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Enhancing Class IX Students' Problem-Solving Skills Through the RME Approach in Reflection Mathematics Material

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Abstract

This study aims to analyze students' problem-solving abilities after participating in learning using the *Realistic Mathematics Education* (RME) approach based on ethnomathematics in the subject of reflection. The research method used is design research with research instruments in the form of *Hypothetical Learning Trajectory* (HLT), Student Activity Sheets (LKPD), written test sheets, and interview guidelines. The data obtained were analyzed using data analysis techniques according to Miles and Huberman, namely: reducing data, presenting data, and drawing conclusion. The written tests used to measure students' problem-solving abilities consisted of two types of questions based on Polya's problem-solving indicators and were given after the learning process. The results showed that after participating in learning using the ethnomathematics-based RME approach, students began to demonstrate the ability to review their answers and understand problems through a reflective context related to the culture of Lamalera ikat weaving. However, students still had difficulty formulating strategies and implementing systematic problem-solving plans. These findings indicate that the ethnomathematics-based RME approach has the potential to foster reflective and contextual thinking skills in mathematics, although students still need more time and practice to optimally develop their problem-solving skills.

Keywords: Ethnomathematics; Hypothetical learning trajectory; Lamalera kwatek ikat woven fabric; Realistic mathematics learning; Problem solving ability.

1. Introduction

Mathematics is a fundamental discipline that plays a crucial role in everyday life and in the development of higher-order thinking skills (Indirwan et al., 2021; Juana et al., 2022). One of the primary objectives of mathematics education is to foster students' problem-solving abilities, as these enable learners to apply mathematical concepts to non-routine situations, identify patterns, generalize results, and communicate ideas effectively (Indriana & Maryati, 2021). Problem solving is widely recognized as a complex cognitive process involving understanding the problem, devising a plan, implementing the strategy, and evaluating the solution (Anwar et al., 2023; Erbilgin & Macur, 2022; NCTM, 2000).

Polya (1973) conceptualizes problem solving into four systematic stages: understanding the problem, planning the solution, carrying out the plan, and looking back. These stages require students to engage in analytical and reflective thinking. However, classroom observations reveal that students still encounter difficulties in executing these processes, particularly in understanding problems and providing logical justifications in geometry reflection topics. This indicates a discrepancy between theoretical expectations and students' actual problem-solving performance.

A substantial body of research on Realistic Mathematics Education (RME) has demonstrated its effectiveness in enhancing students' problem-solving abilities. RME emphasizes mathematics as a human activity and situates learning within meaningful real-life contexts (Pambudi et al., 2022; Wulandari & Sulasmono, 2020). Empirical studies have reported significant improvements in students' problem-solving performance following the implementation of RME (Andzin et al., 2024; Susilawati et al., 2025), with positive effects observed across different educational levels (Rahman & Setyaningsih, 2022; Tantra et al., 2022). However, prior studies have predominantly focused on general learning outcomes and have not sufficiently examined how RME can be systematically integrated with culturally grounded contexts within a structured learning trajectory, particularly in the domain of geometric transformations.

In addition, research in ethnomathematics has emphasized the role of cultural contexts in making mathematics learning more meaningful and relevant. Cultural artifacts and practices can serve as authentic entry points for mathematical understanding (Rahmawati & Hidayah, 2023). Nevertheless, most ethnomathematics studies tend to position cultural elements as supplementary teaching resources rather than as central components of instructional design. Moreover, there is limited research that explicitly integrates ethnomathematical contexts within an RME framework, especially using local cultural artifacts from Eastern Indonesia in geometry learning.

Furthermore, studies on mathematical problem solving based on Polya's framework have provided valuable insights into students' cognitive processes. Problem solving is viewed as an activity that integrates prior knowledge with new situations and requires reflective reasoning (Aljura et al., 2025). While Polya's stages are widely used as analytical tools, few studies have incorporated Polya-based indicators within RME learning environments that are enriched by cultural contexts. Consequently, the interplay between RME, ethnomathematics, and Polya's problem-solving framework remains underexplored.

