

MATHEMATICAL REPRESENTATION ABILITY OF MATHEMATICS EDUCATION STUDY PROGRAM STUDENTS

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Abstract

This research is descriptive because it aims to describe students' mathematical representation ability. Mathematical representation is related to ways of expressing mathematical ideas. Mathematical representation is divided into verbal, visual/pictorial, and symbolic representations. The subjects of this study were 40 new Mathematics Education study program students. Data were collected using test and interview techniques.

Furthermore, the data were analyzed using descriptive and qualitative statistics. The results showed that only 10% of the students had the mathematical representation ability in the good category, 12.5% in the moderate category, 10% in the poor category, and 67,5% in the very poor category. In detail, for verbal skills, there is 2.5% in the very good category, 7.5% in the good category, 17.5% in the moderate category, 15% in the poor category, and 55% in the very poor category. For the ability of visual/pictorial representation, there is 2.5% in the very good category, 7.5% in the good category, 12.5% in the moderate category, 7.5% in the poor category, and 70% in the very poor category. Furthermore, for the symbolic representation ability, 10% is in a good category, 10% in the moderate category, 7.5% in the poor category, and 72.5% in the very poor category. Based on the classroom observation and interview, the low mathematical representation ability is caused by two main things, namely (1) relatively low prerequisite knowledge and (2) mathematics learning, which is dominated mainly by teachers; learning does not provide opportunities for students to develop mathematical representation skills.

Keywords: mathematical representation, students ability



1. Introduction

Mathematics has a strategic position in Indonesian education. Mathematics needs to be given to all students from elementary school to college. Mathematics is intended to equip students with the ability to think logically, analytically, systematically, critically, and creatively, as well as the ability to cooperate. These competencies are needed so students can have the ability to acquire, manage, and use the information to survive in ever-changing, uncertain, and competitive conditions (Ratumanan & Mattitaputty, 2017; Ratumanan & Laurens, 2016).

Several important abilities need to be developed through learning mathematics, including mathematical connections and mathematical representations. National Council of Teachers of Mathematics (NCTM) (2000), describes the existence of 5 (five) standards of mathematical abilities that students must master, namely (1) problem solving, (2) reasoning and proof, (3) communication, (4) connection, and (5) representation.

From the NCTM concept, it is clear that representation is one of the important abilities that must be developed in learning mathematics. Representation is one of the main features of learning mathematics and solving problems. In many mathematics classes, children are often introduced to various representations. Mathematics teachers often use multiple representations, including concrete materials, virtual manipulation, pictures, also written and spoken symbols during instruction (Ahmad., et.al, 2010; Bakar, 2017). Mathematical representation is related to how students identify various aspects of the faced problem, present the problem symbolically or in the form of a mathematical model, choose solving strategies, carry out algorithmic activities, and make interpretations either orally or in writing.

The mathematical representation ability is one of the most important abilities for students and is one of the goals to be achieved in learning mathematics at schools. Student success in problem-solving can not be separated from the role of representation (Bal, 2014; Anwar., et.al, 2016). Representation is very useful in helping students solve a problem more easily. Representation is also useful as a medium to communicate students' mathematical ideas to the teacher or other students. Learning mathematics in class should provide sufficient opportunities for students to practice and develop mathematical representation ability (Sabirin, 2014).

NCTM (2000), further explains that mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, number and letter symbols, spreadsheet views, and so on. The mathematical ideas representation ways are fundamental to how people understand and use those ideas. Many of the representations we now take for granted are the result of the cultural improvement process over the years. When students gain access to mathematically represent their ideas, when they can create representations to capture mathematical concepts or relationships, they acquire a set of tools that significantly expand their capacity to model and interpret physical, social, and mathematical phenomena.

