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Written Mathematical Communication Profiles: Symbolic, Visual, and Metacognitive Features in Inclusion–Exclusion Problem Solving

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Abstract

This study explores profiles of written mathematical communication in solving an inclusion–exclusion problem, with a focus on four interrelated components: writing, mathematical expression, drawing, and metacognitive verification. Although these components have often been examined separately, less is known about how students coordinate and verify them as integrated written communication profiles during problem solving. Employing a qualitative case study design, data were collected from undergraduate mathematics education students through written problem-solving tasks and task-based semi-structured interviews. From 34 initial written responses, two participants were purposively selected as contrasting cases because their written work represented different communication profiles and they were able to articulate their reasoning in follow-up interviews. These profiles were analyzed through iterative coding and cross-case comparison to understand the way the participants organize, express, and justify mathematical ideas in written form. The results suggest that the profiles observed were the results of participants' engagement with multiple representational systems. Symbolic-visual communication, the first of the two profiles, includes the use and combination of formal, sequentially bound systems and multiple representational systems while the second, systematic symbolic communication, involves the use of organized, rational thought structured through sequentially bound systems (i.e., without the use of multiple representational systems). Although each communication profile leads to the same correct answer, they diverge in the degree of engagement with representational systems and the verification process. The study's main contribution is showing that difficulties in written mathematical communication may stem less from procedural failure than from insufficient coordination and verification across written, symbolic, and visual representations. The study's main contribution is showing that difficulties in written mathematical communication may stem less from procedural failure than from insufficient coordination and verification across written, symbolic, and visual representations.

Keywords: Inclusion–Exclusion Principle; Mathematical Expression; Metacognitive Verification; Multiple Representations; Written Mathematical Communication;

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1. Introduction

Communication is a fundamental process through which individuals construct, share, and negotiate meaning across personal and social contexts. Within mathematics education, communication plays a critical role in shaping how learners express, interpret, and validate mathematical ideas. Mathematical communication, therefore, is not merely a supporting skill but a central component in the development of mathematical understanding and problem-solving competence (Annisavitri et al., 2020; Khadka, 2024; Rohid et al., 2019).

According to the National Council of Teachers of Mathematics (NCTM - 2000), the components of mathematical communication are: (a) complex organization of mathematically driven thought, (b) concise and clear communication of mathematical ideas, (c) objective analysis of functioning mathematical thought and strategies, and (d) the precise and appropriate use of mathematical language. These components of mathematical communication show that reasoning, representation, reflection, and mathematical communication are interconnected in pursuing the goals of mathematical education.

Mathematical commutation can further be divided into oral and written communication. Oral mathematical communication requires the use of spoken language to articulate reasoning, explain mathematical procedures, and engage in mathematical discourse and conversation (Johnston-Wilder & Lee, 2008; Stein, 2007). Written mathematical communication requires the use of symbols, language, and diagrams to explain ideas in an organized form and interpretable manner (Azizah et al., 2020).

The current study will focus on written mathematical communication as it acts as an enduring and analyzable manifestation of students' thought processes. In mathematics, written communication encompasses the use of multiple modalities, including visual, symbolic, and language components.

Prior research identifies three main aspects of mathematical communication: (1) writing, where one explains mathematical concepts in one's own words (Meaney et al., 2009; Menezes & Costa, 2020); (2) drawing, where one visually represents ideas via sketches or visual representations (Freeman et al., 2016; Meaney et al., 2009); and (3) mathematical expression, where one conveys practical or contextual problems in the language of formal mathematics (Bahrun & Dasari, 2023).

Beyond these three components, recent perspectives in mathematics education emphasize the importance of *metacognitive verification* as an integral dimension of communication. Written mathematical communication is not only about producing representations but also about monitoring, evaluating, and ensuring the consistency of those representations throughout the problem-solving process (Pugalee, 2001). Thus, effective communication requires the coordination of multiple representations along with reflective processes that validate the coherence of mathematical reasoning (Thompson & Chappell, 2007).

Literature highlights the importance of mathematical communication. It helps develop higher order thinking, it aids to the consolidation of the mathematics, and it can be considered as an indicator of students' conceptual understanding of the topic (Kaya & Aydın, 2016). Communication allows students to construct, refine and justify their mathematical ideas. As competence, communication is one of the essential skills prospective mathematics teachers need to develop. In higher education and specifically in mathematics education, students are expected to not only be able to follow the correct procedures in solving mathematical problems, but are also required to be able to communicate their answers and point of reasoning to their teachers, as well as to their classmates and to an audience beyond their classroom, including the wider community (Toh, 2012).

