p), \tag{4}$$

$$D^{\sigma}N_v = D^{\sigma}(S_v + I_v). \tag{5}$$

Substitute **Equation (3)** to **Equation (4)** with $\alpha_1 = 0$ and $\alpha_2 = 0$, become

$$\begin{gathered} D^{\sigma}N_p \leq \pi_1 - \gamma_pS_p - \gamma_pI_p, \\ D^{\sigma}N_p \leq \pi_1 - \gamma_pN_p. \end{gathered} \tag{6}$$

Take Laplace transform [7] and substitute initial value, become

$$\begin{gathered} \mathcal{L}\{D^{\sigma}N_p(t)\} + \mathcal{L}\{\gamma_pN_p\} \geq \mathcal{L}\{\pi_1\}, \\ N_p(s) \leq \frac{\pi_1}{s(s^{\sigma} + \gamma_p)}. \end{gathered} \tag{7}$$

Furthermore, find the Laplace inverse transform using Mittag-Leffler function [6], become

$$N_p(t) \leq \frac{\pi_1}{\gamma_p}\left(1 - E_{\sigma}(-\gamma_pt^{\sigma})\right). \tag{8}$$

Let $t \to \infty$ and $\gamma_p > 0$, thus $N_p(t) \to \frac{\pi_1}{\gamma_p} \geq 0$. Therefore we have

$$0 \leq N_p(t) \leq \frac{\pi_1}{\gamma_p}. \tag{9}$$

After re-arranging **Equation (9)**, obtained

$$\Omega_p = \left\{(S_p, I_p) \in \mathbb{R}_+^2 : 0 \leq N_p(t) \leq \frac{\pi_1}{\gamma_p}\right\}. \tag{10}$$

By using the same way for **Equation (5)**, obtained

$$\Omega_v = \left\{(S_v, I_v) \in \mathbb{R}_+^2 : 0 \leq N_v(t) \leq \frac{\pi_2}{(\gamma_v + \lambda_v)}\right\}. \tag{11}$$

In general, **Equation (10)** and **Equation (11)**, invariant region of the system of the model is

$$\Omega = \Omega_p \times \Omega_v = \left\{(S_p, I_p, S_v, I_v) \in \mathbb{R}_+^4 : \left(0 \leq N_p(t) \leq \frac{\pi_1}{\gamma_p}\right) \cup \left(0 \leq N_v(t) \leq \frac{\pi_2}{(\gamma_v + \lambda_v)}\right)\right\}.$$

3.3 Positive Solutions

All solutions of the fractional differential equation model of PLRV spread in potatoes are positive for future time if all the initial values are positive.

Theorem 2. *If* $S_{p_0} \geq 0, I_{p_0} \geq 0, S_{v_0} \geq 0, I_{v_0} \geq 0$ *then all solution sets* $(S_p(t), I_v(t), S_p(t), I_v(t))$ *of the system of the model are positive for the future time.*

Proof. Take the first equation in **Equation (3)** with π_i as positive

$$D^\sigma S_p = \pi_1 - aS_pI_v - \gamma_pS_p,$$
$$D^\sigma S_p \geq -S_p(aI_v + \gamma_p). \tag{12}$$

Using Laplace transform, become

$$\mathcal{L}\{D^\sigma S_p\} \geq -(aI_v + \gamma_p)\mathcal{L}\{S_p\},$$
$$S_p(s) \geq \frac{S_{p_0}}{s^\sigma + (aI_v + \gamma_p)}. \tag{13}$$

Next, find the Laplace inverse transform [8] using Mittag-Leffler function, become

$$S_p(t) \geq S_{p_0}t^{\sigma-1}E_{\sigma,\sigma}\left(-(aI_v + \gamma_p)t^\sigma\right) \geq 0; t > 0.$$

By using the same way, the rest equation on **Equation (3)**, obtained

$$\begin{cases} I_p(t) \geq I_{p_0}t^{\sigma-1}E_{\sigma,\sigma}(-kt^\sigma) \geq 0; t > 0, \\ S_v(t) \geq S_{v_0}t^{\sigma-1}E_{\sigma,\sigma}\left(-\left(\delta I_p + (\gamma_v + \lambda_v)\right)t^\sigma\right) \geq 0; t > 0, \\ I_v(t) \geq I_{v_0}t^{\sigma-1}E_{\sigma,\sigma}(-(\gamma_v + \lambda_v)t^\sigma) \geq 0; t > 0. \end{cases}$$