Although previous studies have demonstrated the effectiveness of the Realistic Mathematics Education (RME) approach, several critical gaps remain. First, an empirical gap exists, as only a limited number of studies have specifically examined the application of RME in geometry reflection topics using culturally grounded contexts, with most prior research focusing on arithmetic or general problem-solving domains rather than transformation geometry. Second, a theoretical gap is evident, as previous studies have not sufficiently integrated RME with Polya's problem-solving framework to systematically analyze students' cognitive processes, leaving the relationship between instructional design and problem-solving stages underexplored. Third, a contextual gap persists, particularly in the underrepresentation of local cultural contexts from Eastern Indonesia, where cultural elements from regions such as Lamalera have rarely been incorporated into mathematics learning as a central component of instructional design.

In response to these gaps, this study employs the cultural context of *kwatek ikat* woven fabric from Lamalera, a traditional textile associated with ceremonial and social practices, characterized by distinctive motifs such as whales, stingrays, and *pledang* (traditional boats), which reflect the maritime identity of the Lamalera community. This study integrates the cultural context as a meaningful starting point within an RME-based learning trajectory to support students' understanding of geometric reflection concepts while simultaneously analyzing their problem-solving processes based on Polya's framework.

The novelty of this study lies in its integrative approach, which combines (1) a culturally grounded context positioned as the core component of a Hypothetical Learning Trajectory (HLT) within RME, (2) the systematic application of Polya's problem-solving framework as an analytical lens, and (3) a specific focus on geometric reflection, a topic that remains underexplored in prior RME–ethnomathematics research. Thus, this study contributes theoretically by bridging the gap between instructional design (RME) and cognitive analysis (Polya), empirically by extending RME applications to geometry reflection, and contextually by incorporating an underrepresented cultural setting from Eastern Indonesia into mathematics learning.

The indicators of students' problem-solving abilities in this study are adapted from Polya's stages: (1) understanding the problem by identifying known and unknown information; (2) devising a solution plan; (3) carrying out the plan; and (4) reflecting on and evaluating the solution. Preliminary findings indicate that students experience difficulties in understanding reflection concepts, identifying transformation results, and providing logical reasoning. This highlights the need for a contextual and meaningful instructional approach.

Therefore, this study aims to investigate students' mathematical problem-solving abilities after participating in RME-based learning integrated with the cultural context of Lamalera *kwatek ikat* woven fabric. Furthermore, this study contributes to the existing body of knowledge by providing empirical evidence on the application of culturally grounded RME in geometry reflection, while also offering a theoretical contribution by integrating Polya's problem-solving framework to analyze students' cognitive processes within a structured learning trajectory.

2. Method

The research method used in this study is design research. According to Putrawangsa (Putrawangsa, 2018) design research aims to find effective solutions to complex problems, especially in the field of education. The model used in this study refers to Gravemeijer and Cobb design research model, which consists of three stages, namely: preliminary design, experiment, and retrospective analysis (Indartiningsih et al., 2024). At the preliminary design stage was conducted prior to classroom instruction. At this stage, the researcher developed a Hypothetical Learning Trajectory (HLT) that included learning objectives, learning activity designs, and assumptions about students' thinking processes. The researcher also analyzed students' initial understanding of the concept of reflection and validated learning instruments such as worksheets, test sheets, and interview guidelines. After the HLT is completed, researchers entered the next stage, namely the HLT implementation stage. In the experiment stage was carried out after the preliminary design was completed. This stage involved the implementation of the learning design in class IXA over four meetings, each lasting 80 minutes. Students participated in learning activities using the Realistic Mathematics Education (RME) approach based on ethnomathematics using the context of Lamalera *kwatek* ikat woven fabric. Data were collected through classroom observation, student worksheets, written tests, and interviews. In the retrospective analysis stage was conducted after the teaching experiment had been completed. In this stage, the collected test and interview data were analyzed using the Miles and Huberman framework, which includes data reduction, data presentation, and conclusion drawing to evaluate the extent to which the learning design supports the development of students' problem-solving skills.

