Jurnal Pendidikan Matematika (JUPITEK) | June 2026 | Volume 9 Number 1 | Page. 47–62
ISSN: 2655-2841 (Print); 2655-6464 (Electronic)
DOI https://doi.org/10.30598/jupitekvol9iss1pp47-62

Developing Pedagogical Content Knowledge of Pre-Service Mathematics Teachers in Teaching Using Realistic Mathematics Lessons Approach: A Multiple Case Study

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Submitted: November 08, 2025 *Revised: March 26, 2026* *Accepted: May 27, 2026*

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Abstract

This qualitative study explores the Pedagogical Content Knowledge (PCK) of pre-service mathematics teachers in implementing the Realistic Mathematics Education (RME) approach during microteaching sessions. This research is important because a deep understanding of how pre-service teachers develop and demonstrate PCK within specific pedagogical approaches such as RME can inform the design of more effective teacher education programs that promote meaningful mathematics learning. Participants were selected purposively based on their teaching performance levels (high and moderate). Data were collected through classroom observations and in-depth interviews, then analyzed using an interactive thematic approach involving data reduction, data presentation, and conclusion drawing. The findings reveal that the participants demonstrated emerging PCK characterized by their ability to design contextual problems, guide students' mathematization processes, and foster interactivity and negotiation of mathematical ideas. However, the integration of prerequisite concepts and the facilitation of deeper conceptual connections were not yet optimal. These results highlight that developing comprehensive PCK requires not only mastery of content and pedagogy but also reflective practice in designing contextual, interactive mathematics learning aligned with RME principles.

Keywords: Interactivity; Mathematical Negotiation; Microteaching; Pedagogical Content Knowledge; Realistic Mathematics Education

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1. Introduction

Teachers are a key element in improving students' abilities and play a crucial role in the success of the learning process (Munawir et al., 2022). A teacher is not only required to master the subject matter but also to understand how to deliver it appropriately according to the characteristics of the students. According to Shulman (1986), in the classroom learning process, a teacher requires Pedagogical Content Knowledge (PCK) to effectively integrate understanding of the material and pedagogical strategies.

Pedagogical Content Knowledge (PCK) was first introduced by Shulman (1986) as a form of teacher knowledge that integrates mastery of subject matter (content knowledge) with pedagogical knowledge. Shulman emphasized that mastery of content alone is insufficient; teachers must also be able to transform subject matter into forms that are understandable to students, including organizing, representing, and adapting the material to diverse learners' needs (Matitaputty et al., 2022). This idea is often referred to as the "lost paradigm," highlighting how earlier research on teacher education tended to overlook the importance of content in teaching. Furthermore, Shulman (1987) elaborated that PCK involves understanding students' misconceptions, learning characteristics, and the use of appropriate instructional strategies such as analogies, examples, and demonstrations. Later, Grossman (1990) expanded this concept by conceptualizing PCK as a structured body of knowledge that connects subject matter with pedagogical strategies, including knowledge of students' understanding, instructional methods, and curriculum design.

The study of PCK continued to develop afterward. Ball, Thames, and Phelps (2008) proposed a more detailed concept of Content Knowledge for Teaching (CKT). They identified several subdomains of teacher knowledge, such as Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Specialized Content Knowledge (SCK). This framework suggests that teacher knowledge is not simply a combination of content and pedagogy, but also encompasses specialized forms of knowledge that emerge from teaching practice, such as the ability to predict student misconceptions, select appropriate representations, and provide mathematical reasoning that students can understand (Ball et al., 2008).

Furthermore, recent research has emphasized the development of Pedagogical Content Knowledge (PCK) within broader and more integrated frameworks, such as Technological Pedagogical and Content Knowledge (TPACK), which views teacher knowledge as dynamic and contextual. In this perspective, PCK is not only considered an individual cognitive resource but also develops through interactions with technology, curriculum, and learning environments. This highlights that teacher knowledge evolves through teaching experiences, adaptation to instructional contexts, and the integration of multiple knowledge domains (Mishra, 2019; Koehler et al., 2013). Additionally, the development of PCK is influenced by external factors such as learning environments, including online and offline teaching experiences, which shape how teachers design and implement instruction effectively (Aini & Masrurotullaily, 2024). Therefore, teacher professional development and continuous adaptation to educational changes play a crucial role in strengthening PCK.

Based on these developments, it can be seen that PCK has moved from Shulman's basic concept, which emphasized the integration of content and pedagogy, to a more applicable and contextual framework. Grossman enriched the initial structure, while Ball et al. and the RCM emphasized how PCK is present in real-world practice and continues to evolve according to learning needs. Thus, PCK can be understood as a teacher's knowledge base that connects a deep understanding of the material with pedagogical strategies that are adaptive to student characteristics and current educational demands.

In this study, the indicators of PCK refer to Shulman's theoretical framework, with a slight modification by excluding the school context dimension. This adjustment is made to align with the research context, which involves Pre-service Mathematics Teachers (PSMT) who, as teaching interns, do not yet possess full responsibility or decision-making authority related to school-wide instructional policies. The remaining four dimensions content knowledge, knowledge of students' understanding, knowledge of instructional strategies, and curriculum knowledge are retained because they are highly relevant to mathematics teacher education. These components reflect essential competencies required of prospective mathematics teachers, including conceptual understanding of mathematical content, the ability to identify and address students' learning difficulties, the use of multiple representations, and the design of coherent and meaningful mathematical instruction. Therefore, these dimensions are considered adequate to portray the development of PSMT PCK in planning and implementing effective mathematics teaching.