A specific area where the communication of mathematics in written form is essential is dealing with problems that involve the inclusion-exclusion principle. This principle, commonly dealt with in discrete mathematics, is an extension of the operations of sets, such as union and intersection, and is usually represented by the use of Venn diagrams (Shahriari, 2021). In general, solving inclusion–exclusion problems requires students to reason about set relations, intersections, complements,

cardinalities, and numerical manipulation. The inclusion–exclusion principle is particularly appropriate for examining written mathematical communication because it requires students to coordinate several cognitively demanding processes simultaneously: identifying the universal set, tracking overlapping sets, distinguishing pairwise and triple intersections, avoiding double counting, interpreting complements, and aligning symbolic formulas with possible visual representations such as Venn diagrams. These demands make inclusion–exclusion problems a productive context for revealing how students organize, translate, and verify mathematical ideas across written, symbolic, and visual forms.

Despite its importance, the existing literature has mostly considered the various elements of mathematical communication, such as the use of symbols or diagrams, as separate indicators rather than as coordinated components of an integrated communication process. Previous studies have provided valuable descriptions of students' ability to write explanations, use mathematical symbols, or construct visual representations, yet less attention has been given to how these components interact during problem solving and how students verify the consistency among them. In particular, the role of metacognitive verification in maintaining coherence between symbolic procedures and visual representations remains insufficiently explored. Therefore, further study is needed to understand how students coordinate writing, mathematical expression, drawing, and metacognitive verification as an integrated written communication profile.

Therefore, this study aims to explore the profiles of students' written mathematical communication in solving inclusion–exclusion problems. Rather than focusing on demographic comparisons, this study adopts a representational perspective, examining how different forms of communication emerge and interact during problem solving. Specifically, this study seeks to describe how students organize and express mathematical ideas through writing, construct and apply mathematical expressions symbolically, utilize visual representations such as diagrams, and verify the consistency of their solutions through metacognitive processes.

The main purpose of this study is to examine written mathematical communication in a holistic way. It aims for a deeper understanding of communication as both a type of representational and reflective activity. The novelty of this study lies in identifying written mathematical communication profiles based on the interaction among symbolic, visual, and metacognitive features, rather than evaluating each component separately. Theoretically, this study contributes to mathematics education research by conceptualizing written mathematical communication as a coordinated representational-verification process. Pedagogically, the findings are expected to inform the design of learning activities that support students in integrating multiple representations and systematically checking the coherence of their written mathematical reasoning.

2. Method

2.1 Research Design

This study analyzes students' written mathematical communication when solving an inclusion-exclusion problem. For this purpose, the researchers chose a qualitative case study as the primary research method (Crowe et al., 2011; Yin, 2018). This design was selected because the study aimed to examine communication profiles in depth rather than to measure the frequency or distribution of communication patterns across a large population. A case study approach is appropriate for capturing how written explanations, symbolic expressions, visual representations, and metacognitive verification interact within students' actual problem-solving work. This method provides a holistic understanding of a bounded phenomenon, including the written mathematical communication manifested and orchestrated while students are engaged in solving a mathematical problem. The bounded case in this study was defined as third-semester undergraduate mathematics education students' written mathematical communication while solving one inclusion–exclusion problem within a specific instructional and institutional context. The case was bounded by participant group, mathematical topic, task type, data sources, and data collection period. The case study method enables the researchers to understand students' communication through various dimensions, including the

written word, mathematical language, drawings, and metacognitive checks. The aim of the study is to document the complexity and diversity of communication profiles for a given circumstance, as opposed to making general assertions.

2.2. Participants and Sampling

The participants of this study were chosen from undergraduate students from a Math Education program at a Malang, Indonesia university. Initial data were collected from 34 third-semester students who volunteered to solve a written problem about the inclusion-exclusion principle. From the initial group, two participants, labeled S1 and S2, were selected through purposive criterion-based sampling using a contrasting-case logic. The selection of S1 and S2 was based on three standards: (1) the participant's commitment to attending follow-up data collection, (2) the participant's capacity to verbalize and clearly articulate their reasoning in interviews, and (3) the participant's alignment in the written response to illustrate the diverse characteristics of the corresponding communication profiles. The 34 written responses were first reviewed using the components of written mathematical communication. From this preliminary review, S1 and S2 were identified as information-rich contrasting cases: S1 represented a symbolic–visual profile involving both symbolic calculation and a Venn diagram, whereas S2 represented a systematic symbolic profile relying primarily on formal notation and sequential symbolic reasoning. Thus, the two participants were not selected to represent the whole population statistically, but to provide in-depth evidence of contrasting communication profiles that emerged from the broader pool of responses. Using two participants from the larger group is appropriate, as the qualitative case study approach emphasizes the importance of in-depth analysis.