To ensure the quality of the research instruments, all components of the Hypothetical Learning Trajectory (HLT) which includes learning objectives, activity design, assumptions about student thinking processes, worksheets, written tests, and interview guidelines, were validated through expert judgment by mathematics education lecturers. The validators assessed the content appropriateness, clarity of the instruments, cultural relevance, and the instruments' relevance to Polya's problem-solving indicators. Revisions were made until all instruments were deemed valid for use in the classroom. Research ethics approval was obtained from the university's Ethics Committee, and permission to conduct the research was granted by the school. In addition, the supervising lecturer helped review the test scores to ensure consistency in the use of the rubric and supported the proportional selection of interview subjects.

The research subjects were 28 students in class IX A at SMPK Sta. Theresia. The data collection methods used were written tests and interviews. The written test consisted of two open-ended questions developed based on Polya's problem-solving ability indicators and contextualized with the Lamalera *kwatek* ikat weaving culture. Semi-structured interviews were conducted to explore the students' thinking processes, clarify their written answers, and understand their problem-solving abilities after participating in RME approach.

The written test consisted of two open-ended questions designed to explore students' problem-solving processes in depth. Only two questions were selected because each question required step-by-step reasoning according to Polya's four stages and used the cultural context of Lamalera kwatek ikat weaving, which encouraged students to provide long and analytical answers. Despite the limited number of questions, scoring consistency was maintained through the use of a scoring rubric that had been validated by experts and through double-checking of scores with the supervising lecturer. The continuity of the analysis was also strengthened through data triangulation using interviews to further understand the students' thought processes. This combination of procedures ensured the credibility of the data and the accuracy of the analysis process in this study.

After obtaining the students' written test results from both the pre-test and post-test, the researchers conducted a quantitative analysis to examine the improvement in students' mathematical problem-solving abilities. The analysis began by calculating the mean score ($\bar{X}$) and standard deviation (S) for both pre-test and post-test results.

To measure students' learning improvement, the normalized gain ($N - gain$) was calculated using the following formula:

$$N - gain = \frac{post\ test\ score - Pre\ test\ score}{Maximum\ score - pretest\ score}$$

The$N - gain$ values were then interpreted to determine the level of improvement in students' problem-solving abilities (low, medium, or high).

Furthermore, students' problem-solving abilities were classified into three categories—high, medium, and low—based on their post-test scores. This classification was determined using the mean and standard deviation through the following steps: (1) calculating the mean score ($\bar{X}$), (2) calculating the standard deviation (S), and (3) determining the category boundaries as presented in Table 1.

Table 1. Student Categories Based on Written Test Results (Arikunto, 2010)

Student Group	Student Category
High	$X \geq \bar{X} + S$
Medium	$\bar{X} - S < X < \bar{X} + S$
Low	$X \leq \bar{X} - S$

Description: X = Student Scores, $\bar{X}$ = Mean Scores, S = Standard Deviation

To strengthen the quantitative analysis, the distribution of students across categories and the extent of their improvement were analyzed using descriptive statistics and presented through tables and bar charts comparing pre-test and post-test scores. After grouping students based on their post-test problem-solving scores, the researcher selected interview participants proportionally from each category (high, medium, and low) to ensure representative qualitative data.

3. Results and Discussion

3.1 Results

The results of the study indicate that learning using a *Hypothetical Learning Trajectory* (HLT) based on the cultural context of Lamalera *kwatek ikat* woven fabric was implemented with 28 ninth-grade students who were divided into seven groups.

Based on the written test results, pre-test and post-test score data were obtained to measure students' mathematical problem-solving abilities. The mean pre-test score was 45.46 with a standard deviation of 14.90, while the mean post-test score increased to 58.91 with a standard deviation of 15.09. Thus, a mean difference of 13.45 was obtained, indicating an increase in scores after the learning process. The distribution of pre-test and post-test scores, as well as the level of improvement, is presented in tables and visualized using bar charts to illustrate the comparison of learning outcomes before and after the implementation of the learning process.