3.4 Equilibrium point and basic reproduction number

There are two equilibrium points, disease free equilibrium point and endemic equilibrium point associated with basic reproduction number. The disease free equilibrium is the condition with no infection $(I_p = I_v = 0)$. **Equation (3)** has a disease free equilibrium point if it does [9]:

$$D^\sigma S_p = 0, D^\sigma I_v = 0, D^\sigma S_p = 0, D^\sigma I_v.$$

Disease free equilibrium represented as $E^0 = (S_p^0, I_v^0, S_p^0, I_v^0)$, so it become

$$E^0(S_p^0, I_v^0, S_p^0, I_v^0) = \left(\frac{\pi_1}{\gamma_p}, 0, \frac{\pi_2}{(\gamma_v + \lambda_v)}, 0\right).$$

Basic reproduction numbers are obtained by using the next generation matrix method [10].

$$G = FV^{-1}, \tag{14}$$

where F and V are the results of linearization using the Jacobian at the disease free equilibrium point which is $(S_p, I_v, S_p, I_v) = \left(\frac{\pi_1}{\gamma_p}, 0, \frac{\pi_2}{(\gamma_v+\lambda_v)}, 0\right)$,

$F = \begin{bmatrix} 0 & a\frac{\pi_1}{\gamma_p} \\ \delta\frac{\pi_2}{(\gamma_v+\lambda_v)} & 0 \end{bmatrix}$ and $V = \begin{bmatrix} k & 0 \\ 0 & (\gamma_v + \lambda_v) \end{bmatrix}$. The next generation matrix is

$$G = FV^{-1} = \begin{bmatrix} 0 & \frac{a\pi_1}{\gamma_p(\gamma_v + \lambda_v)} \\ \frac{\delta\pi_2}{k(\gamma_v + \lambda_v)} & 0 \end{bmatrix}. \tag{15}$$

The basic reproduction number is the dominant of eigenvalues. Therefore

$$R_0 = \sqrt{\frac{a\pi_1\delta\pi_2}{k\gamma_p(\gamma_v+\lambda_v)^2}}.$$

Furthermore, the endemic equilibrium is the condition with infection $(I_p \neq 0,\ I_v \neq 0)$. Endemic equilibrium represented as $E^* = (S_p^*, I_v^*, S_p^*, I_v^*)$ and it becomes

$$E^* = (S_p^*, I_v^*, S_p^*, I_v^*)$$
$$= \left(\begin{array}{c} \frac{\pi_1}{aI_v^*+\gamma_p}, \frac{a\pi_1 I_v^*}{k(aI_v^*+\gamma_p)}, \\ \frac{\pi_2 k(aI_v^*+\gamma_p)}{aI_v^*(\delta\pi_1+(\gamma_v+\lambda_v)k)+k\gamma_p(\gamma_v+\lambda_v)}, \frac{k(\gamma_v+\lambda_v)\gamma_p(R_0^2-1)}{a(\delta\gamma_1+(\gamma_v+\lambda_v)k)} \end{array}\right).$$

3.5 Stability of equilibrium point

There are two main types of stability associated with equilibrium points: local stability and global stability. Local stability and global stability of the equilibrium points is given in the following theorem.

Theorem 3. *The disease free equilibrium point is locally asymptotical stable if* $R_0 < 1$ *and unstable if* $R_0 > 1$.

Proof. Jacobian matrix of **Equation (1)** at disease free equilibrium point is **[11]**

$$J(E^0) = \begin{bmatrix} -\gamma_p & 0 & 0 & -a\frac{\pi_1}{\gamma_p} \\ 0 & -k & 0 & a\frac{\pi_1}{\gamma_p} \\ 0 & -\delta\frac{\pi_2}{(\gamma_v+\lambda_v)} & -(\gamma_v+\lambda_v) & 0 \\ 0 & \delta\frac{\pi_2}{(\gamma_v+\lambda_v)} & 0 & -(\gamma_v+\lambda_v) \end{bmatrix}. \tag{16}$$

Eigen values of **Equation (16)** is $\lambda_1 = -\gamma_p \leq 0, \lambda_2 = -(\gamma_v+\lambda_v) \leq 0$ and polynomial equation

$$\lambda^2 + \underbrace{((\gamma_v+\lambda_v)+k)}_{b_1} + \underbrace{\left(k(\gamma_v+\lambda_v) - \frac{a\pi_1\delta\pi_2}{\gamma_p(\gamma_v+\lambda_v)}\right)}_{b_2} = 0.$$

