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Big-Concept-Based Design of High School Mathematics Review Lessons: Function-Based Approaches to Inequalities

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Abstract

High school mathematics review lessons often emphasize repeated exercises and isolated techniques, which may limit students' ability to connect previously learned knowledge and transfer problem-solving methods to unfamiliar situations. This study aimed to develop and preliminarily examine a big-concept-based design for a Grade 12 review unit on function-based approaches to inequalities. An exploratory, post-test-only classroom comparison was conducted with 95 students from two classes at K High School in City N. One class received three lessons based on the proposed design, while the other received conventional review instruction. Data were collected through a nine-item post-test and semi-structured student interviews. Descriptive results showed that the experimental class had a lower failure rate than the comparison class (2% vs. 10%) and generally higher performance in knowledge mastery, comprehensive understanding, and transfer and application. The available interview summaries suggested clearer connections among functions, derivatives, monotonicity, equations, and inequalities, as well as improved awareness of auxiliary-function construction and reverse reasoning. Unlike conventional review instruction organized around separate question types, the proposed design integrates fragmented knowledge around the transferable idea of auxiliary-function construction. The findings provide preliminary evidence of feasibility; however, because standardized pre-test data and inferential analyses were unavailable, the results should be interpreted cautiously.

Keywords: Big-concept-based teaching; High School; Review lessons; Instructional design; Mathematics

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1.Introduction

Grade 12 mathematics review lessons are intended to help students reorganize prior knowledge and apply it flexibly to integrated problems. In practice, however, review instruction is often organized around isolated question types, repeated exercises, and fixed solution procedures. This fragmentation is especially evident in function-based inequality problems, where students may be able to differentiate a given function but still struggle to work backward from an inequality and construct an appropriate auxiliary function. The General Senior High School Mathematics Curriculum Standards (2017 Edition, 2020 Revision) emphasize disciplinary big concepts, unit-based instruction, and the development of core mathematical competencies (Ministry of Education of the People's Republic of China, 2020).(Ministry of Education of the People's Republic of China., 2020) Big-concept-based teaching offers a way to connect related knowledge around core concepts and principles rather than present isolated skills or algorithms. It directs attention to mathematical structures, methods, and connections, thereby helping students overcome fragmented learning(Zhou& Wu., 2023)Studies of high school mathematics teaching further suggest that organizing instruction around big concepts can support the implementation of mathematical core competencies (Zhang, 2023)Big concepts can be understood as general concepts or principles that explain a range of related phenomena and organize more specific knowledge within a domain(Xue, 2024) .From a cognitive perspective, advance organizers provide an inclusive conceptual framework for connecting new material with learners' prior knowledge, while a spiral curriculum supports revisiting central ideas at increasing levels of depth and complexity (Ausubel, 1960; Bruner, 1960). In mathematics education, big ideas connect multiple mathematical understandings into a coherent whole, enabling students to view mathematics as a system of related ideas rather than a collection of isolated facts and procedures (Charles, 2005).In unit teaching, this requires teachers to clarify learning objectives, consider students' prior knowledge, analyze teaching requirements, and construct a coherent knowledge framework (Zhou & Wu., 2023).Unit objectives and teaching activities should be organized around the big concept so that students' understanding can develop continuously and transfer to new situations (Zhu & Liu., 2020) However, existing studies mainly discuss general unit design or new-content instruction, while less attention has been paid to Grade 12 review lessons, where students must reorganize previously learned knowledge. Classroom-based evidence on using auxiliary-function construction to integrate functions, derivatives, equations, and inequalities also remains limited.

Function-based inequality solving is a suitable context because it requires students to coordinate multiple concepts and use reverse reasoning to infer an auxiliary function from an inequality or derivative expression. Therefore, this study develops a big-concept-based review design organized around auxiliary-function construction and examines its classroom implementation. It addresses two questions: (1) How can a Grade 12 review unit on function-based approaches to inequalities be organized around the big concept of auxiliary-function construction? (2) What descriptive learning outcomes and student perceptions are observed after implementation?

The study contributes a practical framework for organizing review instruction around conceptual connections rather than isolated question types and provides preliminary classroom evidence for future research.