For Pre-service Mathematics Teachers (PSMT), mastery of Pedagogical Content Knowledge (PCK) is an essential foundation for teaching mathematics meaningfully and contextually. A mathematics teacher

is not only required to master the material to be taught but is also expected to have the understanding and skills to convey it appropriately according to the characteristics of the students (Sutamrin et al., 2022). PCK is crucial because it is the main provision for becoming a professional teacher. According to Gess-Newsome (1990), PCK is knowledge that helps teachers combine understanding of the subject matter with teaching methods that are easily understood by students. Teachers are not only smart about the material but also must know how to deliver it according to the abilities and needs of students. This is supported by Depaepe et al. (2013), who found that teachers with well-developed pedagogical content knowledge are better equipped to connect mathematical content with instructional strategies, enabling them to design learning environments that address students' diverse needs and foster meaningful conceptual understanding. Therefore, PCK is key for PSMT to be able to teach in an interesting, understandable, and appropriate way to diverse classroom conditions.

The Realistic Mathematics Education (RME) approach emphasizes the use of real-world contexts, active student participation, and guided concept discovery to make mathematics more meaningful and understandable (Purba et al., 2022). RME was first developed in the Netherlands by Hans Freudenthal in response to mathematics learning methods that were too mechanistic and disconnected from students' real-life situations. Freudenthal viewed mathematics as a human activity, so learning must begin with contexts that students can understand (Phan et al., 2022). In this approach, students are encouraged to engage in mathematization through problem-solving related to everyday life. Thus, they not only understand concepts abstractly but also develop critical thinking and problem-solving skills in real-life situations (Asmaarobiyah et al., 2025).

According to Holisin (2007), the Realistic Mathematics Education approach has several key characteristics. Learning begins with simple, contextual problems that are close to students' lives, rather than directly starting with formal concepts. The problems used can be real-life situations or something students can imagine. Models or representations, such as diagrams or symbols, serve as a bridge from concrete situations to abstract concepts. Student contributions are highly valued; they are given the opportunity to find solutions independently or with teacher assistance. This approach also emphasizes the importance of interaction, both between students and teachers and between students, as discussions help strengthen thinking processes and problem-solving. Furthermore, mathematical concepts are presented in an integrated manner with other topics, so that students' understanding becomes more comprehensive and meaningful.

In this context, Pedagogical Content Knowledge (PCK) becomes particularly crucial for the successful implementation of RME. Teachers are required not only to understand mathematical content but also to anticipate students' thinking processes and possible learning trajectories as they construct their own models. Strong PCK enables teachers to design appropriate contextual problems, select effective representations, and provide scaffolding that aligns with students' evolving understanding. Without well-developed PCK, the principle of guided reinvention where students rediscover mathematical concepts through teacher guidance may not be effectively realized. Therefore, PCK plays a central role in bridging instructional intentions with students' actual learning processes, ensuring that RME principles are implemented in a meaningful and pedagogically sound manner.

Microteaching practice is a strategic course for PSMT to develop and hone their Pedagogical Content Knowledge (PCK) in a controlled, gradual, and reflective atmosphere. In the Mathematics Education Study Program at Sanata Dharma University, the Microteaching course is implemented as a practicum course that aims to equip PSMT with basic teaching skills systematically. PSMT are trained to master various teaching skills, such as opening and closing lessons, explaining material, varying stimuli, asking questions, providing reinforcement, guiding discussions, and teaching small groups and individuals. The implementation of the lecture process in this course is complemented by the process of compiling teaching modules, creating learning media, implementing innovative approaches in learning, and a process of reflection and evaluation for each PSMT whose practice is carried out through direct observation, video recordings, and reflections from lecturers and colleagues (Sarkim, 2022). In line with this, Mukuka and Alex (2024) emphasize that microteaching is an effective tool in the education of prospective mathematics teachers because it is able to integrate pedagogical theory and learning practices in a concrete manner. Thus, the Micro-Teaching course at Sanata Dharma University is not just a technical teaching exercise, but also a process of forming teachers who are reflective, critical, and ready to apply innovative and meaningful learning approaches in real classrooms.

Various previous studies have examined the Pedagogical Content Knowledge (PCK) of teachers and prospective teachers in the context of mathematics learning. While previous studies have extensively mapped PCK in general mathematics, there is a lack of empirical evidence on how pre-service teachers' PCK manifests and evolves when constrained by the specific design principles of Realistic Mathematics Education (RME). More importantly, although PCK has been widely discussed, only a limited number of studies have examined at a micro level how its core components knowledge of content, knowledge of students, and knowledge of instructional strategies interact dynamically when teachers implement specific pedagogical principles. In the context of RME, such interaction becomes crucial, as teaching is guided by principles such as didactical phenomenology and the use of self-developed models, which require teachers to continuously connect mathematical concepts, students' thinking, and instructional decisions in a coherent manner. However, existing research tends to focus on general aspects of PCK without explicitly linking it to these kinds of approach-specific practices. For instance, Khumaeroh and Pagiling (2020) emphasized the importance of mastering PCK in geometry teaching, yet did not address how particular learning approaches shape such knowledge. Similarly, Sutamrin et al. (2022) described PCK profiles of prospective teachers based on academic categories, highlighting cognitive aspects and general teaching strategies rather than examining how PCK supports meaningful and context-based mathematical learning.