According to Routh-Hurtwitz criteria **[12]**, a polynomial equation has a negative real root if and only if $b_1 > 0$ and $b_2 > 0$. It is clearly $b_1 > 0$ because it is the sum of positive parameters. But $b_2 > 0$ if

$$\begin{aligned} b_2 = (\gamma_v+\lambda_v)k - \frac{a\pi_1\delta\pi_2}{\gamma_p(\gamma_v+\lambda_v)} &> 0, \\ (\gamma_v+\lambda_v)k &> \frac{a\pi_1\delta\pi_2}{\gamma_p(\gamma_v+\lambda_v)}, \\ 1 &> \frac{a\pi_1\delta\pi_2}{k\gamma_p(\gamma_v+\lambda_v)^2}, \end{aligned} \tag{17}$$

Equation (17) shows that

$$\frac{a\pi_1\delta\pi_2}{\gamma_p k(\gamma_v+\lambda_v)^2} < 1. \tag{18}$$

Let $R_0 = \sqrt{\frac{a\pi_1\delta\pi_2}{\gamma_p(\gamma_v+\lambda_v)^2 k}}$, then **Equation (18)** shows $R_0^2 < 1$, it is means $R_0 < 1$. Thus, the disease free equilibrium point is locally asymptotically stable if $R_0 < 1$.

Theorem 4. *The disease free equilibrium point is locally asymptotical stable if all the eigenvalues of the* $J(E^0)$ *satisfy* $|arg(\lambda_i)| > \frac{\sigma\pi}{2}$ *where* $i = 1, 2, 3, 4$.

Proof. In the sub-section before, the eigen values are

1. $\lambda_1 = -\gamma_p.$

2. $\lambda_3 = -(\gamma_v + \lambda_v)$.
3. λ_2 and λ_4 is.

$$\lambda^2 + \underbrace{((\gamma_v + \lambda) + k)}_{b_1} + \underbrace{\left(k(\gamma_v + \lambda_v) - \frac{a\pi_1\delta\pi_2}{\gamma_p(\gamma_v + \lambda_v)}\right)}_{b_2} = 0. \tag{19}$$

Because γ_p, γ_v and λ_v are a positive parameter, then $\lambda_1 < 0$ and $\lambda_3 < 0$, thus satisfying $|arg(\lambda_i)| = \pi > \frac{\sigma\pi}{2}$ for any $0 < \sigma \leq 1$ [13].

Previously, it has shown that $b_1, b_2 > 0$ if $R_0 < 1$, thus polynomial **Equation (19)** has the negative real root and satisfies $|arg(\lambda_i)| = \pi > \frac{\sigma\pi}{2}$ for any $0 < \sigma \leq 1$. Thus, disease free equilibrium point is locally asymptotically stable if $R_0 < 1$.

Theorem 5. *The disease free equilibrium point is globally stable if* $R_0 < 1$.

Proof. Consider Lyapunov function

$$V(S_p, I_p, S_v, I_v) = \left(S_p - S_p^0 - S_p^0\, ln\left(\frac{S_p}{S_p^0}\right)\right) + \left(I_p - I_p^0 - I_p^0\, ln\left(\frac{I_p}{I_p^0}\right)\right) + \left(S_v - S_v^0 - S_v^0\, ln\left(\frac{S_v}{S_v^0}\right)\right) + \left(I_v - I_v^0 - I_v^0\, ln\left(\frac{I_v}{I_v^0}\right)\right) \tag{20}$$

Thus $V > 0$ for all $(S_p, I_p, S_v, I_v) \neq (S_p^0, I_p^0, S_v^0, I_v^0)$ and $V = 0$ if and only if $(S_p, I_p, S_v, I_v) = (S_p^0, I_p^0, S_v^0, I_v^0)$. Next, calculate $V^\sigma(S_p, I_p, S_v, I_v)$ to show $D^\sigma V \leq 0$ at disease free equilibirum point [14].

$$D^\sigma V \leq \left(\frac{S_p - S_p^0}{S_p}\right) D^\sigma S_p + \left(\frac{I_p - I_p^0}{I_p}\right) D^\sigma I_p + \left(\frac{S_v - S_v^0}{S_v}\right) D^\sigma S_v + \left(\frac{I_v - I_v^0}{I_v}\right) D^\sigma I_v. \tag{21}$$