2 Methods

This study used an exploratory, post-test-only classroom comparison to examine the implementation of the proposed review design. The instruction was conducted over three class periods with 95 Grade 12 students from two classes at K High School in City N. Evidence was collected through a nine-item post-test and semi-structured interviews. Because standardized pre-test data and inferential analyses were unavailable, the findings are interpreted descriptively rather than as evidence of a causal treatment effect.

2.1 Theoretical Foundations

Cheng Yingping(Cheng, 2023), proposed a staged process for big-concept-based unit design that includes identifying the overarching concept, organizing instruction around it, and assessing students' understanding. Bu Mengdan(Bu, 2023) follows the big concept teaching theory and incorporates the fundamental theory of deep learning in mathematics into the curriculum through three steps: clarifying teaching objectives, designing teaching evaluation, and designing teaching process. Based on these principles, the present study aligned learning objectives, assessment evidence, and instructional activities. This sequence is consistent with backward design, which begins with the desired learning outcomes and then determines appropriate assessment evidence and instructional activities (Wiggins & McTighe, 2005).

2.2 Instructional Design

In this review unit, functions, inequalities, derivatives, and algebraic transformations were organized around the big concept of auxiliary-function construction.The unit was organized around a transferable sequence: identifying the structural features of an inequality, constructing an auxiliary function that makes the hidden relationship explicit, determining the function's properties, and using those properties to solve the inequality. A concept hierarchy diagram was then used to connect this overarching idea with specific knowledge and problem-solving strategies., and construct new functions appropriately and quickly. On the other hand, organize the subdivision goals of unit knowledge to help students grasp the idea and refine the problem-solving methods.

Build a concept hierarchy diagram. The review unit's big concepts are distilled as the "construction method," which is the way of thinking that students acquire when learning to solve problems with constructors. It involves constructing a new function based on the problem, visualizing the hidden conditions, discussing the properties of the new function using derivatives, and then solving inequality problems.

Before introducing problem-solving methods, it is necessary to interpret and review the content of the unit. First, guide students to recall the specific content of the constructor, including the concept of the function, the four arithmetic operations, properties and graphs, to help students intuitively perceive and understand the graph of the function and its monotonicity. 2 Use the function differentiation to study the monotonicity, extremum, maximum and minimum values of the function, and further study the zeros of the function for the purpose of comparing magnitudes.

How to construct auxiliary functions is the key to solving this type of problem. The main obstacle for students to construct functions is: The transformation and reduction between functions, equations and inequalities, in the unit of solving inequalities with constructors, the three main principles of direct transformation, visualization, and equivalent transformation(Zhao, 2023) are mainly used. The "construction method" is mainly reflected in the three problem-solving ideas of similarity, visualization, and equivalent transformation, which is a generalization(Nie & Luo, 2023) of the thinking methods obtained after the specific mathematical knowledge such as core concepts and principles is processed by thinking. Based on the analysis of the content of the big concepts "constructor method" and "Constructor for solving inequalities", as well as the specific thinking methods they embody, the following concept hierarchy diagram is constructed.