Recent studies have begun to explore pedagogical knowledge in more specific contexts; however, their limitations can be grouped into key areas that reveal an important research gap. First, some studies focus on analyzing pedagogical knowledge without explicitly linking it to instructional approaches that foster meaningful mathematical learning. For instance, Zayyadi et al. (2024) examined pedagogical knowledge using a commognitive framework in microteaching and identified components such as word use, visual mediators, routines, and narratives, yet these were not connected to approaches that support students' conceptual understanding through contextualization, such as RME.

Second, other studies have situated PCK within innovative instructional approaches but still highlight challenges in its practical implementation. Maharani and Julie (2025), for example, found that although prospective teachers could design contextual problems within a STEAM approach, they struggled to guide students' problem-solving processes and facilitate meaningful conclusions. Similarly, Dyastika and Julie (2023) reported that most prospective teachers demonstrated only moderate levels of PCK in a flipped classroom setting, with ongoing difficulties in connecting mathematical concepts and supporting students' thinking. Taken together, these findings indicate that prior research has not sufficiently examined how PCK operates at a micro level within a specific instructional approach that systematically promotes contextual understanding and mathematical abstraction, such as RME, thereby underscoring the need for a more integrated and practice-based framework.

Although prior research has provided valuable insights into teachers' and pre-service teachers' PCK, there remains a limited understanding of how PCK is enacted within specific pedagogical approaches that support meaningful mathematics learning. In particular, little is known about how the components of PCK interact in real-time teaching practices when guided by RME principles such as didactical phenomenology and self-developed models. Moreover, existing studies often describe PCK in general terms without examining how it is manifested in practice-based settings such as microteaching. Therefore, this study addresses this gap by analyzing the PCK of pre-service mathematics teachers within the context of microteaching using the RME approach. In addition, this study contributes to the literature by developing observation and interview instruments specifically designed to capture the interaction of PCK components as enacted in RME-based teaching practices, thereby offering a more contextualized, practice-oriented, and analytically refined perspective on PCK development.

Based on this background, this study aims to describe the Pedagogical Content Knowledge (PCK) profiles of pre-service mathematics teachers (PSMTs) who took a Microteaching course in the even semester of the 2024/2025 academic year, particularly in teaching high school mathematics using the Realistic Mathematics Education (RME) approach. In this study, RME is positioned as the pedagogical context in which PCK is enacted, while PCK serves as the analytical lens to examine how pre-service teachers integrate content knowledge, pedagogy, and students' thinking in practice. Special attention is given to Knowledge of Content and Students (KCS), as it enables teachers to anticipate students' learning trajectories and support the development of meaningful mathematical understanding, especially in relation to the principle of self-generated models. PCK is analyzed through four interrelated dimensions curriculum, content, pedagogy, and learners adapted from Shulman's model and aligned with key RME principles such

as contextualization, mathematization, interactivity, and the interconnectedness of mathematical ideas. To clarify the conceptual linkage between PCK and RME, the following framework is proposed.

Table 1. Conceptual Framework of the Linkage between PCK Dimensions and RME Principles

Dimension	Knowledge of Content and Students	RME Principle: Self-Generated Models	Conceptual Link
Curriculum	Understanding concept progression, students' prior knowledge, and potential learning obstacles	Models evolve progressively from informal, situational representations to more formal mathematical forms	Anticipating students' model development along the learning trajectory and aligning it with curriculum progression
Pedagogy	Understanding students' thinking, misconceptions, and problem-solving strategies	Students actively construct and refine their own models through meaningful contextual problems	Designing and sequencing tasks that elicit, support, and progressively refine students' self-generated models
Content	Understanding mathematical structures, representations, and their conceptual connections	Models function as bridges between real-world contexts and abstract mathematical concepts	Identifying and leveraging potential models that students may develop to represent and generalize mathematical ideas
Students	Understanding students' characteristics, prior experiences, and cognitive readiness	Models emerge from students' lived experiences and intuitive reasoning	Anticipating diverse student responses and facilitating meaningful construction and negotiation of models

This study contributes to the professional development of future mathematics teachers by going beyond general descriptions of Pedagogical Content Knowledge (PCK) and identifying how specific profiles of pre-service teachers' PCK can facilitate or hinder students' mathematization processes within the Realistic Mathematics Education (RME) framework. Building on the conceptual framework presented above, this study develops and employs observation and interview instruments specifically designed to capture the interplay between PCK dimensions and RME principles within microteaching contexts. In this regard, microteaching is positioned not merely as a reflective activity, but as a structured setting for investigating how these integrated constructs are enacted in practice. Through this lens, the study seeks to answer two research questions: (1) how do pre-service mathematics teachers demonstrate their PCK when teaching mathematics using the RME approach, and (2) how are the principles of RME—specifically self-developed models, didactical phenomenology, and interactivity—reflected in and supported by pre-service teachers' PCK dimensions during microteaching practice?