Mathematical representations can be divided into visual representations and non-visual representations. Visual representations include graphs, tables, sketches/drawings, and diagrams; Non-visual representations include numerical representations and mathematical equations or mathematical models (Minarni, et.al, 2016). Representations can also be divided into verbal, numerical, graphical, and algebraic representations. Verbal representations are usually used in proposing a problem and also required in the final interpretation of the results obtained in the solution-finding process. Numerical representations are familiar to students in the early algebraic stage. The numerical approach provides a convenient and effective bridge to algebra and often precedes the use of other representations. The use of numbers is important in gaining the first understanding of a problem and in the specific case investigation. A graphical representation is effective in providing a clear picture of the real value function of real variables.

The graphics are intuitive and very attractive to students who like a visual approach. On the other hand, graphical representations may be less accurate, influenced by external factors (such as scaling), and often only represent part of the problem's domain or range. Its strength as a mathematical tool varies according to the faced task. Algebraic representation is succinct, general, and effective in presenting mathematical patterns and models, therefore, it is a powerful tool. Algebraic objects manipulation is sometimes the only method to justify or prove general statements. However, the algebraic symbols exclusive use (at any stage of learning) can obscure or obstruct the represented objects' mathematical meaning or nature and cause difficulties in some students' interpretations of their results (Friedlander and Michal, 2001). NCTM in (Loc and Phuong, 2019),

divided mathematical representations into visual, symbolic, verbal, contextual, and physical representations.

Villegas, et.al (Villegas, et.al, 2009), divided external representations into 3 (three) types, as follows:

- Verbal representation: consists of problems that are stated either in writing or orally
- Pictorial representation: consists of pictures, diagrams, or graphs, as well as all kinds of related actions; and
- Symbolic representation: consists of numbers, operation signs and relations, algebraic symbols, and all kinds of actions that refer to it.

The relationship between these three representations is described as follows:

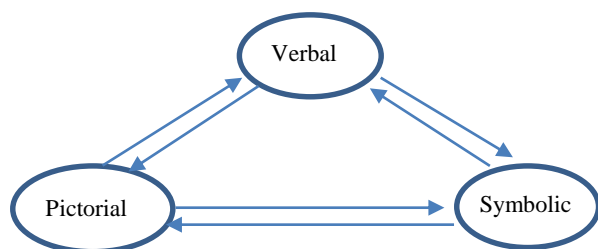


Figure 1. Types of Representation System (Villegas, et.al, 2009)

Mathematical representation clearly is an important process and capability. It allows students to have a clearer picture in a problem solving process. For example, finding a maximum solution in a linear programming problem will be easier if students are able to express the information provided in graphical forms and solve the problem by using various methods. Thus, it is also easier for the students to determine an area between two curves, if they are able to make a graph correctly and express it in a certain integral form (symbolic representation). In conclusion, the study of mathematical representation is one of the important studies in mathematics education.

This study will further examine the mathematical representation ability of students in a mathematics education study program. It will also examine the factors that influence the representation ability. The results of this study will be useful in terms of a more effective mathematics learning arrangement in relation to the development of mathematical representation ability.

2. Method

This research is descriptive research because it aims to describe student's mathematical representation ability and the causing factors. The intended mathematical representation ability will be studied and described more specifically, including (1) verbal representation, (2) visual/pictorial representation, and (3) symbolic representation.

The population of this study was the 1st semester 2020/2021 academic year Mathematics Education study program's new students. The research samples are new students who are Maluku Province's high school graduates. From 47 new students, only 40 came to take the test, so the sample of this research was 40 (85,11%). Data are collected using test and interview techniques. The test is intended to identify the mathematical representation ability. There are 3 (three) questions on the test. On question 1, given the absolute value function, students are asked to draw and determine the value of the function at time t . On question 2, given the center and the radius of two circles, students are asked to draw and determine the length of the tangent of the two circles. On question 3, given two functions, students are asked to describe the two functions and determine the area bounded by the two functions.

The students' works are then assessed using a specially developed rubric to assess verbal, visual, and symbolic representation skills.

Furthermore, referring to the test results, 3 (three) students were selected for further study. They are selected by considering their communication ability. 1 (one) student represented the high or very high representation score group, 1 (one) student represented the moderate representation score group, and 1 (one) student represented the low or very low score group.