By Equilibrium condition, $\pi_1 = S_p^0\gamma_p, \pi_1 = S_v^0(\gamma_v + \lambda_v), S_p^0 = \frac{\pi_2}{\gamma_p}$ and $S_v^0 = \frac{\pi_2}{(\gamma_v + \lambda_v)}$, **Equation (21)** become

$$D^\sigma V \leq -\gamma_p \left(\frac{S_p - \frac{\pi_1}{\gamma_p}}{S_p}\right)^2 - (\gamma_v + \lambda_v)\left(\frac{S_v - \frac{\pi_2}{(\gamma_v + \lambda_v)}}{S_v}\right)^2 + I_v(\gamma_v + \lambda_v)(R_0^2 - 1). \tag{22}$$

Equation (22) shows that $D^\sigma V \leq 0$ if $R_0 < 1$ for all $(S_p, I_p, S_v, I_v) \in \mathbb{R}_+^4$ and $D^\sigma V = 0$ if and only if $S_p = S_p^0, I_p = I_p^0, S_v = S_v^0, I_v = I_v^0$. Therefore, the disease free equilibrium point is globally stable if $R_0 < 1$.

Theorem 6. *Let* $\sigma \in (0,1]$ *and* $R_0 > 1$. *Then the endemic equilibrium of the fractional order model is globally stable in the interior of* Ω.

Proof. Consider Lyapunov function

$$V(S_p, I_v, S_p, I_v) = \left(S_p - S_p^* - S_p^*\, ln\left(\frac{S_p}{S_p^*}\right)\right) + \left(I_p - I_p^* - I_p^*\, ln\left(\frac{I_p}{I_p^*}\right)\right) + \left(S_v - S_v^* - S_v^*\, ln\left(\frac{S_v}{S_v^*}\right)\right) + \left(I_v - I_v^* - I_v^*\, ln\left(\frac{I_v}{I_v^*}\right)\right) \tag{23}$$

Function V is defined as continuous and positive definite for all $S_p > 0, I_p > 0, S_v > 0, I_v > 0$.
Next, calculate $V^\sigma(S_p, I_p, S_v, I_v)$ to shows $D^\sigma V \leq 0$ at the endemic equilibrium point [14].

$$D^\sigma V \leq \left(\frac{S_p - S_p^*}{S_p}\right) D^\sigma S_p + \left(\frac{I_p - I_p^*}{I_p}\right) D^\sigma I_p + \left(\frac{S_v - S_v^*}{S_v}\right) D^\sigma S_v + \left(\frac{I_v - I_v^*}{I_v}\right) D^\sigma I_v \tag{24}$$

By equilibrium condition $\pi_1 = S_p^* a I_p^* + S_p^*\gamma_p, \pi_2 = S_v^*\delta I_p^* + S_v^*(\gamma_v + \lambda_v)$ and let $I_p^* = S_p^* a I_v^*$ and $I_v^*(\gamma_v + \lambda_v) = S_v^*\delta I_p^*$. **Equation (24)** becomes

$$\begin{aligned} D^{\sigma}V \leq & -\gamma_p\left(\frac{S_p - S_p^*}{S_p}\right)^2 - kI_p^*\left(\frac{S_p^*}{S_p} - 2\right) - kI_p - aI_v\left(\frac{I_p^* S_p}{I_p} - S_p^*\right) \\ & - (\gamma_v + \lambda_v)\left(\frac{S_v - S_v^*}{S_v}\right)^2 - I_v(\gamma_v + \lambda_v)\left(\frac{S_v^*}{S_v} - 2\right) - I_v(\gamma_v + \lambda_v) \\ & - \delta I_p\left(\frac{I_v^* S_v}{I_v} - S_v^*\right). \end{aligned} \tag{25}$$

Equation (25) shows that $D^{\sigma}V \leq 0$ and $D^{\sigma}V = 0$ if and only if $S_p = S_p^*, I_p = I_p^*, S_v = S_v^*, I_v = I_v^*$. Therefore, $D^{\sigma}V = 0$ for all $(S_p^*, I_p^*, S_v^*, I_v^*) \in \Omega$ is endemic equilibrium, this shows that endemic equilibrium is globally assymptotically stable in Ω.

3.6 Numerical Solution

The numerical solutions were calculated using initial values from the fractional differential equation model for the spread of PLRV on potatoes.

Table 2. Parameter Values for PLRV Model.