2. Method

a) Research Types and Participants

This study employed a qualitative case study approach to explore phenomena in depth within their real-life context and from the participants' perspectives (Sugiyono, 2016). This approach was considered appropriate because the purpose of the study was not to measure variables statistically, but to provide an in-depth description of the Pedagogical Content Knowledge (PCK) of pre-service mathematics teachers (PSMT) when implementing the RME approach in microteaching settings. Through this approach, the study examined teaching processes and how PCK was manifested in lesson planning and teaching practices.

The study involved 14 pre-service mathematics teachers enrolled in a microteaching course. Participant selection was conducted using purposive contrasting case sampling (Palinkas et al., 2015) based on their teaching performance scores, which were derived from a combined evaluation of their lessons plane and microteaching practice scores. The categorization was established using statistical distribution thresholds based on the group mean ($\bar{x} = 85.40$) and standard deviation ($s = 2.71$): (a) high category if the score more than $\bar{x} + s = 88.12$; (b) moderate category if $\bar{x} - s \leq$ score $\leq \bar{x} + s$ i.e. 82.69 to 88.12; and (c) low category if scoreless than $\bar{x} - s$ = 82.69. Based on these categories, the researcher found that no students fell into the high category, 12 students fell into the medium category, and 2 students fell into the low category.

From these categories, contrasting cases were purposively selected for in-depth qualitative analysis: Participant A representing the high-performing spectrum within the moderate group with a score of 87.33,

who designed a digital credit installment context and Participant B representing the moderate-to-lower spectrum, including participants from the low category with a score of 81.66, who implemented paper-folding and pattern tasks. Pre-service teachers from the extreme low category were included to examine how pedagogical challenges and limitations manifest in PCK enactment during RME implementation. This contrasting selection aimed to explore diverse manifestations of PCK across teaching performance levels.

b) Ethical Considerations

This study was conducted with formal permission from the lecturer responsible for the microteaching course and/or the relevant study program. All participants were informed about the study's purpose and procedures and voluntarily provided informed consent for observation, interviews, and the collection of their teaching-related work. Participation was voluntary, and participants were assured that their involvement or withdrawal would not affect their academic grades. Confidentiality was maintained by using pseudonyms, namely Participant A and Participant B, and protecting participants' identifying information throughout data collection and reporting.

c) Instrument and Data Collection Technique

The instruments used in this study consisted of field notes and semi-structured interview guides developed based on Shulman's (1986) concept of Pedagogical Content Knowledge (PCK). From Shulman's original framework, four key dimensions curriculum, student, pedagogical, and content were adopted and adapted to suit the context of this study. The school context dimension was excluded because the participants, as pre-service teachers, had not yet gained real teaching experience in schools, making this aspect unobservable within the research setting.

The observation instrument was designed to capture how the participants demonstrated these four PCK dimensions during their microteaching sessions. Meanwhile, the interview guide aimed to explore the participants' reasoning, understanding, and pedagogical considerations in designing and implementing mathematics lessons using the RME approach. Both instruments were reviewed and validated by an expert in mathematics education and qualitative research to ensure their theoretical alignment and content validity.

Data collection was conducted through classroom observations and semi-structured interviews. The classroom observations served as the primary data source to identify the participants' PCK profiles during their microteaching practice. The semi-structured interviews were conducted with the two selected participants to further explore their reasoning processes, understanding, and decision-making in planning and conducting mathematics lessons.

d) Trustworthiness

To ensure the rigor and quality of the qualitative research, the trustworthiness framework proposed by Lincoln and Guba (1985) was systematically applied through four criteria: (1) credibility: Achieved through the triangulation of methods and data sources, involving cross-referencing video recordings, lesson plans, student worksheets, and interview transcripts. Credibility was further strengthened through formal "member checking" with students A and B, who reviewed their interview transcripts and interpretive summaries to verify the accuracy of the data; (b) dependability: Supported by an audit trail that documents data collection protocols, raw transcripts, coding schemes, and analysis matrices; (c) confirmability: Reinforced through peer debriefing sessions involving two senior researchers in the field of mathematics education. They independently reviewed portions of the raw data and verified the emergent thematic codes and cross-case matrices; and (d) transferability: Established through rich, in-depth descriptions of the micro-teaching course context, participant backgrounds, task structures, and detailed mathematical episodes, enabling readers to evaluate the applicability of the findings to similar teacher education contexts. Note on Instrument Validation: The observation and interview instruments underwent content validation by an expert—a senior lecturer in mathematics education with extensive expertise in the Realistic Mathematics Education (RME) approach and qualitative research methodology. Validation by a single expert is considered methodologically sufficient and defensible, given that the primary purpose of the expert review was to qualitatively align the content and verify the validity of the research instruments employed in this study (Yin, 2018).

e) Data Analysis Techniques

Data were analyzed using Miles and Huberman's (1994) interactive model, comprising data reduction, data display, and conclusion drawing and verification. During data reduction, observation records and interview transcripts were selected, organized, and deductively coded based on four PCK dimensions: knowledge of students, curriculum, subject matter, and pedagogy, using indicators relevant to RME. The coded data were presented in descriptive narratives and comparative matrices to identify patterns, similarities, and differences between the PCK profiles of Participant A and Participant B through cross-case analysis. Conclusions were verified by comparing interview responses with observed teaching practices.

The observation sheet and interview guide were developed based on the four PCK dimensions and reviewed by one expert, the lecturer responsible for the microteaching course. The expert assessed the relevance of the indicators, their alignment with the theoretical framework, and their coverage of the intended PCK aspects. Based on the feedback, the interview questions were revised to be more probing and open-ended to explore participants' knowledge and reasoning in greater depth. Trustworthiness was further supported through technique triangulation and peer debriefing to review the data analysis and emerging interpretations.