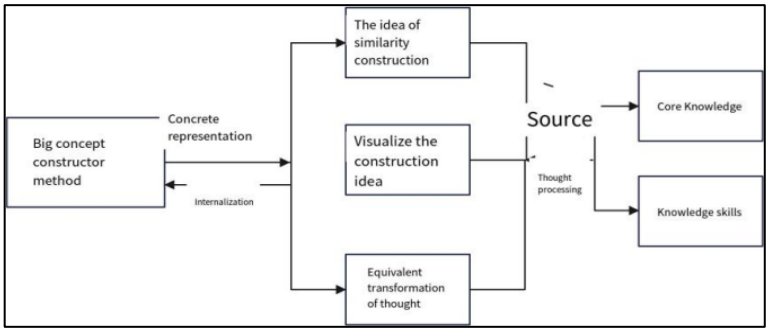


Figure 1. Conceptual Structure of Auxiliary-Function Construction

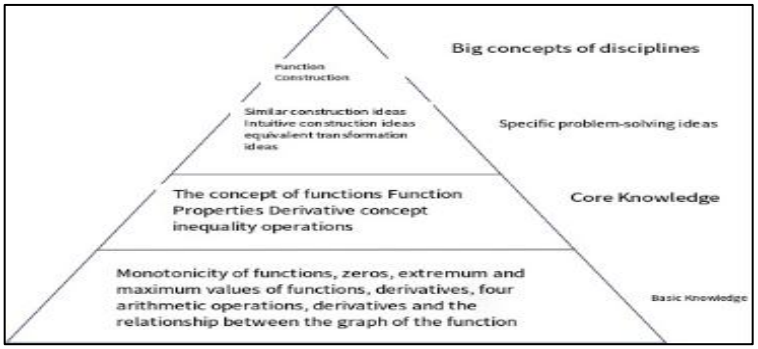


Figure 2. Concept Hierarchy for Function-Based Inequality Solving

a) Learning Situation Analysis

The teaching is for senior three students who have learned the concept of functions, the four arithmetic rules, monotonicity, have a basic grasp of derivative formulas, the arithmetic rules of derivatives, and the relationship between function monotonicity and derivatives, and are proficient in using given functions to find the derivative. The difficulty lies in the fact that constructing an original function with a known derivative function or a part of the derivative function is reverse thinking based on the original knowledge. Students who are familiar with a single knowledge point have difficulty focusing on the core knowledge of constructing functions.

b) Unit Structure and Learning Objectives

The unit was divided into three sub-units: (1) constructing the knowledge system and consolidating core knowledge; (2) examining the mathematical structure of problems and explaining specific construction strategies; and (3) selecting and adapting auxiliary functions to solve complex problems. The sequence was designed to move from recognition to explanation and flexible application(Nie & Luo, 2023) will be clarified. The unit is divided into three major sub-units: Sub-unit 1 "Constructing the Knowledge system, breaking through Core Knowledge", and sub-unit 2 "Digging into the essence of Knowledge, Comprehending specific problem-solving ideas"; Subunit 3 "Construct to Solve Complex Problems Flexibly".

Example 1 (Qiu Pingyuan, 2024). Let $f:\Re\to\Re$ satisfy $f(-1)=3$ and， $f'(x)>3$ for every $x\in\Re$. Determine the solution set of $f(x)>3x+6$.

A. $\{x|-1<x<1\}$ B. $\{x|x>-1\}$ C. $\{x\,|\,x<-1\}$ D. $\Re$

Solution. Define $g(x)=f(x)-3x-6$. Then $g'(x)=f'(x)-3>0$, so g is strictly increasing on $\Re$. Moreover, $g(-1)=f(-1)-3\times(-1)-6=0$.

Therefore, $f(x)>3x+6\Leftrightarrow g(x)>0\Leftrightarrow g(x)>g(-1)\Leftrightarrow x>-1$ Hence, the correct answer is **B**.

Example 2. Let f be differentiable on $\mathbb{R}$ and suppose that $2f(x)+xf'(x)>x^2$ for every . $x\in\Re$ Which of the following statements must be true?
A. $f(x)>0$ B. $f(x)<0$ C. $f(x)>x$ D. $f(x)<x$

Solution. Define $h(x)=x^2f(x)-\frac{x^4}{4}$. Then $h'(x)=x\left[2f(x)+xf'(x)-x^2\right]$. By the given condition, $h'(x)<0$ for $x<0$ and, $h'(x)>0$ for $x>0$. Hence h is strictly decreasing on $(-\infty,0)$ and strictly increasing on $(0,+\infty)$. Since $h(0)=0$, $h(x)<0$ for $x\neq0$, so $f(x)>\frac{1}{4}x^2\geq0$. At $x=0$, the original condition gives $2f(0)=0$. Therefore $2f(0)>0$ for all x∈ℝ, and the correct answer is A.

Example 3. Let $f:\Re\to\Re$ satisfy $f(2)=7$ and $f'(x)<3$ for every $x\in\Re$. Determine the solution set of $f(\ln x)>3\ln x+1$.

Solution. Since ln x is defined for $x>0$ let $t=\ln x$ and define $g(t)=f(t)-3t-1$. Then $g'(x)=f'(x)-3<0$, so g is strictly decreasing on $\Re$ Moreover, $g(2)=f(2)-6-1=0$. Therefore, $f(\ln x)>3\ln x+1$ is equivalent to $g(\ln x)>g(2)$. Since g is strictly decreasing, $\ln x<2$. Combining this with $x>0$ gives $0<x<e^2$

Example 4. Let $f:\Re\to\Re$ satisfy $\frac{f(x_1)-f(x_2)}{x_1-x_2}>2$ for all $x_1\neq x_2$ 2, and suppose that $f(3)=8$. Determine the solution set of $f(2x-1)<4x$.