Parameters	Values	Source	Parameters	Values	Source
S_{p_0}	600	[15	I_{p_0}	200	[15]
S_{v_0}	100	[15]	I_{v_0}	10	[15]
π_1	0.8	[15]	π_2	0.19	[15]
a	0.00022	[15]	δ	0.0025	[15]
α_1	0.033	[15]	α_2	0.01	[15]
γ_v	0.0028	[15]	γ_p	0.04	[15]
λ_v	0.003	Asummed			

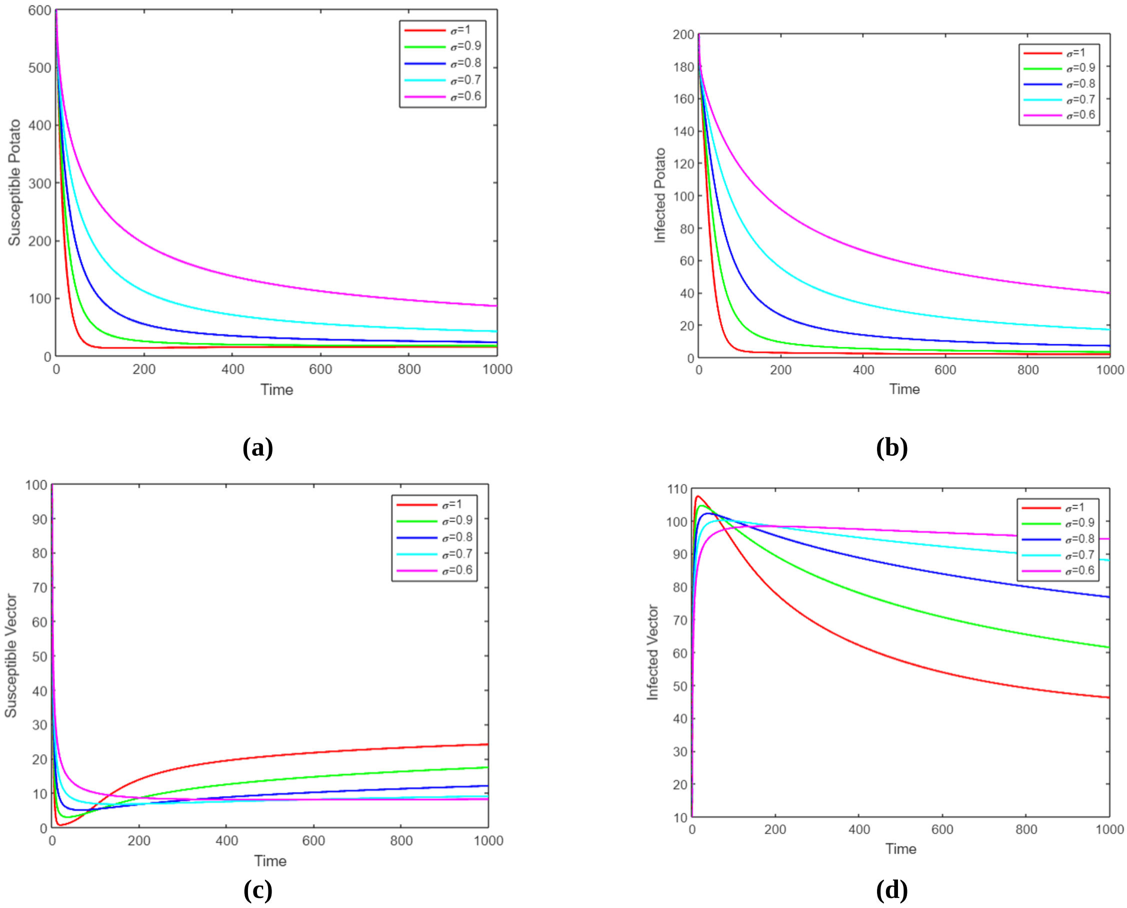

Figure 2. Numerical Solution of Endemic PLRV Condition, (a) Endemic PLRV on Susceptible Potato, (b) Endemic PLRV on Infected Potato, (c) Endemic PLRV on Susceptible Vector, (d) Endemic PLRV on infected vector

Figure 2 shows that fractional order which is σ related to how fast the system is heading towards an endemic PLRV point on potatoes, that is $E^* = (16.257, 1.804, 67.857, 41.856)$. Figure 2 also shows that the susceptible potato population and infected potato population is decreasing steadily. The susceptible vector population gets an extreme decrease at initial time and then increase of susceptible vector population follows. Otherwise, infected vector population gets an extreme increase at initial time and then the infected vector population getting decrease in future time.