Table 2. Pre-test and Post-test results

Jenis Tes	Rata-rata	Simpangan Baku
Pre-test	45,46	14,90
Post-test	58,91	15,09

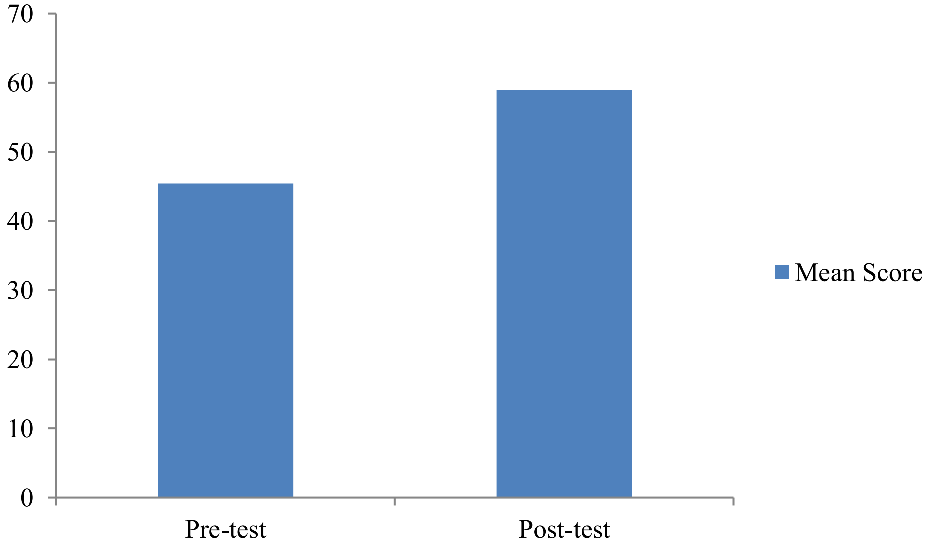

Figure 1. Pre-test and Post-test results

Table 2 presents a comparison of the mean scores and standard deviations between the pre-test and post-test, while Figure 1 illustrates the increase in students' mean scores through a bar chart visualization. To quantitatively measure the level of improvement, the *normalized gain* (N-gain) was calculated. The results showed that the average N-gain was 0.25, which falls into the low category.

3.1.1 Premilinary Design

At this stage, the researcher designed a Hypothetical Learning Trajectory (HLT) consisting of subject identity, class, core and basic competencies (KI and KD), learning objectives, contextual problems based on the Lamalera kwatek ikat woven fabric, and hypothesized student learning activities aligned with the principles of the Realistic Mathematics Education (RME) approach.

The HLT was implemented in five face-to-face meetings, each consisting of two lesson hours. The learning objectives for each meeting are summarized in Table 3.

Table 3. Learning Objectives for Each Meeting

Meeting to-	Learning Objectives
1	a. Students are able to define the concept of reflection. b. Students are able to find the properties of reflection.
2	Students are able to discover the concept of reflection on $x -$ axis dan $y -$ axis
3	Students are able to find the concept of reflection on the $x = k$ line and on the $y = h$ line.
4	Students are able to find the concept of reflection on the $y = x$ line and on the $y = -x$ line.
5	Students conduct an evaluation to measure the level of problem solving ability.

In each meeting, students were given contextual problems designed to stimulate idea development and support the achievement of learning objectives.

To provide a clearer synthesis of the learning design, the sequence of activities and contexts used in each meeting is presented in Table 4.