Solution. Define $g(x)=f(x)-2x$. For any $t_1\neq t_2$, $\frac{g(t_1)-g(t_2)}{t_1-t_2}=\frac{f(t_1)-f(t_2)}{t_1-t_2}-2>0$ .Thus g is strictly increasing on $\mathbb{R}$. Moreover, $g(2)=f(2)-6-1=0$. The inequality $f(2x-1)<4x$ can be rewritten as $g(2x-1)<2=g(3)$. Since g is strictly increasing, $2x-1<3$, so $x<2$. The solution set is $(-\infty,2)$.

2.3 Classroom Implementation and Data Collection

The classroom implementation was examined through a post-test and student interviews to describe learning outcomes and students' perceptions of the proposed design. t

a) Participants and Procedure

The participants were 95 Grade 12 students from Class 1 and Class 5 at K High School in City N. The school regarded the two classes as broadly comparable in overall academic level. One class received the big-concept-based review instruction, while the other received conventional review instruction; both completed three class periods of explanation and practice. Because standardized pre-test data were unavailable, baseline equivalence cannot be independently verified and the comparison should be interpreted cautiously.

b) Research Instruments

The post-test consisted of nine items, including multiple-choice, fill-in-the-blank, and proof questions, with a total score of 110 points. Items 1–4 assessed knowledge mastery, Items 5–7 assessed comprehensive understanding, and Items 8–9 assessed transfer and application.

Table 1. Question Design

Assessment level	Question Number	Requirements	Question Design
Knowledge Mastery	1-4	The flexible application of the derivative formula of basic elementary functions and the four arithmetic rules of derivatives, as well as the mastery of the derivative rules of composite functions and the properties of functions, cultivate students' mathematical abstraction ability.	Be able to construct functions using the derivative method Be able to construct functions using the isomorphism method Be able to solve simple problems such as comparing sizes and solving function inequalities using the basic constructor method
Comprehensive Understanding	5-7	Using the idea of transformation and reduction, construct abstract function models by classifying functional polynomials, concrete exponential functions, trigonometric functions and f (x).	Be able to solve ... using a basic auxiliary-function method, transform the essence of the problem, combine the function graph, and use the four arithmetic operations of derivatives, the parity of functions, monotonicity and other properties to solve integer solutions of parametric inequalities and problems that always hold true.
Transfer and application	8, 9	Develop students' ability to apply and master the mathematical idea of "variable normalization"	Be able to further apply mathematical ideas such as classification and integration, transformation and conversion, and combination of numbers and shapes to solve two-variable inequality problems.

A total of 95 valid post-test papers were collected. Item difficulty was calculated as the mean item score divided by the maximum item score, and the overall test difficulty was calculated as the weighted sum of the item difficulty coefficients. The coefficients were interpreted using the classification criteria proposed by Cui Guosheng et al.(Cui et al., 2008) aBased on the values reported in Table 3, the overall difficulty coefficient was 0.77, which was classified as easy under the criteria in Table 2. j. $p_j = \frac{\frac{1}{n}\sum_{i=1}^{n} x_i}{k}$ $P = \sum_{i=1}^{n} k_j p_j$ k_j p_j

Table 2. Difficulty Classification Criteria

Difficulty coefficient	Degree of difficulty
Above 0.85	Easy
0.75 ~ 0.84	Easy
0.55 ~ 0.74	Moderate
0.34 ~ 0.54	difficult
Below 0.34	Hard

Measure the difficulty of the questions by this standard. The overall difficulty of the test is 0.7, which is moderate. The formula for calculating the difficulty of the question and the formula for calculating the overall difficulty of the test paper: where P is the overall difficulty of the test paper, is the weight of question j in the total score of the test paper, is the difficulty of question j.

Table 3. Overall Learning situation of Unit

Question number	Full marks	Difficulty	Difficulty level
1	10	0.84	Easy
2	10	0.83	Easy
3	10	0.83	Easy
4	10	0.78	Easy
5	10	0.78	Easy
6	10	0.69	Moderate
7	10	0.65	Moderate
8	20	0.66	Moderate
9	20	0.52	difficult
Overall difficulty	110	0.77	Easy

Semi-Structured Interviewse

Q1. Do you think there are any differences between the teacher's teaching this time and the regular classes?