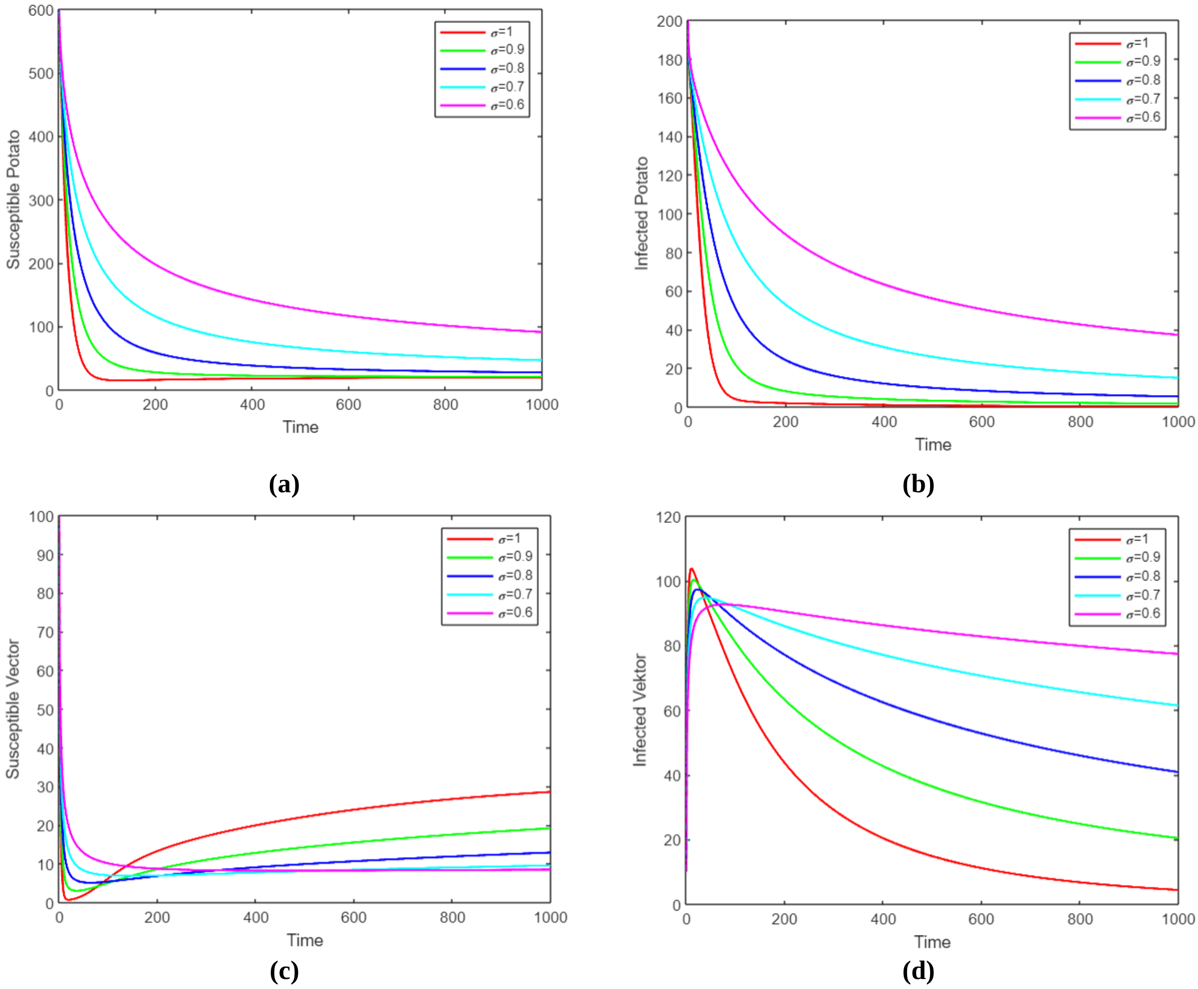

Figure 3. Numerical solution of free-PLRV condition, (a) Free-PLRV on susceptible potato, (b) Free-PLRV foninfected potato, (c) Free-PLRV on susceptible vector, (d) Free-PLRV on infected vector

Figure 3 shows that fractional order which is σ related to how fast the system is heading towards a free PLRV point on potatoes, that is $E^0 = (20, 0, 32.7586, 0)$. Fractional order σ changes will affect the complexity interaction in the system. Figure 3 also shows that susceptible potato population and infected potato population is getting decreasing steadily. Susceptible vector population getting extreme decrease at an initial time and then increase of susceptible vector population followed. Otherwise, infected vector population gets an extreme increase at initial time and then in future time, infected vector population gets decrease faster than the endemic PLRV condition on potatoes.