Table 2. Data Analysis Based on Miles and Huberman's Interactive Model

Stage	Procedure
Data Reduction	Selecting and coding observation and interview data based on the four PCK dimensions and relevant RME indicators.
Data Display	Presenting coded data in descriptive narratives and comparative matrices for Participant A and Participant B.
Conclusion Drawing and Verification	Identifying patterns and differences through cross-case analysis and verifying findings through observation–interview triangulation.

3. Results and Discussion

3.1. Results

This section presents the empirical findings from classroom observations, interview accounts, and available student work for Participant A (Case 1) and Participant B (Case 2). The findings are organized by case and interpreted through four PCK dimensions: curriculum knowledge, pedagogical knowledge, content knowledge, and knowledge of learners. A cross-case analysis follows the two case accounts. The descriptions below report observed practices and participants' stated reasoning; interpretive comparisons are reserved for the cross-case synthesis.

A. Case 1: Participant A

(1) Curriculum Knowledge

Participant A selected a mobile-phone purchase through ShopeePay Later as a context for learning about annuities and decreasing interest. Students were guided to identify relevant information, including the item price, installment amount, and interest rate. Participant A also identified sequences, compound interest, and annuities as prerequisite topics. However, in the interview, the participant acknowledged not explicitly connecting those prerequisites to the lesson: "No, because I assumed that students already understood the prerequisite materials, so I didn't link them again." The observed lesson therefore used a relevant context, while the explicit connection to prior curricular content remained limited.

(2) Pedagogical Knowledge

At the beginning of the lesson, Participant A greeted students, checked their condition, invited a student to lead a prayer, communicated the learning objectives, and introduced the ShopeePay Later context with the question, "Why do some people prefer to pay in installments rather than in full?" During problem solving, the participant asked students to reread the situation and identify what was known and what was being asked. Participant A explained: "I asked students to re-identify what was known and what was being asked in the problem. But first, I had them reread the situation presented." When students did not volunteer to present, the participant assigned groups to present and gave appreciation for correct answers. The participant also used guiding questions during the conclusion. These practices show the use of structured explanation, questioning, and participation support; however, observed discussion across groups and comparison of alternative strategies were limited.

(3) Content Knowledge

During the lesson, Participant A addressed a misconception about the principal loan. When a student treated the principal as the total debt including interest, the participant clarified that principal is the amount borrowed before interest is added. The interview account stated: "I tried to explain that the concept was wrong; the principal loan should be the amount borrowed without interest." Participant A also described the intended mathematization process through two related credit contexts: "Both are credit systems, but the difference is in the interest type. From there, I led a discussion so students could first grasp the structure of the problem—that's what we call a model of. Then we transformed it into a mathematical form, which became a model for solving the problem." This account indicates an explicit intention to move from a contextual situation to a mathematical representation. In practice, the participant's explanations helped clarify key information, although some conceptual confusion between principal and interest remained among students. This account indicates an explicit intention to move from a contextual situation to a mathematical representation, as demonstrated by the student's written work presented in **Figure 1**.

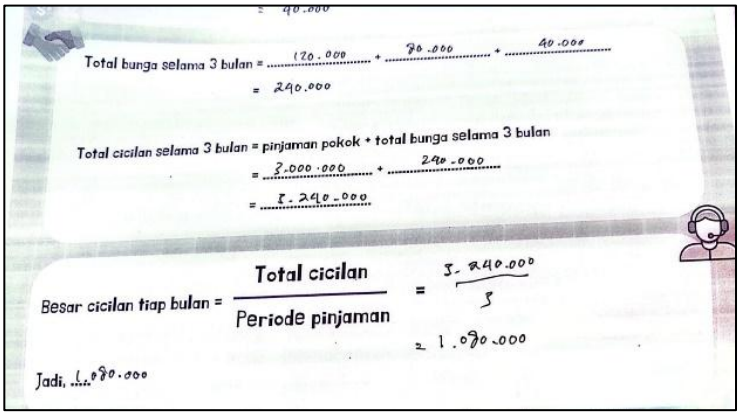

Figure 1. Students' answers

Critical Mathematical Analysis:

The procedure shown in Figure 1—calculating total decreasing interest = Rp120.000 + Rp80.000 + Rp40.000 = Rp240.000, adding it to the principal loan = Rp3.000.000, and dividing the sum equally by $n = 3$ periods to obtain a flat monthly payment of Rp1.080.000—is mathematically inconsistent with standard financial mathematics:

Annuity Scheme Inconsistency: An actual annuity scheme with periodic interest rate $i = 4\%$ per month for $n = 3$ months yields a fixed monthly payment of: $A = P \cdot \frac{i(1+i)^n}{(1+i)^n - 1} = 3.000.000 \cdot \frac{0{,}04(1{,}04)^3}{(1{,}04)^3 - 1} \approx$ Rp1.081.046

Decreasing Interest Schedule Inconsistency: Under a true decreasing interest schedule, monthly total payments are not constant; rather, monthly payments decrease over time (Month 1: Rp1.120.000; Month 2: Rp1.080.000; Month 3: Rp1.040.000).