Table 4. Summary of Learning Activities in Each Meeting

Meeting	Context Used	Main Activities	Expected Concept
1	Ink experiment and woven fabric motifs	Folding paper, observing symmetry, relating to motifs	Concept and properties of reflection
2	Fabric folding and coordinate plane	Vertical and horizontal folding, plotting points	Reflection on $X - axis$ and $Y - axis$
3	Whale and pledang motifs	Reflection on shifted lines	Reflection on $x = k$, $y = h$
4	Stingray and pledang motifs	Reflection on diagonal lines	Reflection on $y = x$, $y = -x$
5	Written test	Problem-solving evaluation	Measurement of ability

In the first meeting, students engaged in a concrete activity using ink and paper folding to explore reflection informally. They observed similarities in shape, size, position, and distance from the fold line without prior introduction to formal terminology. This understanding was then connected to woven fabric motifs, allowing students to contextualize reflection within their cultural environment.

In the second meeting, students began to connect folding activities with formal representations on the Cartesian coordinate plane. Vertical and horizontal folds were interpreted as reflections on the $x - axis$ and $y - axis$, respectively, enabling students to determine image points systematically.

In the third and fourth meetings, learning progressed toward more formal and generalized concepts. Students explored reflections on lines parallel to the axes ($x = k$, $y = h$) and later on diagonal lines ($y = x$, $y = -x$). These activities emphasized the relationship between point coordinates and the characteristics of the mirror line, leading to a more structured and formal understanding of reflection.

In the fifth meeting, a written test was conducted to evaluate students' problem-solving abilities after participating in the learning process.

Overall, the learning trajectory was designed progressively, moving from concrete experiences to abstract mathematical reasoning while maintaining the cultural context as a meaningful representation.

To better illustrate the differences in students' understanding levels, the development of solution strategies is summarized in Table 5.

Table 5. Comparison of Students' Strategies

Level	Characteristics	Strategy Development
High	Consistent and integrated understanding	From visual observation → analogy → spatial reasoning → coordinate transformation
Medium	Partial understanding, procedural	Visual imitation → rule memorization → inconsistent application
Low	Fragmented understanding	Surface observation → guessing → inconsistent or incorrect strategies

Students with high understanding demonstrated a consistent and holistic learning progression. Initially, they relied on direct observation and visual comparison during the folding activity. This strategy then developed into analogy when connecting ink experiments with woven fabric motifs. In subsequent meetings, their strategies evolved into spatial reasoning and finally into formal coordinate transformation. These students were able to systematically determine reflection results using distance relationships and coordinate rules.

Students with intermediate understanding were able to follow the learning trajectory but showed incomplete conceptual understanding. Their strategies were dominated by imitation and procedural approaches. Although they could apply reflection rules, errors frequently occurred due to inconsistent understanding of concepts such as axis direction and distance from mirror lines.

Students with low understanding experienced difficulties throughout the learning process. Their strategies remained at the level of surface observation and guessing. They struggled to connect concrete activities with formal representations, resulting in inconsistent and incorrect reflection results.

The series of problems given at each meeting are arranged in a connected and gradual manner, starting from concrete experiences to formal mathematical understanding, with clear learning objectives at each stage. In solving problems in the first and second meetings, students are still guided to use contextual motifs as a means of solving problems. In the first meeting, students are focused on discovering the concept and properties of reflection through ink experiments on paper as a concrete folding activity. Through this activity, students can directly observe the similarities in shape, size, position, and distance of the dots to the fold line without first being introduced to the formal term of reflection. The intuitive understanding gained was then transferred to the problem of motifs on kwatek woven fabric, where ink dots were represented as fabric motifs and paper fold lines were replaced by stitch lines or fabric folds. Thus, students strengthened their understanding of the concept and properties of reflection in a cultural context close to their lives. Next, in the second meeting, students were presented with the problem of designing a whale motif based on vertical and horizontal fabric folds, so that students began to understand the relationship between the direction of the folds and the position of the shadows. This understanding is then formalized through the placement of the pledang and stingray motifs on the Cartesian coordinate plane, so that the process of folding fabric is interpreted as a reflection on the $Y - axis$ (vertical folds) and the $X - axis$ (horizontal folds), and students are able to determine the shadows of the points systematically.