Ethical considerations were addressed before data collection. Participants were informed about the purpose of the study, the voluntary nature of their participation, and their right to withdraw at any stage. Consent was obtained before the written task and interviews were conducted. To protect confidentiality, participants' identities were anonymized using the codes S1 and S2, and the data were used only for research purposes.

2.3. Data Sources and Instruments

This study utilized two primary sources of data: students' written responses to a problem involving the inclusion–exclusion principle, and task-based semi-structured interview transcripts. The written task was designed to elicit students' ability to coordinate multiple mathematical representations. The problem required students to determine the number of integers between 1 and 100 that are not divisible by 2, 3, and 5. The mathematical statement of the task as follows: Determine the positive integer n, where $1 \leq n \leq 1001$, and n is not divisible by 2, 3, and 5. This task was chosen because it requires the application of the inclusion–exclusion principle, which inherently involves the coordination of symbolic, numerical, and potentially visual representations (e.g., Venn diagrams).

Students' written responses were examined using an analytical rubric based on four components of written mathematical communication: writing, mathematical expression, drawing, and metacognitive verification. The writing component referred to how students stated known information, the target of the problem, and explanatory statements. The mathematical expression component referred to the use of set notation, cardinality, intersections, complements, formulas, and numerical operations. The drawing component referred to the use and relevance of visual representations, particularly Venn diagrams. The metacognitive verification component referred to evidence of checking, consistency monitoring, and alignment among symbolic, numerical, written, and visual representations.

The interviews were developed with a semi-structured approach in mind which allowed the researcher the flexibility to ask questions to students to elucidate their reasoning, response, and communication processes that were only made clear in the written response. The interviews were developed to understand the participants as they interpreted the problem, chose solution paths, created

representations, and determined the accuracy of their answers. To ensure the quality of the instruments, a mathematics education expert reviewed the written task, interview guide, and communication indicators in terms of mathematical correctness, clarity of wording, relevance to the inclusion–exclusion principle, and alignment with the four communication components. Revisions were made based on the expert's feedback before the instruments were used in data collection.

2.4. Data Collection Procedures

The data collection process was conducted in several stages. First, the written task was administered to 34 students, who were given approximately 30 minutes to complete the problem individually. Their responses were then analyzed using a predefined rubric focusing on components of written mathematical communication. Based on this preliminary analysis, two participants (S1 and S2) were selected to represent different communication profiles. The selection was also supported by recommendations from the course instructor to ensure that the participants were capable of articulating their reasoning during interviews.

Following the selection, individual task-based interviews were conducted on the same day. During these interviews, participants were asked to explain their written solutions, clarify their reasoning, and reflect on their problem-solving processes. The interviews were directly linked to each participant's written work; therefore, the researcher used the written response as a stimulus to ask why particular symbols, calculations, diagrams, or checking strategies were used. All interviews were audio-recorded to ensure accuracy in data transcription and analysis.

2.5. Data Analysis

Data analysis used an iterative qualitative framework divided into three core components: data reduction, data display, and conclusion drawing (Creswell, 2012; Mezmir, 2020). Data reduction began by transcribing all interview audio. The researcher then read through written interview data and transcripts multiple times to build an overall perspective of the data. Coding was conducted through a combination of deductive and inductive procedures. Deductive coding was guided by the four components of written mathematical communication: writing, mathematical expression, drawing, and metacognitive verification. Inductive coding was then used to identify emerging patterns, especially representational coordination, consistency across solution stages, and differences in how participants connected symbolic and visual forms.

Examples of data from written and oral (interview) work were annotated and categorized, respectively, to their corresponding indicators. The communication profiles were established by comparing each participant's dominant representational features across four problem-solving stages: understanding the problem, planning the solution, executing the solution, and reviewing the solution. A profile was assigned only when consistent evidence appeared across the written response and interview data. For example, the symbolic–visual profile was identified when symbolic expressions were accompanied by visual representation, whereas the systematic symbolic profile was identified when the solution was organized mainly through formal notation and sequential symbolic reasoning without diagrammatic support.

Data display was organized into structured narratives integrating written responses and interview excerpts for each participant. The four problem-solving stages—(1) understanding the problem, (2) planning the solution, (3) executing the solution, and (4) reviewing the solution—were used to describe the development of each participant's written mathematical communication.

At the final stage, the researcher analyzed the data to detect patterns and develop communication profiles for each participant. The data interpretation examined the communication components and the participants' ability to handle multiple representations during the problem-solving activity. The researcher used investigator triangulation to bolster the findings' credibility. The first author's preliminary findings were assessed independently by the second author. When disagreements arose, evidence from the data was used to reach an agreement. This triangulation

demonstrated that the findings were data-driven and reduced the author's bias (Korstjens & Moser, 2018).