Participant A blurred the conceptual distinction between fixed annuity installments and average flat payments of decreasing interest schedules. While the arithmetic resulted in Rp1.080.000, equating this average flat figure directly to annuity concepts demonstrates a limitation in his Specialized Content Knowledge (SCK) regarding financial mathematical models.

(4) Knowledge of Learners

Participant A recognized that most students understood the context but that some confused principal and interest. The participant reported asking questions during group work "to check whether they really understood it or to see if they had difficulties." The participant also responded to participation patterns by assigning groups to present when no students volunteered. The available student work shows students following a structured procedure to determine installment amounts. At the same time, the lesson account indicates that the participant did not substantially compare different solution strategies or extend students' responses through inter-group discussion. Thus, the participant noticed some learner difficulties and participation needs, while opportunities to probe diverse mathematical thinking were less evident.

B. Case 2: Participant B

Participant B taught geometric sequences using a paper-folding activity and a contextual problem involving patterns and the position of billiard balls. The lesson included guided observation of folding patterns, student problem solving, a group presentation, and a brief reflection.

(1) Curriculum Knowledge

Participant B introduced geometric sequences through paper folding, intending the repeated folds to provide a concrete starting point for noticing patterns. The participant also used questions such as "How many balls are in the 10th position?" to orient students toward pattern recognition. During the activity, students expressed uncertainty about the shape of the folds. The teacher initially indicated that the folds could take "any shape" and later clarified that they should be square and symmetrical. Near the end of the lesson, a student compared geometric and arithmetic sequences, but the comparison was not developed through further questioning. The lesson therefore included contextual and curricular entry points, while connections to related concepts were not explored in depth. The lesson therefore included contextual and curricular entry points, while connections to related concepts were not explored in depth, as illustrated in the paper-folding activity shown in **Figure 2**.



Figure 2. Paper folding as a teaching manipulative

(2) Pedagogical Knowledge

Participant B began by greeting students, checking attendance, and inviting a student to lead a prayer. The participant asked a student to read the problem aloud and invited others to express difficulties. During the folding activity, however, some explanations were initially unclear. When students asked about the shape of the folds, the response "It's up to you, son" did not immediately resolve their uncertainty; the instruction was subsequently made more specific. During the presentation phase, only one group presented its solution, and the interaction primarily involved validating the answer. The participant later asked questions about students' understanding, but the reflection was brief and did not consistently invite students to explain their reasoning.

(3) Content Knowledge

The paper-folding task was intended to represent geometric sequences and support students in observing patterns. Students completed the task, but the observed account indicates that the transition from the physical activity to a generalized mathematical model was not explicitly guided. When students were confused about the shape of the folds, the initial response did not explain the mathematical purpose of the representation. A student also raised a question about zero exponents; Participant B responded, although the account describes the explanation as not fully structured. These episodes show that the lesson addressed the intended mathematical content, while the explanation of why the representation worked and how it connected to formal generalization was less explicit. These episodes show that the lesson addressed the

intended mathematical content, while the explanation of why the representation worked and how it connected to formal generalization was less explicit, as displayed in the student's solution in **Figure 3**.

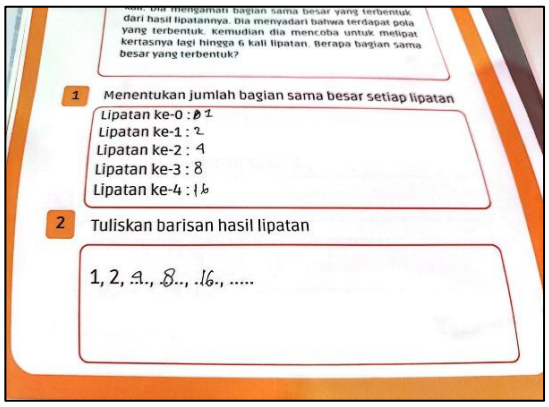

Figure 3. Students' answer

During the activity, a student questioned why "Fold 0" equaled 1 (Lipatan ke-0 = 1). Participant B addressed this by linking the physical state of unfolded paper ($2^0 = 1$ sheet) to exponent rules. In the interview, Participant B explained her pedagogical reasoning:

"When students asked why fold 0 yields 1, I asked them to look at the flat paper before any folding occurred. There is 1 whole region. Mathematically, $2^0 = 1$. I wanted them to see that exponent zero is not zero, but represents the starting state." (Interview, Participant B)

This explanation successfully bridged physical observation with mathematical notation ($2^0 = 1$), serving as clear evidence of Knowledge of Content and Students (KCS) and Specialized Content Knowledge (SCK).

(4) Knowledge of Learners

Participant B invited students to identify difficulties and responded when a student questioned the folding instructions. The participant also acknowledged student contributions by inviting one group to present and validating a correct answer. A student's question about zero exponents and another student's comparison of geometric and arithmetic sequences provided evidence of learners raising conceptual questions and making connections. However, only one group presented, and the participant did not extend the comparison between sequence types through further probing. The evidence therefore shows attention to students' questions, but limited use of those questions to explore differences in reasoning or broaden participation.

Across the two cases, both participants used contexts intended to make mathematical ideas accessible and included opportunities for students to solve problems and share answers. Their practices differed in how explicitly they connected the context to mathematical reasoning, responded to learner thinking, and developed links across topics.