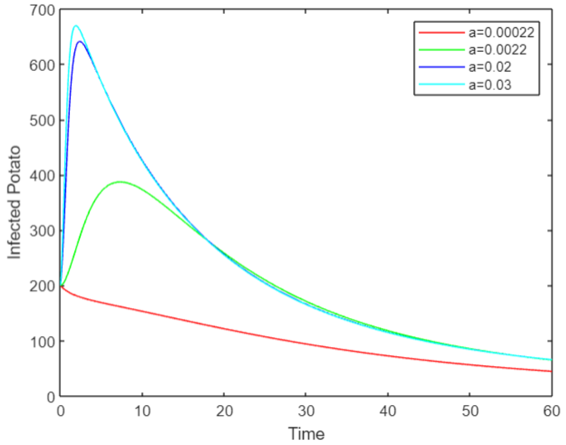

Figure 4. Numerical Solution of Variation of a on PLRV model

Figure 4 shows that as the infection rate of potato (a) rises, so does the infected potato population. It means, the virus will spread faster as the infection rate of potato increases.

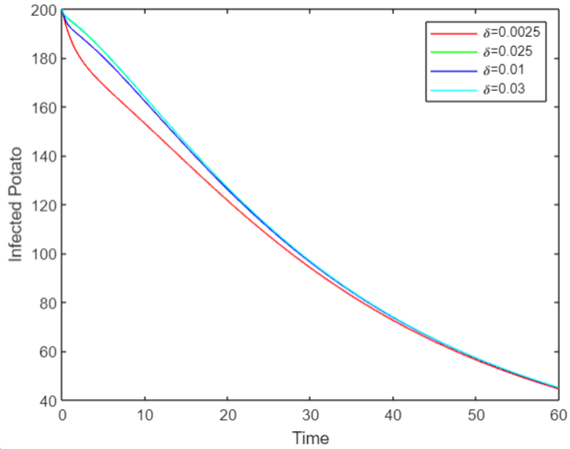

Figure 5. Numerical Solution Of Variation of δ on PLRV model

Figure 5 shows that as infection rate of the vectors (δ) rises, so does the infected potato population.

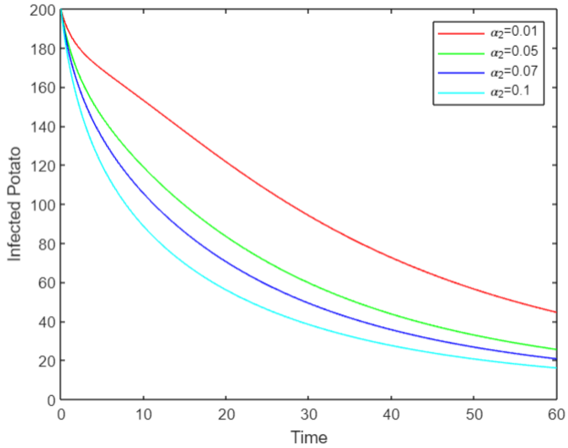

Figure 6. Numerical Solution Of Variation of α_2 on PLRV model

Figure 6 shows that when the elimination rate of infected potato (α_2) rises, the infected potato population is reduced. It means, increasing the elimination rate of infected potato can be control the spread of PLRV.

CONCLUSIONS

Based on the numerical solution, we conclude that:

1. Endemic conditions of PLRV on potatoes are obtained at the rate of death vectors by predators being ignored ($\lambda_v = 0$) and $R_0 = 1.7922 > 1$.
2. PLRV-free conditions on potatoes are obtained at the rate of death vectors by predators (λ_v) is 0.03 and $R_0 = 0.8652 < 1$.
3. In the absence of PLRV, the rate of potato infection (a) and the rate of the infected potato elimination (α_2) can be better controlled the spread of the PLRV on potatoes by reducing the rate of potato infection (a) and increasing the pace of the elimination of the infected potato (α_2).

REFERENCES

[1] K. Ummah and A. Purwito, "Budidaya Tanaman Kentang (Solanum tuberosum L.) dengan Aspek Khusus Pembibitan di Hiikmah," *IPB (Bogor Agric. Univeristy)*, 2009.