3. Results and Discussion

3.1. Results

A. Profiles of Written Mathematical Communication in Solving Inclusion–Exclusion Problems

This section provides student examples of written mathematical communication used to solve an inclusion–exclusion problem. The analysis focuses on four components of written mathematical communication: communication through writing, communication through expression, communication through drawing, and communication through metacognitive verification. Mathematical expression refers to the use of set notation, cardinality, intersections, complements, formulas, and numerical operations. Drawing refers to the use of visual representations, particularly Venn diagrams, to represent set relationships. Metacognitive verification refers to evidence of checking and maintaining consistency among written, symbolic, numerical, and visual representations.

Two participants, S1 and S2, were selected to illustrate contrasting written mathematical communication profiles. The data were analyzed through the method of triangulation by comparing students' written work and task-based interview data. S1 was identified as representing a symbolic–visual written mathematical communication profile because the participant combined symbolic calculation with a Venn diagram. S2 was identified as representing a systematic symbolic written mathematical communication profile because the participant organized the solution primarily through formal notation and sequential symbolic reasoning without using a diagram.

B. Profile of S1: Symbolic–Visual Written Mathematical Communication

1) Understanding the Problem

At the stage of understanding the problem, S1 recognized the essential goal of the problem, that is, the determination of integers from the range of 1 to 100 that are not divisible by 2, 3, and 5. The partial understanding was, however, not clear in the written solution. No clear statements of known or asked that would have begun the answer were evident at the top of the written response. S1 did not clearly state the known information, the universal set, or the target of the problem at the beginning of the solution. Based on the writing indicator, this suggests that S1's problem understanding was present but remained only partially externalized in written form.

During the interview, S1's understanding of the problem was clearer in the written response. The understanding can be evidenced from the interview excerpt below.

R : What information is provided in the problem?
S1 : There are numbers from 1 to 100, and the problem asks us to determine how many numbers are not divisible by 2, 3, and 5.
P : How did you present this information in your written answer?
S1 : I rewrote the information from the question and then determined how many numbers are divisible by 2, 3, and 5.

The interview excerpt confirms that S1 understood the structure of the problem, but the written response did not fully reflect this understanding. Thus, the writing component at the problem-understanding stage was coded as implicit rather than explicit.

2) Planning the Solution

During the planning stage, S1 began to demonstrate more systematic mathematical representation. S1 used the symbols C_1, C_2, and $C_3$to represent the sets of numbers divisible by 2, 3, and 5, respectively. Next, S1 determined the cardinality of each set and the intersections among the

sets as the basis for applying the inclusion–exclusion principle. The planning process involved the following calculations: $N(C_1) = 50$, $N(C_2) = 33$, and $N(C_3) = 20$, as well as $N(C_1 \cap C_2) = 16$, $N(C_1 \cap C_3) = 10, N(C_2 \cap C_3) = 6$ and $N(C_1 \cap C_2 \cap C_3) = 3$.

Based on the mathematical expression indicator, S1 was able to translate contextual information about divisibility into formal mathematical structures involving set notation, intersections, and cardinality. However, the written explanation accompanying these symbolic steps was still limited. Therefore, S1's communication at this stage was dominated by symbolic expression, while explanatory writing remained minimal.

3) Executing the Solution

At the execution stage, S1 substituted all calculated values into the inclusion–exclusion principle to determine the number of integers not divisible by 2, 3, and 5. The mathematical process can be represented as follows:

$$100 - [(50 + 33 + 20) - (16 + 10 + 6) + 3] = 26$$

This process shows that S1 correctly integrated the relationships among unions, and both the intersections and complements. Mathematical expressions came to the forefront through the symbol use, equations, and numerical placements that were consistent with the inclusion-exclusion principle. Along with the use of symbols, S1 drew a Venn Diagram, in line with the principle, to further illustrate the problem.