Q2. After this big concept teaching, talk about how the unit review class in the big concept perspective has helped you?

Q3. Would you like the teacher to continue using the big concept unit review teaching model in the future?The post-test results were summarized using score-category distributions, item difficulty coefficients, and mean performance across the three assessment dimensions. The available interview responses were reviewed for recurring perceptions. Because the source materials did not retain the number of interviewees, the sampling procedure, complete transcripts, or a formal coding record, the interview findings are treated as supplementary evidence.

3.2 Analysis of Practical effects

A. Analysis of Test results

The post-test had a maximum score of 110 points. Performance was classified as excellent (102–110), good (88–101), pass (66–87), or fail (below 66). The score distributions of the two classes were compared descriptively. The experimental class had a larger proportion of students in the excellent and good categories and a lower failure rate than the comparison class (2% vs. 10%). Because no inferential test was conducted, these differences should not be described as statistically significant.

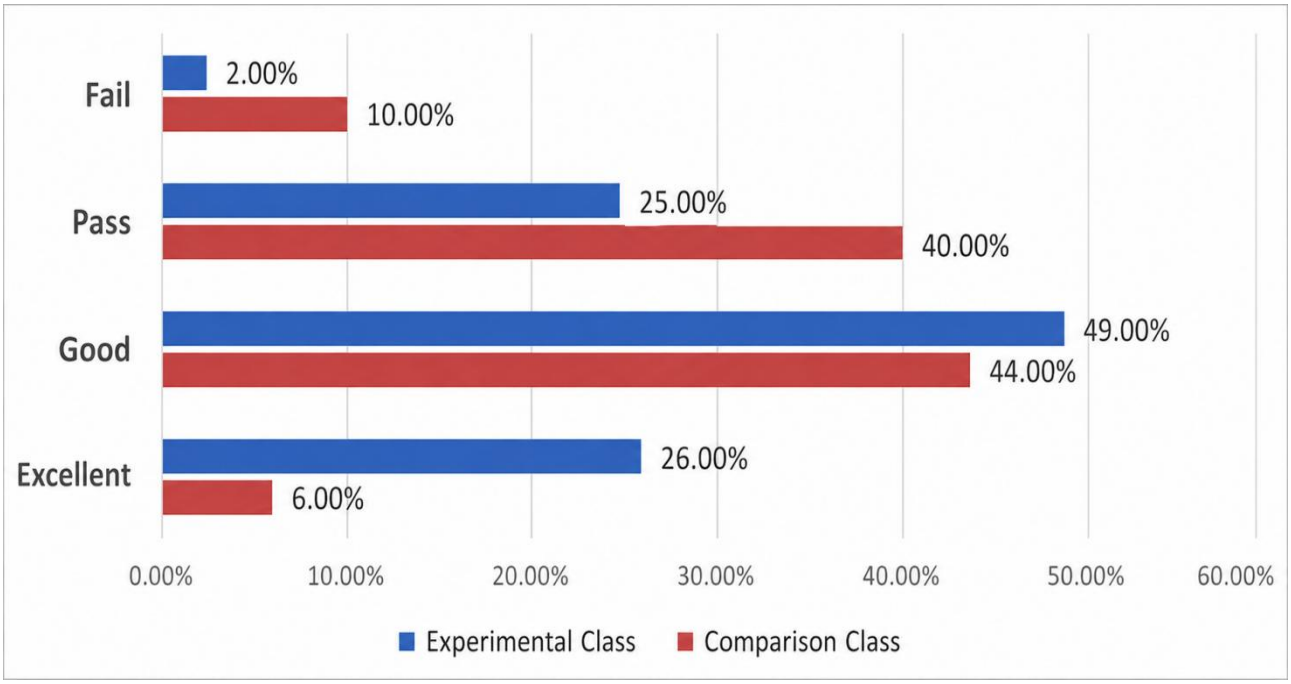


Figure 3. Comparison of total scores

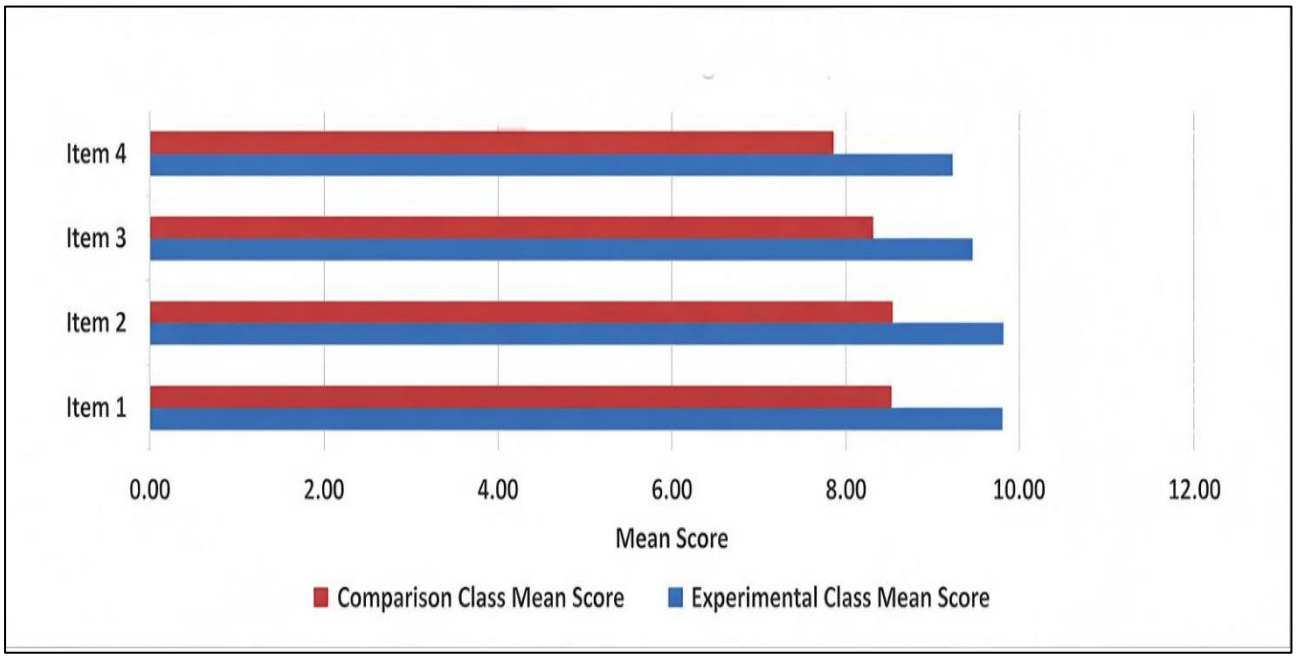