[2] I. A. Astarini, D. Margareth, and I. G. R. Maya Temaja, "*In Vivo* Thermoterapy: Attempt to Eliminate Virus in Potato Tuber," *IOP Conf. Ser. Earth Environ. Sci.*, vol. 130, p. 012021, Mar. 2018, doi: 10.1088/1755-1315/130/1/012021.

[3] D. Moller, "Primary and Secondary Infection of Potatoes," *Int. J.*, vol. 8, 2008.

[4] J. Joel Mapinda, G. Godson Mwanga, and V. Grace Masanja, "Modelling the Transmission Dynamics of Banana Xanthomonas Wilt Disease with Contaminated Soil," *J. Math. Informatics*, vol. 17, pp. 113–129, May 2019, doi: 10.22457/jmi.146av17a11.

[5] Z. U. A. Zafar, Z. Shah, N. Ali, E. O. Alzahrani, and M. Shutaywi, "Mathematical and Stability Analysis of Fractional Order Model for Spread of Pests in Tea Plants," *Fractals*, vol. 29, no. 01, p. 2150008, Feb. 2021, doi: 10.1142/S0218348X21500080.

[6] C. Milici, G. Drăgănescu, and J. Tenreiro Machado, *Introduction to Fractional Differential Equations*, vol. 25. Cham: Springer International Publishing, 2019. doi: 10.1007/978-3-030-00895-6.

[7] M. R. Spiegel, *Transformasi Laplace Terjemahan Bahasa Indonesia*. Jakarta: Erlangga, 1999.

[8] S. L. Ross, *Differential Equations*, 3rd ed. New York: John Willey & Sons, 2010.

[9] C. H. Edwards and D. E. Pennery, *Elementary Differential Equations with Boundary Value Problems*, 4th ed. New Jersey: Prentice Hall, 2002.

[10] P. van den Driessche and J. Watmough, "Reproduction Numbers and Sub-threshold Endemic Equilibria for Compartmental Models of Disease Transmission," *Math. Biosci.*, vol. 180, no. 1–2, pp. 29–48, Nov. 2002, doi: 10.1016/S0025-5564(02)00108-6.

[11] J. K. Hale and H. Koçak, *Dynamics and Bifurcations*, vol. 3. New York, NY: Springer New York, 1991. doi: 10.1007/978-1-4612-4426-4.

[12] G. J. Olsder and J. W. van der. Woude, *Mathematical Systems Theory*. Netherlands: Delft University Press, 2004.

[13] K. Diethelm, *The Analysis of Fractional Differential Equations*. New York: Springer, 2004.

[14] H. Khalil, *Nonlinear System*, 3rd ed. New Jersey: Pearson Prentice Hall, 2002.

[15] G. T. Tilahun, G. A. Wolle, and M. Tofik, "Eco-epidemiological Model and Analysis of Potato Leaf Roll Virus Using Fractional Differential Equation," *Arab J. Basic Appl. Sci.*, vol. 28, no. 1, pp. 41–50, Jan. 2021, doi: 10.1080/25765299.2020.1865621.

[16] H. Ahmad, T. A. Khan, I. Ahmad, I, P. S. Stanimirovic, and Y. M. Chu, "A New Analyzing Technique for Nonlinear Time Fractional Cauchy Reaction-Diffusion Model Equations," *Results in Physics*, vol. 19, 103462, 2020, doi:10.1016/j.rinp.2020.103462.

[17] J. Singh, "Analysis of fractional blood alcohol model with composite fractional derivative," Chaos, Solitons & Fractals, 140, 110127, 2020, doi : 10.1016/j.chaos.2020.110127, doi: 10.1080/25765299.2020.1865621.

[18] Y. Ali, A. Raza, H. M. Aatif, M. Ijaz, S. Ul-Allah, S. ur Rehman, S. Y. M. Mahmoud, E. S. H. Farrag, M. A. Amer, and M. Moustafa, "Regression Modeling Strategies to Predict and Manage Potato Leaf Roll Virus Disease Incidence and Its Vector ," *Agriculture*, 12, 550, 2022, doi: 10.3390/agriculture12040550.

[19] E. Bonyah, "A Fractional Dynamics of a Potato Disease Model," *Commun. Math. Biol. Neorosci*, ID 88, 2022, doi: 10.28919/cmbn/7599.