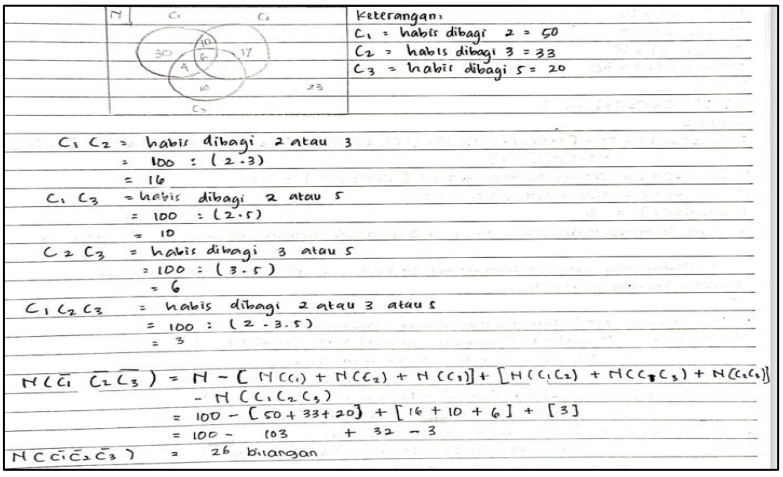


Figure 1. S1's Written Work in Solving the Inclusion–Exclusion Problem.
Note. The Indonesian Term "Keterangan" Means "Explanation/Notes," and "Habis Dibagi" Means "Divisible By."

The figure shows S1's attempt to connect the sets of numbers divisible by 2, 3, and 5 with a Venn diagram and symbolic cardinality calculation. The Venn diagram supports the classification of S1 as a symbolic–visual communicator because the participant used both symbolic notation and visual representation to organize the set relationships. However, the diagram did not merely function as an illustration; it became part of the communication structure through which S1 attempted to represent intersections among the three divisibility sets.

4) Reviewing the Solution

At the review stage, one inconsistency was identified between S1's symbolic calculation and the associated visual representation. Even though the symbolic calculation produced the correct answer of 26, the Venn diagram showed a symbol for a different number. This inconsistency indicates that S1's metacognitive verification was incomplete. In terms of the coding indicators, S1 demonstrated strong mathematical expression and emerging drawing, but the coordination between symbolic and visual representations was not fully verified.

C. Profile of S2: Systematic Symbolic Written Mathematical Communication

1) Understanding the Problem

Unlike S1, S2 demonstrated more systematic written mathematical communication from the beginning of the solution process. S2 explicitly represented the given information using set notation, see Figure 2.

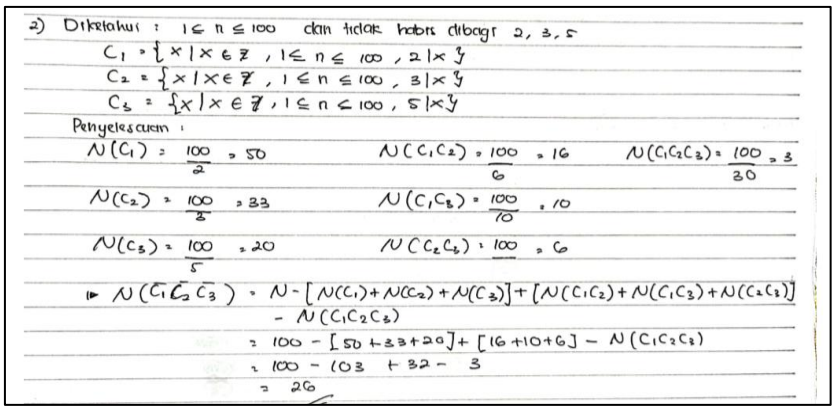


Figure 2. S2's Written Answer in Solving the Inclusion–Exclusion Problem. Note. the Indonesian Term "Diketahui" Means "Given," While "Habis Dibagi" Means "Divisible By."

Information regarding numbers divisible by 2, 3, and 5 was represented using the symbols C_1, C_2, and C_3. This was confirmed during the interview session.

R : How did you present the information in your written answer?
S2 : I wrote the information using sets of numbers divisible by 2, 3, and 5.
P : Why did you use this notation?
S2 : So that the writing would be shorter but still clear.

The interview excerpt shows that S2 understood mathematical notation not only as a procedural tool but also as a communication tool that supports concise and structured representation. Based on the writing and mathematical expression indicators, S2's problem understanding was coded as explicit because the written response directly transformed the verbal problem into formal set notation.

2) Planning the Solution

At the planning stage, S2 selected the inclusion–exclusion principle as the solution strategy by determining the cardinality of each set and their intersections. S2 calculated: $N(C_1)$, $N(C_2)$, and $N(C_3)$ as well as all pairwise intersections and the three-set intersection before substituting the values into the formula. At this stage, S2's written mathematical communication was predominantly symbolic and formal. The entire planning process was constructed through symbolic representation without additional visual representation. Thus, the drawing indicator was not observed in S2's written work, while the mathematical expression indicator appeared strongly.