Students' representational abilities are divided into 3 (three) groups, namely (1) very high and high categories, (2) moderate categories, and (3) low or very low category. In each group, 1 (one) person was selected for an in-depth interview. A student with a good communication skill was what the researchers needed so that the interview would run well.

The data obtained were then analysed using descriptive statistics and qualitative analysis. The data on the mathematical representation ability test results were analysed using the five scales conversion as follows:

Table 1. Five Scale Value Conversions

Score Interval	Letter	Category
$85\% \leq x$	A	Very High
$70\% \leq x < 85\%$	B	High
$55\% \leq x < 70\%$	C	Moderate
$40\% \leq x < 55\%$	D	Low
$x < 40\%$	E	Very Low

(Ratumanan & Laurens, 2015)

Data from learning observations and interviews then analyzed using qualitative analysis techniques according to Creswell (2014), namely by the stages (1) preparing and organizing data, (2)

Table 2. Representation Ability

Category	Verbal		Pictorial/Visual		Symbolic		Total	
	f	%	f	%	f	%	f	%
Very Good	1	2.5	1	2.5	0	0	0	0
Good	4	7.5	3	7.5	4	10	4	10
Moderate	7	17.5	5	12.5	4	10	5	12.5
Poor	6	15	3	7.5	3	7.5	4	10
Very Poor	22	55	28	70	29	72.5	27	67.5
Total	40	100	40	100	40	100	40	100

Table 2 above showed that the new students mathematical representation ability is relatively low. Only 10% of students had the mathematical representation ability in the good category, 12.5% in the moderate category (enough); the largest percentage was in the very poor category, namely 67.5%, and 10% in the poor category. In detail, for verbal skills, there are 2.5% in the very good category, 7.5% in the good category, 17.5% in the moderate category, 15% in the poor category, and 55% in the very poor category. For the ability of visual/pictorial representation, there are 2.5% in the very good category, 7.5% in the good category, 12.5% in the moderate category, 7.5% in the poor category, and 70% in the very poor category. Furthermore, for the symbolic representation ability, 10% is in a good category, 10% in the moderate category, 7.5% in the poor category, and 72.5% in the very poor category.

Furthermore, from the results of the interviews, qualitative analysis with 3 (three) selected subjects, the following results were obtained:

3.1 Subject M.S

In question 1, M.S used the definition of absolute value to describe $f(t) = -2|t - 10| + 20$. M.S described the function $f(t)$ for $t \geq 10$ and for $t < 10$ separately, then combines the two figures.

data reduction, (3) data presentation, and (4) drawing conclusions. To ensure the validity of the data, triangulation was carried out by comparing the results of the students' work and the results of their interviews.

3. Result and Discussion

From the new students' mathematical representation test, the ability of verbal, pictorial/visual, and symbolic representations can be identified as presented in Table 2 below:

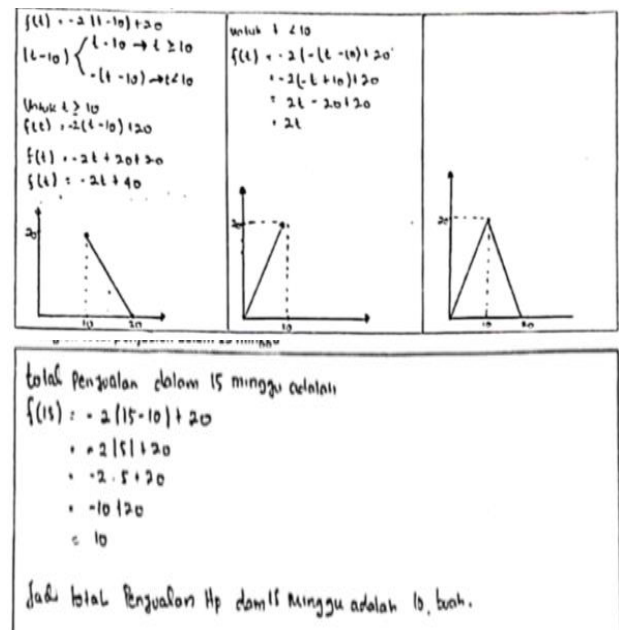


Figure 2. M.S' Solution for Question 1

From the interview, it was known that M.S could correctly explain what was known and asked in the questions. M.S is able to explain the definition of absolute value so that he can use it to elaborate $f(t) = -2|t - 10| + 20$, namely into $t \geq 10$ and $t < 10$.