Table 3. Cross-Case Synthesis of PSMTs' Implementation of RME

PCK dimension	Participant A (Case 1)	Participant B (Case 2)	Cross-case finding
Curriculum	Used a ShopeePay Later or installment context for annuities; identified sequences and compound interest as prerequisites but did not explicitly reconnect them.	Used paper folding and pattern contexts for geometric sequences; a student's comparison with arithmetic sequences was not explored further.	Both selected contextual starting points. Explicit connections to prior or related topics were limited, particularly when students offered opportunities to make them.
Pedagogy	Used stimulus questions, guided rereading, checks for understanding, assigned presentations when volunteers were absent, and a closing reflection.	Used reading aloud, invitations to report difficulties, one group presentation, answer validation, and brief reflection; some instructions were initially unclear.	Both structured the lesson and used questions, but Participant A's reported scaffolding and participation support were more sustained. Neither account shows extensive comparison of strategies.

Content	Clarified principal versus interest and described moving from contextual models ("model of") to mathematical forms ("model for").	Used folding to represent geometric patterns, but the transition from concrete manipulation to a generalized model was not explicitly supported; a zero-exponent question received a response described as insufficiently structured.	Participant A made the intended mathematical reasoning and model transition more explicit. In Participant B's lesson, the link between representation and formal generalization was less clearly articulated.
Learners	Noticed confusion about principal and interest, checked for difficulties, and adjusted presentation arrangements; student work followed a structured installment procedure.	Responded to questions about folding and zero exponents; students raised questions and topic comparisons, but participation and follow-up probing were limited.	Both showed awareness of students' needs and questions. The evidence of using learner responses to elicit and compare diverse reasoning was limited in both cases, more noticeably in Participant B's account.

3.2. Discussion

The findings of this study provide in-depth insight into how pre-service mathematics teachers (PSMTs) demonstrate their Pedagogical Content Knowledge (PCK) within the framework of the Realistic Mathematics Education (RME) approach during microteaching sessions. By analyzing the students' PCK across four key dimensions—curriculum, pedagogy, content, and students—this discussion interprets empirical evidence in light of existing literature, highlights the critical interplay between PCK dimensions and RME principles, and outlines theoretical and practical implications for future microteaching instruction. Manifestation of PCK Dimensions in the RME Context

A. Curriculum Knowledge and Contextualization

Both participants demonstrated a foundational level of curriculum knowledge, as they were able to select meaningful starting points rooted in real-life situations or students' experiences—a core principle of RME. Participant A utilized the context of the digital economy (ShopeePay Later) to ground the concept of annuities, whereas Participant B engaged students in a concrete paper-folding activity to introduce geometric sequences. These choices reflect an understanding of didactical phenomenology: the idea that mathematical concepts are constructed through solving problems embedded in contexts that students can realistically visualize.

However, a notable limitation in both cases was the failure to explicitly link prerequisite material with the new mathematical concepts. Participant A acknowledged not explicitly connecting prerequisite material regarding sequences and compound interest, operating under the assumption that students had already mastered those concepts. Meanwhile, Participant B missed an opportunity to explore the differences between arithmetic and geometric sequences that had been raised by the students themselves. These findings align with Grossman's (1990) view that curriculum knowledge encompasses not only the selection of appropriate material but also an understanding of the horizontal and vertical interconnections between concepts over time. In RME, a failure to link prerequisite concepts can hinder the mathematization process—specifically vertical mathematization. This may prevent students from constructing a coherent mathematical structure where concepts are interconnected.

B. Pedagogical Knowledge and Scaffolding Strategies

Observations of the two participants' pedagogical practices revealed varying levels of scaffolding provided by the prospective teachers. Participant A demonstrated a relatively more structured pedagogical approach, using guiding questions to prompt students to reread the problem and addressing misconceptions regarding principal amounts and interest. In contrast, Participant B encountered difficulties in sustaining the lesson when technical uncertainties arose during a paper-folding task; the initial explanations provided were unclear and overly procedural in nature.

This disparity highlights the need for general pedagogical skills to be closely integrated with subject-specific strategies to form effective Pedagogical Content Knowledge (PCK) (Shulman, 1986). While the microteaching environment allows prospective teachers to practice fundamental teaching

skills—such as opening a lesson, questioning, and classroom management (Sarkim, 2022; Mukuka & Alex, 2024) translating these skills into adaptive scaffolding during RME-based instruction remains a challenge. As Maharani and Julie (2025) found in their research on STEAM-based learning, prospective teachers often succeed in designing contextual problems but struggle during implementation—particularly in guiding students through the problem-solving process, facilitating the drawing of profound conclusions, and linking the problem-solving steps to the underlying mathematical concepts.

C. Content Knowledge and Progressive Mathematization

A crucial aspect of RME is the transition from context-bound, informal representations ("model of") to formal mathematical structures ("model for") (Van den Heuvel-Panhuizen, 2000). In this study, Participant A explicitly stated an intention to guide students through this transition by comparing two different credit systems. In contrast, Participant B tended to treat the physical paper-folding activity as an end in itself, rather than using it as a bridge toward formal generalization and algebraic representation.