3) Executing the Solution

At the execution stage, S2 systematically substituted numerical values into the inclusion–exclusion principle. The process can be represented as follows.

$$100 - (50 + 33 + 20) + (16 + 10 + 6) - 3 = 26$$

This representation demonstrates that S2 was able to organize the mathematical procedure systematically and consistently. The *mathematical expression* component became dominant through the use of symbols, formulas, numerical substitution, and arithmetic operations. Unlike S1, S2 did

not use diagrams or additional visual representations during the solution process. The entire planning process was constructed through symbolic representation without additional visual representation. Thus, the drawing indicator was not observed in S2's written work, while the mathematical expression indicator appeared strongly.

4) Reviewing the Solution

At the reviewing stage, S2's written work did not show inconsistencies between procedures and the final result. The final answer was consistent with the substitution and arithmetic operations performed. Although S2 did not write an explicit checking statement, the alignment among notation, substitution, operation, and final result indicates implicit metacognitive verification. Thus, S2's metacognitive verification was coded as implicit but stable because consistency was maintained across the symbolic solution structure.

D. Cross-Case Comparison of S1 and S2

A cross-case comparison shows that S1 and S2 differed not in the correctness of the final answer, but in how they organized, represented, and verified their mathematical ideas. S1 combined symbolic notation and visual representation; therefore, the symbolic–visual profile was characterized by the presence of both mathematical expression and drawing. However, S1's written communication also showed a verification weakness because the visual representation was not fully consistent with the symbolic calculation. By contrast, S2 relied on concise symbolic notation and sequential formal reasoning without using a diagram. The systematic symbolic profile was characterized by explicit written organization, strong symbolic expression, absence of drawing, and relatively stable consistency across the solution steps.

These two profiles indicate that written mathematical communication can follow different representational pathways. The symbolic–visual pathway may enrich communication by making set relationships visible, but it also requires careful verification between symbolic and visual forms. The systematic symbolic pathway may support procedural consistency, but it provides less visual access to the structure of set intersections. Therefore, the main difference between the two profiles lies in representational coordination and verification, rather than in procedural correctness alone.

3.2. Discussion

This study shows that written mathematical communication in solving inclusion–exclusion problems is not determined only by the correctness of the final answer. Both participants obtained the correct answer, yet they differed in how they organized information, transformed contextual information into symbolic form, used or avoided visual representation, and verified the consistency of their solution. This finding supports the view that written mathematical communication should be understood as a coordinated representational process involving writing, mathematical expression, drawing, and metacognitive verification.

The two identified profiles, symbolic–visual and systematic symbolic, demonstrate different representational pathways. S1's symbolic–visual profile shows that a student may use multiple representations to communicate mathematical ideas, but the use of multiple representations also increases the need for consistency checking. S2's systematic symbolic profile shows that a student may communicate effectively through formal notation and sequential symbolic reasoning without relying on diagrams. Therefore, the central issue is not whether students use many representations, but whether the representations they use are coordinated, coherent, and verified.

A. Writing as Externalized Mathematical Organization

One of the major findings concerns the role of *writing* in organizing mathematical thinking. Although both participants were able to solve the problem correctly, the structure and explicitness of their written communication differed substantially. S1 demonstrated an implicit form of writing. The participant appeared to understand the problem conceptually, as evidenced by the interview data, yet this understanding was not fully externalized in the written solution. Important information such as

the known conditions and the target of the problem was not explicitly organized at the beginning of the response. This indicates a gap between internal understanding and written representation.

In contrast, S2 explicitly structured the problem information using mathematical notation from the beginning of the solution process. The participant intentionally transformed verbal information into symbolic set representations to create a shorter yet clearer explanation. This finding suggests that writing in mathematics functions not merely as documentation but also as a cognitive tool for organizing mathematical reasoning.

The comparison between these profiles suggests that written mathematical communication varies in the degree of representational explicitness. There may be students who understand the concepts and can provide reasoning whereby the only hurdle is the lack of written structure. Therefore, the evaluation for the mathematical communication should be beyond the answer and capture the reasoning process that is presented in clear and organized manner. These findings endorse those that view math literacy as the combination of cognitive and communicative aspects (Lim & Pugalee, 2004; Pugalee, 2001). An organized response enables students to understand the relationships among different concepts and helps maintain sequential logic, and aids in tracing the progression of reasoning (Goldsby & Cozza, 2002). Therefore, the primary absence of the ability of students to solve mathematical problems should not be interpreted as the lack of understanding of the concepts, but as the lack of ability to convert the mental conceptualization of the problems into an organized response (Khairunnisa et al., 2020; Wandari & Anggara, 2021).

B. Mathematical Expression as Representational Transformation

Another important finding concerns the role of mathematical expression in transforming contextual information into formal mathematical structures. Both participants represented divisibility relationships using set notation, cardinality symbols, intersections, and numerical operations associated with the inclusion–exclusion principle. However, the nature of mathematical expression differed between the two profiles. S1 coordinated mathematical symbols with a visual representation, whereas S2 relied primarily on formal symbolic procedures.