Figure 4. Comparison of Knowledge Mastery

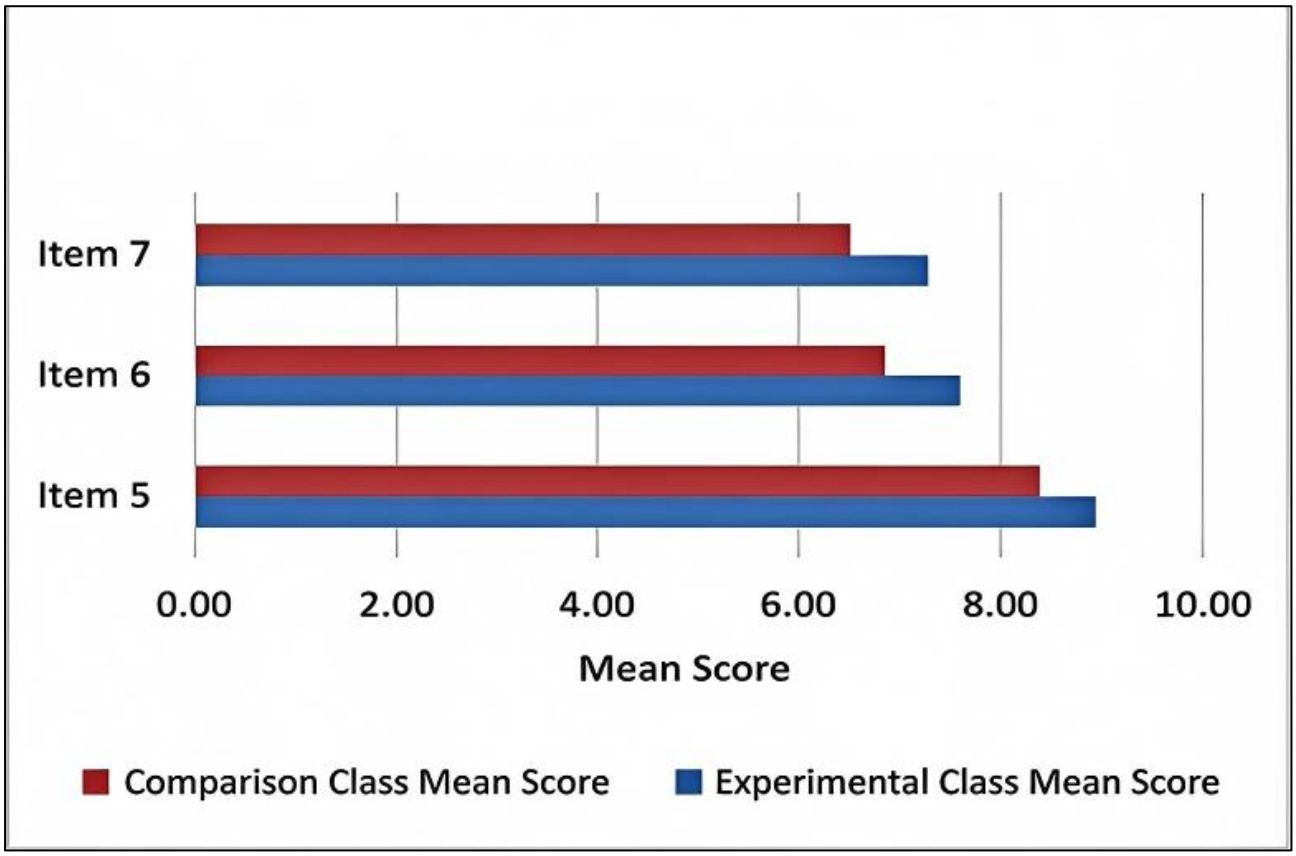


Figure 5. Comparison of Comprehensiveness

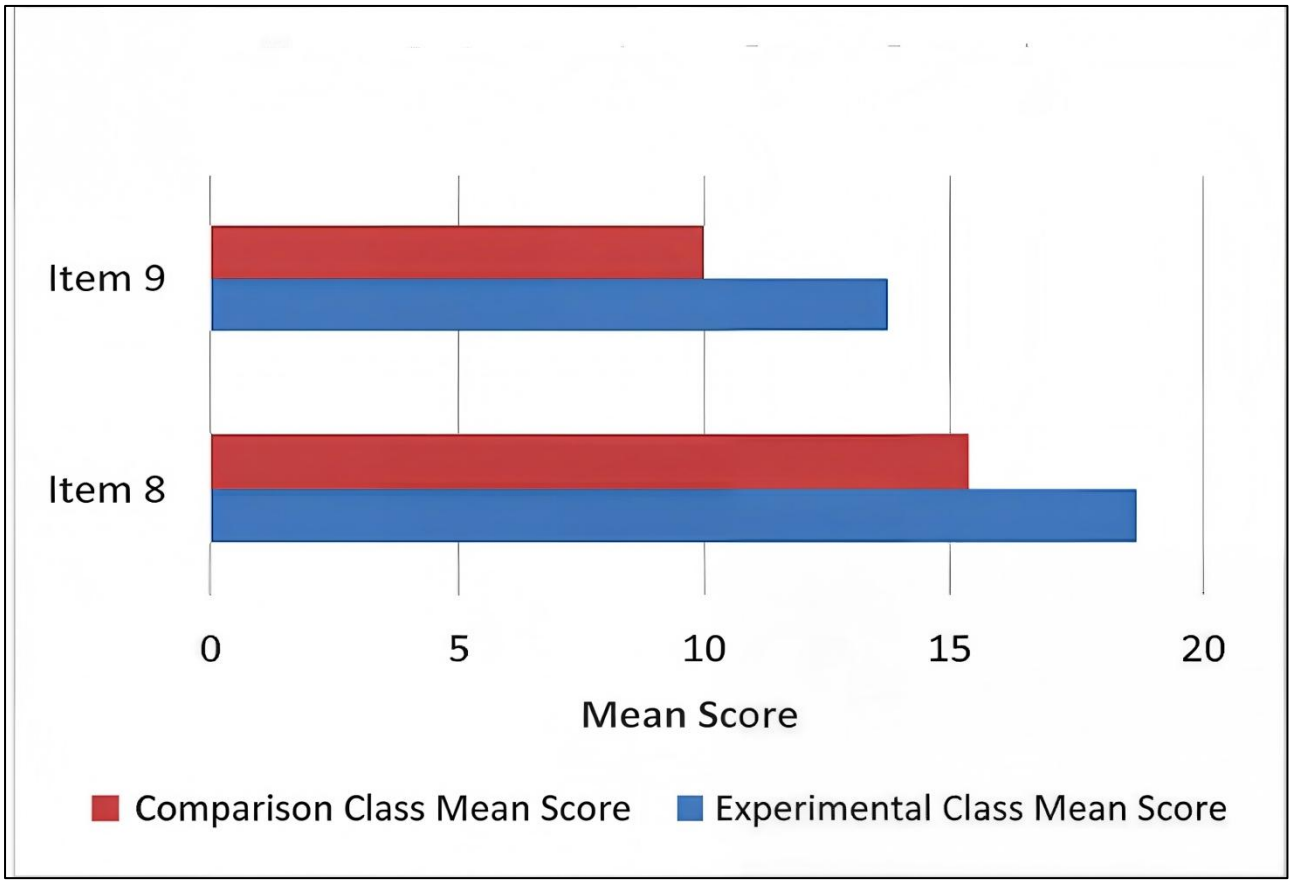


Figure 6. Comparison of Migration Applications

At the item level, the experimental class generally achieved higher mean scores than the comparison class, with the clearest descriptive difference appearing in transfer and application tasks. This pattern is consistent with the instructional aim of helping students connect knowledge and select auxiliary-function strategies in less familiar situations.This interpretation is also consistent with international recommendations that mathematics instruction should promote conceptual connections, mathematical reasoning, problem solving, and the flexible application of knowledge in unfamiliar situations (National Council of Teachers of Mathematics, 2014; National Research Council, 2001). However, the post-test-only design does not rule out pre-existing differences between the classes and does not support claims of statistically significant improvement in mathematical literacy

B. Interview Content analysis

The descriptive results suggest that organizing a review unit around auxiliary-function construction may help students treat functions, derivatives, and inequalities as a connected system rather than as separate topics. This