This disparity highlights the distinction between general content knowledge and Specialized Content Knowledge (SCK) within the framework proposed by Ball et al. (2008). SCK requires teachers to understand mathematical representations, recognize the mathematical structures underlying student activities, and strategically facilitate the students' transition toward abstraction. The difficulty in supporting this progressive mathematization reinforces the findings of Dyastika and Julie (2023), who reported that prospective teachers with moderate PCK levels employing a flipped classroom approach often struggle to link concrete representations to formal concepts. Without explicit teacher mediation, physical manipulations or real-world contexts risk becoming isolated procedural tasks rather than catalysts for conceptual understanding. If this occurs during the learning process, students fail to engage in the process of abstraction.

D. Knowledge of Learners and Mathematical Interactivity

Regarding the dimension of knowledge of learners, both participants demonstrated concern for student participation and emotional readiness. Participant A responded to classroom dynamics by assigning groups to present when no volunteers came forward, while Participant B addressed student questions concerning zero exponents. These practices reflect the early development of Knowledge of Content and Students (KCS) (Ball et al., 2008; Hill et al., 2008).

Nevertheless, interactivity in both classrooms remained largely teacher-centric and evaluative in nature. Group presentations were primarily used to confirm the correctness of answers rather than to compare alternative strategies, facilitate peer-to-peer discussion, or enable students to construct new concepts. In RME, interaction and negotiation hold significant social importance, allowing students to refine their mathematical thinking (Holisin, 2007; Purba et al., 2022) and develop their ability to communicate mathematical ideas. The tendency of prospective teachers to limit interaction to answer verification aligns with findings by Zayyadi et al. (2024), who observed that prospective teachers often rely on visual aids and closed routines during microteaching, generally failing to foster open mathematical discourse.

E. Theoretical and Practical Implications

The findings of this study offer three significant implications for mathematics teacher education programs and the professional development of prospective educators:

1) Integration of PCK and Learning Approach Principles:

The results indicate that PCK cannot be developed or evaluated in isolation; it is heavily influenced by the specific pedagogical approach employed by the participants. Teacher education programs must move beyond teaching general instructional strategies and explicitly train prospective teachers on how PCK components interact dynamically within specific frameworks such as applying the Realistic Mathematics Education (RME) approach to the learning process.

2) Supporting the Transition from "Model-of" to "Model-for":

Teacher preparation curricula need to place greater emphasis on training prospective secondary mathematics teachers (PSMTs) to facilitate the process of progressive mathematization, enabling students to construct new mathematical concepts linked to the problems they are solving. Particular

attention should be paid to helping prospective teachers design probing questions that encourage students to move from informal manipulation toward formal mathematical abstraction.

3) Strengthening Microteaching as a Reflective Platform:

Microteaching courses should not be treated merely as routine technical exercises in teaching mechanics. Integrating video-based self-reflection, peer debriefing, and mentoring—with a specific focus on analyzing actual student thinking—can help PSMTs identify overlooked conceptual connections and hone their adaptive questioning skills (Mukuka & Alex, 2024; Sarkim, 2022).

4. Conclusion

This study concludes that pre-service mathematics teachers (PSMTs) demonstrated an emerging understanding of Pedagogical Content Knowledge (PCK) in implementing Realistic Mathematics Education (RME) during microteaching. Both participants were able to introduce mathematical concepts through meaningful contexts: Participant A used installment-payment situations to teach annuities, while Participant B used paper folding and visual patterns to introduce geometric sequences. These practices reflected their initial ability to connect mathematical content with real-world or experiential situations and to organize learning activities that encouraged student participation.

However, the findings revealed differences and limitations across the four dimensions of PCK. In the curriculum dimension, both participants selected relevant contexts, but connections between prerequisite knowledge and new mathematical concepts were not always made explicit. In the pedagogical dimension, Participant A provided more structured guidance to help students identify relevant information and address misconceptions, whereas Participant B experienced difficulties in clarifying initial instructions and supporting students' transition from concrete activities to mathematical representations. In the content dimension, the progressive mathematization process remained insufficiently developed, particularly in making explicit how contextual or concrete situations could be transformed into formal mathematical models. In the learners dimension, both participants responded to students' questions and contributions, but opportunities for comparing strategies, negotiating mathematical ideas, and exploring students' reasoning in greater depth remained limited. Overall, the findings suggest that the integration of content knowledge, pedagogical strategies, and an understanding of students' thinking the core of PCK was still developing.

These findings highlight the potential of RME-based microteaching as a platform for developing and reflecting on PCK in mathematics teacher education. However, the findings also indicate that using contextual problems and organizing group activities are not sufficient on their own. PSMTs need further support in connecting prerequisite and new concepts, scaffolding progressive mathematization, responding systematically to students' reasoning, and facilitating discussion that enables students to compare and justify different mathematical strategies. Structured reflection, peer feedback, and focused mentoring may help PSMTs identify these areas for improvement and develop more coherent RME-based teaching practices.

This study is limited by the small number of participants and the use of microteaching settings, which may not fully represent the complexity of classroom teaching. Future research could involve a more diverse group of PSMTs, longitudinal observations, and teaching practices in authentic school settings. Further studies could also investigate how structured reflection, peer feedback, and targeted mentoring influence the development of PCK across different mathematical topics and pedagogical approaches. Such research may provide a deeper understanding of how teacher education programs can support prospective mathematics teachers in designing meaningful learning experiences and facilitating students' mathematical understanding.

Acknowledgements

"This research was funded by the Institute for Research and Community Service of Sanata Dharma University with grant number 016 PeneI./LPPM-USD/II/2025."

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