These differences indicate that symbolic representation is not constructed uniformly across students. In inclusion–exclusion problems, students must coordinate set membership, divisibility, intersections, unions, complements, and cardinality. Therefore, mathematical expression involves more than placing symbols correctly; it also requires students to preserve the meaning of relationships among sets throughout the solution process. This interpretation is consistent with the view that mathematical symbols function within a broader communicative and semiotic system, rather than as isolated procedural marks (Sfard, 2002; Sfard & Kieran, 2001).

C. Drawing as Conceptual Visualization

The emergence of *drawing* in S1's written work provides important insight into the role of visual representation in mathematical communication. The use of a Venn diagram allowed S1 to spatially organize relationships among sets and visualize intersections among numbers divisible by 2, 3, and 5. The diagram functioned not merely as decoration or supplementary illustration but as a representational tool for structuring mathematical relationships. This finding supports the view that visual representation can serve as an important mediator between abstract symbolic concepts and conceptual understanding.

Through the Venn diagram, S1 attempted to coordinate numerical relationships spatially. The visual representation appeared to support the participant's reasoning about overlaps among sets, which is a central conceptual component of the inclusion–exclusion principle. In this sense, drawing functioned as a cognitive scaffold that facilitated the organization of abstract relationships.

The findings also suggest an important constraint on visual representation. While the diagram offered support in organizing concepts, it also created a lack of consistency during the final stage of the problem solving process. The difference between the symbolic output and the numbers written in

the diagram evidences that visual representations demand a more detailed synchronization of symbolic procedures. This corroborates (Kulpa, 2003), claiming that the self-consistency of diagrammatic representations is, at some points, ignored due to the lack of precision or the structural restrictions of the visual language. (Cheng, 2003), points out that the limitations of representation mostly manifest themselves in the problems of a conceptual nature and the inability to solve a problem, while (Koedinger and Nathan, 2004), is of the opinion that the variations of the external representations affect function and learning to a great extent.

The research studies encourage the use of self-consistency to relate coherent communication to suggest representation, and to balance representational systems. This, naturally, presents a potentially important sustaining factor, and self-validation, in visual representation. The diagrams cannot, of themselves, fail or succeed in communicating the mathematical ideas unless the students integrate the diagrams and representational systems cohesively. S2 solved the problem symbolically in a completely diagram-free environment, leaving S2 to represent a pathway that is justified outside and, in some aspects, negates the requirement for visual representation to be adopted for solving the underlying problem. The communication was twofold, in that the self-validation and representational system moved to balance and, in some aspects, negated the visual representation to solve the problem. The students adopted representational pathways for solving problems, of which drawing was one of the components, but in some aspects, it faded to be the most important factor. S2 left the, in some aspects, “representational system” to exist for visual representation.

Lowrie (2001) notes that the way individuals conceptualized representations did not impact their problem-solving ability, although students who leaned more heavily toward visual representations as compared to more conceptual representations experienced different results. This indicates that the use of representations is a preference of individuals, not a requirement that is experienced by all. On the other hand, the gap between the dual profiles indicates that the way students are allowed to choose representations affects the way students are able to articulate their thinking in mathematics, as well as the way students reflect over their reasoning.

D. Metacognitive Verification and Representational Consistency

Perhaps the most significant finding of this study concerns the role of metacognitive verification in maintaining consistency across representations. S1 correctly obtained the final numerical answer through symbolic procedures but failed to maintain consistency in the visual representation. This inconsistency indicates that the verification process did not extend from checking the final answer to checking the coherence among representations. In this sense, metacognitive verification should not be understood merely as confirming arithmetic correctness, but as monitoring whether written explanations, symbolic procedures, numerical results, and visual representations communicate the same mathematical meaning (Pugalee, 2001; Bellon et al., 2020)

The verification gap in S1’s work may be explained by procedural closure after obtaining the correct symbolic answer. Once the numerical result was reached, S1 appeared to treat the problem as solved and did not re-examine whether the supporting Venn diagram communicated the same set relationships as the symbolic calculation. In this case, the correct numerical result functioned as a closure point, while the diagram was treated more as a supplementary illustration than as a representation requiring mathematical accountability. This interpretation is consistent with the view that multiple representations support mathematical understanding only when learners are able to coordinate their meanings and functions across representational forms (Ainsworth, 2006; Duval, 2006).