interpretation is consistent with prior studies emphasizing the role of big concepts in organizing mathematical knowledge and supporting transfer (Zhou & Wu, 2023; Zhu & Liu, 2020; Nie& Luo, 2023; Wu & Li, 2022). The present design operationalized this principle by repeatedly asking what function should be constructed, why its properties can be determined, and how those properties transform the original inequality.

The remaining 2% failure rate in the experimental class also requires attention. The available records do not identify the individual causes, so a definitive diagnosis is not possible. Two task-related obstacles are plausible: some students may still have fragmented prerequisite knowledge, and reverse reasoning places a high cognitive demand on students because they must reconstruct a useful function from a derivative pattern or inequality instead of differentiating a given function. A three-lesson intervention may be insufficient for students whose prior knowledge or procedural habits are strongly established.

These obstacles can be addressed through a staged scaffold. Teachers may begin with forward recognition of derivative structures, proceed to partial reverse-matching tasks, ask students to compare alternative auxiliary functions and justify which one yields a derivative with a determinable sign, and then gradually withdraw support in unfamiliar problems. This sequence treats reverse reasoning as an explicit and learnable process rather than an intuitive leap.

The approach also presents practical challenges. Teachers need sufficient mathematical and pedagogical knowledge to select an appropriate overarching concept and examples that reveal mathematical structure, while Grade 12 review schedules impose time constraints. The study involved only two classes at one school, used a post-test-only comparison, did not report inferential statistics or effect sizes, and lacked documented validity evidence for the post-test and a complete interview coding record. Therefore, the findings indicate feasibility and favorable descriptive patterns, not a causal effect.

4. Conclusion

This study developed a big-concept-based design for a Grade 12 review unit on function-based approaches to inequalities. By positioning auxiliary-function construction as the organizing idea, the design connected functions, derivatives, equations, inequalities, and recurring problem-solving strategies. The classroom evidence showed favorable descriptive outcomes and student perceptions, particularly in knowledge organization and transfer. These findings support the feasibility of the design but should not be interpreted as causal proof.

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