3.1.2 Experiment

At this stage, the researchers tested the learning trajectory with students of class IX A. In the first meeting until the fourth meeting, learning will be carried out in the form of groups with many groups of students there are seven groups and each group consists of four students. Activities in this trial follow the following RME teaching steps: understanding the real problem, explaining the real problem, solving the real problem, comparing and discussing answers, and concluding.

3.1.3 Retrospective Analysis

The following describes the results of analyzing students' problem solving ability for written tests after experiencing the learning process using the RME approach:

a. Indicators of Understanding the Problem

The written test results show that 60.71% of students were able to write down the known and unknown elements in the first question, but only 35.71% were able to do so in the second question. On average, only 48.21% of students were able to understand the problem correctly. This difficulty shows that when the context of the question changed from a concrete situation (folds in a piece of fabric) to a symbolic representation (coordinate plane), some students failed to identify the essential information in the problem.

Some error patterns that emerged at this stage indicate fundamental difficulties in students' conceptual understanding. Several students made errors in identifying known elements by writing down irrelevant information or misunderstanding the position of the mirror line. In addition, students often misread the coordinate context, confusing the position of the motif on the fabric with the position of the corresponding point on the Cartesian plane. Students also tended to ignore key terms such as "distance to the line" and "shadow point," even though these concepts are essential for correctly understanding and solving reflection problems.

From interviews, high-ability students can explain the elements of the problem by mentioning the relationship between the visual motif and the characteristics of the transformation. Conversely, students with moderate and low abilities show partial reasoning: they understand the context of the problem intuitively but cannot extract its mathematical structure. This shows that the students' mental processes are still at the situational representation stage (according to Gravemeijer), not yet advancing to a more abstract model-of.

b. Indicators of Formulating the Problem and Implementing the Plan

At this stage, student achievement is lower. On average, only 41.13% of students are able to formulate problems and implement appropriate plans. Although some questions show high percentages (85–89%), most questions that require generalization of concepts or symbolic representation actually produce very low achievement (10–35%).

Consistent patterns of errors were identified in students' work. These included overgeneralization, in which students assumed that reflection always involves "changes the sign" without considering the position of the mirror line; Mirror rule misapplication particularly when applying the reflection formulas to the lines $x = k$ or $y = h,$ especially when the point is not in a simple symmetrical position; coordinate swapping errors, where students inconsistently exchanged coordinates such as $(x, y) \rightarrow (-y, -x) \, or \, (y, x)$, especially for the lines $y = -x \, and \, y = x$; misconceptions about distance, in which students incorrectly determine the distance of a point to a line, resulting in an asymmetrical image; and representational disconnects, where students were able to identify the reflected pattern on the woven fabric but struggled to translate it accurately into coordinate representations.

The finding reveal clear differences in students' thinking patterns across ability groups. High-ability students tend to employ analytical mapping strategies by consistently connecting the concept of distance with appropriate coordinate rules. Students with moderate abilities demonstrate a predominantly procedural yet fragile pattern of thinking, as they attempt to apply formulas without fully understanding their underlying geometric meaning. In contrast, low-ability students are more likely to rely on trial-and-error strategies, often guessing sign changes without supporting their solutions with sound mathematical reasoning. The discrepancy between performance in concrete activities (when using fabric) and performance on written problems indicates that some students fail to make the transition from model-of to model-for, which is an important stage in RME.

c. Indicator of Reviewing Answers

This indicator is measured through written test numbers 1g and 2h. For questions number 1g and 2h, there were 25 out of 28 or 89.29% students were able to review the answers well. Therefore, the researcher can conclude that 89.29% of students have been able to review the answers well.

Based on the results of the written test for problem solving ability, researchers grouped students' problem solving ability into three classes. From the written test results, the average of the written test is 58.91 and the standard deviation of the written test is 15.0895. Therefore, the problem solving ability categories for each class are presented in table 3 below:

Table 6 Category of Students' Problem Solving Ability

Category	**Value Range**	**Many Students**
High	$X \geq 74{,}0007$	6 students
Medium	$43{,}821 < X < 74{,}0007$	14 students
Low	$X \leq 43{,}8217$	8 students

Based on the categorization above, it was obtained that there were six students in the high problem solving ability class, 14 students in the medium problem solving ability class, and eight students in the low problem solving ability class. Then, researchers determined the number of students interviewed from each class proportionally as follows: (1) one student was selected for the high class, (2) three students were selected for the medium class, and (3) one student was selected for the low class. The purpose of the interviews in this study was to explore how students' problem-solving abilities in each group.