Another possible explanation is the cognitive demand involved in coordinating multiple representations. Inclusion–exclusion problems require students to manage the universal set, individual sets, pairwise intersections, triple intersections, complements, and the final subtraction from the universal set. When attention is concentrated on formula application and arithmetic accuracy, fewer cognitive resources may remain for verifying whether the diagram accurately represents the same relationships. This may explain why a university-level student can produce a

correct symbolic solution while still leaving an inconsistency in the visual representation. From a cognitive-load perspective, problem solving can impose substantial demands on working memory, particularly when learners must coordinate several elements simultaneously (Sweller, 1988).

This finding also suggests a tendency to privilege symbolic procedures over diagrammatic accuracy. In formal mathematics learning, students may regard symbolic computation and the final answer as the primary evidence of correctness, whereas diagrams may be treated as secondary or illustrative. However, when a diagram is included in written mathematical communication, it becomes part of the mathematical argument and must be verified. This is especially important in inclusion–exclusion problems because the Venn diagram is not merely visual support; it represents set membership, overlap, and complement relations that should be consistent with the symbolic inclusion–exclusion formula. Thus, the present finding challenges the assumption that correct symbolic computation alone is sufficient evidence of coherent mathematical communication.

In contrast, S2 demonstrated relatively stable consistency throughout the solution process. Although no explicit written reflection or checking strategy was identified, the alignment among procedures, substitutions, and final results suggests stronger procedural monitoring. In this case, *metacognitive verification* appeared implicitly through representational stability. This finding is consistent with Goswami et al. (2020), who showed that learners can detect inconsistencies and incoherence in problem-solving processes without explicit reflection, with behavioral patterns reliably predicting task outcomes. Similarly, Mata (2023) demonstrated that metacognitive monitoring enables individuals to distinguish between correct and incorrect answers, indicating that verification can operate implicitly through the consistency of representations and procedural alignment.

Overall, the findings extend the understanding of written mathematical communication by emphasizing representational verification as a central component. Effective written mathematical communication requires students not only to produce written, symbolic, or visual representations, but also to align and validate them as a coherent whole. This supports the view that the quality of mathematical communication is closely related to students' ability to form, coordinate, and verify representations across different components of a solution (Masfingatin et al., 2020). Overall, the findings extend the understanding of written mathematical communication by emphasizing representational verification as a central component. Effective written mathematical communication requires students not only to produce written, symbolic, or visual representations, but also to align and validate them as a coherent whole. This supports the view that the quality of mathematical communication is closely related to students' ability to form, coordinate, and verify representations across different components of a solution

4. Conclusion

This study examined profiles of written mathematical communication in solving an inclusion–exclusion problem, focusing on four components: *writing*, *mathematical expression*, *drawing*, and *metacognitive verification*. The findings show that written mathematical communication is a multidimensional process involving the coordination of multiple representations rather than a purely procedural activity. Two distinct communication profiles were identified. The first reflects a symbolic–visual orientation, in which symbolic procedures are supported by visual representations such as Venn diagrams. The second reflects a systematic symbolic orientation, characterized by the structured use of formal notation without visual support. Both profiles led to correct solutions; however, they differed in how mathematical ideas were organized, represented, and verified. A key finding is the role of *metacognitive verification* in maintaining consistency across representations.

A key finding of this study is that correctness of the final answer does not necessarily indicate coherence in written mathematical communication. In S1's case, the symbolic procedure produced the correct numerical answer, but the visual representation was not fully consistent with the symbolic solution. In S2's case, the solution was symbolically consistent, although it did not involve visual representation. These findings indicate that metacognitive verification plays a central role in ensuring

that written, symbolic, numerical, and visual representations are aligned throughout the problem-solving process.

Theoretically, this study positions written mathematical communication as an integrated representational-verification process. Written mathematical communication is not limited to the production of explanations, formulas, or diagrams; rather, it involves the ability to coordinate and validate these representations as a coherent mathematical argument.. From a pedagogical stance, representational-verification process. Written mathematical communication is not limited to the production of explanations, formulas, or diagrams; rather, it involves the ability to coordinate and validate these representations as a coherent mathematical argument.

Representational-verification process. Written mathematical communication is not limited to the production of explanations, formulas, or diagrams; rather, it involves the ability to coordinate and validate these representations as a coherent mathematical argument. Future research may extend this work by representational-verification process. Written mathematical communication is not limited to the production of explanations, formulas, or diagrams; rather, it involves the ability to coordinate and validate these representations as a coherent mathematical argument.

Overall, The study demonstrates the quality of written mathematical communication representational-verification process. Written mathematical communication is not limited to the production of explanations, formulas, or diagrams; rather, it involves the ability to coordinate and validate these representations as a coherent mathematical argument.

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