The analysis of interview data after the implementation of the Realistic Mathematics Education (RME) approach reveals clear differences in students’ problem-solving abilities across achievement levels. Students in the high-achievement group demonstrated strong mastery of all problem-solving indicators. They were able to clearly explain the known and unknown elements of the problems, formulate appropriate solution plans by transforming contextual situations into formal mathematical representations, and apply prior learning experiences to develop effective strategies. These students were also able to implement their plans coherently and reflect on their solution processes by reviewing each step to ensure consistency with mathematical principles.

In contrast, students in the medium-achievement group showed partial mastery of problem-solving skills. While they were generally able to understand the problems presented, difficulties emerged in the stages of planning and implementation. Several students were unable to clearly explain the strategies they intended to use or how those strategies were applied to solve the problems. These difficulties were often caused by imprecise use of mathematical concepts and procedures. Moreover, during the review stage, many students struggled to identify errors or inconsistencies in their solutions, indicating limited reflective ability in evaluating their own problem-solving processes.

Students in the low-achievement group experienced the greatest challenges across all stages of the problem-solving process. They had difficulty fully identifying the known information and the questions posed in the problems, which hindered their ability to formulate appropriate solution plans. As a result, these students were unable to clearly explain the steps taken to solve the problems. Additionally, they showed limited ability to reflect on their solution processes or recognize incorrect procedures, largely due to insufficient mathematical understanding.

Overall, the findings indicate that there was an improvement in students’ mathematical problem-solving abilities after the learning process, as evidenced by an increase in the mean score from 45.46 in the pre-test to 58.91 in the post-test, with a normalized gain ($N - gain$) value of 0.25, which falls into the low category. In addition, the qualitative analysis reveals differences in problem-solving abilities across student categories, where students in the high-achievement group were able to meet all problem-solving indicators, while students in the medium- and low-achievement groups still experienced difficulties at several stages, particularly in planning, implementing, and evaluating solutions. These findings suggest that the improvement in problem-solving abilities has not been evenly distributed among all students, indicating the need for more targeted instructional support at each stage of the problem-solving process.

3.2 Discussion

The findings indicate that students' patterns in discovering reflection rules develop progressively from concrete experiences to symbolic understanding, with clear differences among high-, medium-, and low-ability students. High-ability students are able to construct understanding consistently up to the formal level, whereas medium- and low-ability students demonstrate partial and fragmented development. These findings support theories of geometric thinking development, which suggest that students' understanding evolves hierarchically from visual to formal levels and is strongly influenced by meaningful learning experiences (Fujita et al., 2025; Seah & Horne, 2020). However, the results of this study further reveal that although these stages are generally applicable, the rate of development is significantly influenced by students' prior knowledge. This finding slightly differs from those of Bicer, 2021, who emphasize the role of instructional interventions in general, whereas in this study, initial ability appears to play a more dominant role in determining the quality of students' conceptual construction.

The findings also show that the use of contextual problems, such as paper folding, fabric folding, and woven patterns, helps students develop an initial intuitive understanding of reflection before transitioning to formal representations. This supports the principles of Realistic Mathematics Education (RME), which emphasize that learning should begin with meaningful contexts to promote progressive mathematization (Gravemeijer et al., 2017; Muhammad et al., 2025). In addition, the integration of cultural contexts has been shown to enhance student engagement and help them connect real-life experiences with mathematical concepts. However, the findings of this study indicate that contextualization does not automatically guarantee deep understanding for all students. This contrasts with the findings of Wikasari et al., 2025; Yanti, 2025, who argue that ethnomathematical approaches consistently improve conceptual understanding, whereas in this study, low-ability students still experienced difficulties in transforming contextual experiences into formal representations.

The next finding indicates that the transition from concrete models to visual and symbolic representations is a crucial stage in learning reflection. High-ability students are able to integrate multiple representations consistently, whereas medium- and low-ability students tend to experience discontinuities in meaning across representations. This finding is consistent with studies highlighting that the ability to connect visual, spatial, and symbolic representations is a key factor in understanding geometric transformations (Elia et al., 2021; Weigand et al., 2025) . However, the results of this study show that even when representations are introduced gradually, some students still apply rules mechanically without understanding the underlying concepts. This reinforces the findings of Van Breukelen et al., 2021 regarding the gap between procedural and conceptual knowledge, while also indicating that transitions between representations require more explicit scaffolding.

Furthermore, the findings reveal that the use of artifacts such as woven patterns, fold lines, and mirror lines serves as a bridge in constructing the meaning of reflection. Students are able to understand concepts such as symmetry and equal distance through these visual representations. These findings support the view that the use of concrete and visual artifacts is effective in helping students achieve conceptual understanding before reaching formal abstraction (Bakker et al., 2021; Pepin et al., 2017). In addition, spatial ability has been shown to play an important role in understanding geometric transformations (Lowrie et al., 2021). However, this study also finds that the effectiveness of these artifacts largely depends on students' ability to construct meaning. This extends previous findings, which tend to emphasize the effectiveness of instructional media, by showing that without adequate guidance, low-ability students tend to remain at a superficial visual level without achieving formal generalization.

Finally, the findings show that there is a variation in problem-solving strategies, reflecting differences in students' abilities to understand problems, plan solutions, implement strategies, and evaluate results. High-ability students demonstrate systematic and concept-based thinking patterns, whereas low-ability students tend to rely on trial-and-error approaches or imitation of procedures. These findings are consistent with theories of mathematical problem solving, which emphasize the interaction between reasoning, representation, and metacognition (Schukajlow et al., 2017; Verschaffel et al., 2020). However, the results of this study indicate that the evaluation phase, or "looking back," remains a major challenge, particularly for low-ability students. This differs from the findings of Hanin & Van Nieuwenhoven, 2020; Suseelan et al., 2022, which suggest that instructional interventions can significantly improve metacognitive skills. In contrast, this study indicates that additional scaffolding is needed to develop students' reflective habits more consistently.

4. Conclusion

Based on the results of written tests and interviews, it can be concluded that students have begun to demonstrate the ability to review their answers, yet they still face difficulties in understanding problems, formulating plans, and implementing solution strategies. These challenges arise from their limited experience with systematic problem-solving processes and their insufficient comprehension of mathematical sentences in the tasks provided. The findings of this study emphasize the importance of designing mathematics learning that explicitly cultivates reflective thinking and problem-solving habits through guided instruction, the use of contextual tasks, group discussions, and prompting questions that stimulate reasoning prior to the teacher's formal explanation.

Beyond the scope of this study, these results provide broader implications for mathematics education, particularly in supporting the shift toward learning approaches that align with the goals of Kurikulum Merdeka, which encourages student autonomy, reasoning, and deep conceptual understanding. The patterns of student difficulties identified here can serve as important input for teachers, schools, and curriculum developers in improving instructional design—especially in topics that require transitions between visual, contextual, and symbolic representations such as geometric transformations. Moreover, the study contributes to the growing body of evidence that sustained exposure to reflective and problem-based learning has the potential to strengthen students' reasoning, mathematical communication, and adaptability when facing more complex and unfamiliar problems.

For future research, it is recommended that long-term investigations be conducted to examine how continuous implementation of problem-solving-oriented instruction influences students' cognitive development, metacognitive awareness, and mathematical resilience across different grade levels and school contexts. Such studies will help further validate the broader impact of reflective and problem-based learning not only on immediate performance but also on students' long-term mastery and confidence in mathematics.

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