But to calculate the total sales in 15 weeks, M.S made a mistake. M.S misinterpreted the question purpose, so M.S was wrong in making mathematical modeling, which is only looking for the value of $f(15)$. In the interview, M.S was able to explain what was asked, namely counting the total sales in 15 weeks, but was doing it wrong again, M.S was only looking for the value of

$f(15)$. When asked what was meant by total sales, then M.S realized the mistake. The following is an excerpt from the interview results:

P	: Why $f(15)$ is sought?
M.S	: in the question, it was asked to calculate the total sales in 15 weeks
P	: What is meant by total sales?
M.S	: Oh, yes, it should be added up
P	: How to add it up?
M.S	: $f(1) + f(2) + f(3) + \dots + f(15)$
P	: Good, what form or concept that fits it?
M.S	: arithmetic sequence, Sir
P	: What formula should be used to solve this?
M.S	: arithmetic sequence formula, Sir

From the interview above, it can be concluded that M.S made the wrong mathematical modelling and solution because M.S misinterpreted the problem. After being guided through the interview, M.S was able to realize his mistake and explain the concepts and rules that should be used in determining the total sales in 15 weeks.

In question 2, M.S could correctly identify the question, the known, asked and must be done components. M.S was able to make the drawing correctly. It was just incomplete, because M.S did not draw a tangent. However, from the solution and interview, it was known that M.S understood well the steps to solve the problem. When question 2's solution was shown in the interview, M.S realized

that the drawing he had made was incomplete, and he could make the outer and inner alliance tangents. From the solutions done by M.S, it was clear that M.S understands and could write the formula to determine the length of the outer alliance tangent correctly and solved question 2 correctly. Therefore, in question 2, it was clear that M.S shows mathematical representation ability, either verbal, visual/pictorial, or symbolic representations.

In question 3, M.S could identify the problem well, M.S could also graph $y = x^2 + 4x + 3$ and $y = 2x + 6$, as well as the area bounded by the two curves. M.S could also write down in detail the steps in making a picture of the two functions. M.S also understood the rules for determining the area bounded by the two functions, namely by using a finite integral. M.S could write the formula correctly, namely $\int_{-3}^{-1} (y_1 - y_2) dx$; but in substituting y_1 and y_2 , M.S made a mistake, M.S wrote $y_1 = x^2 + 4x + 3$ and $y_2 = 2x + 6$ instead of $y_1 = 2x + 6$ and $y_2 = x^2 + 4x + 3$. M.S did not pay attention to the position of the image that $y_2 > y_1$ is in the interval $[-1, -3]$, thus a mistake was created in the making of the symbol representation.

As a result, the further solution to determine the area was wrong. M.S also did not make the result interpretation of the integral operation obtained.

$$y = x^2 + 4x + 3 \text{ dan } y = 2x + 6$$

$$\rightarrow y = y$$

$$x^2 + 4x + 3 = 2x + 6$$

$$x^2 + 4x + 3 - 2x - 6 = 0$$

$$x^2 + 2x - 3 = 0 \rightarrow x_1 = -3, x_2 = 1$$

$$\rightarrow \int_{-3}^{-1} (y_1 - y_2) dx$$

$$= \int_{-3}^{-1} (x^2 + 4x + 3) - (2x + 6) dx$$

$$= \int_{-3}^{-1} (x^2 + 2x - 3) dx$$

$$= \left[\frac{1}{3}x^3 + \frac{1}{2} \cdot 2x^2 - 3x \right]_{-3}^{-1} dx$$

$$= \left[\frac{x^3}{3} + x^2 - 3x \right]_{-3}^{-1} dx$$

$$= \left[\left(\frac{(-1)^3}{3} + (-1)^2 - 3(-1) \right) - \left(\frac{(-3)^3}{3} + (-3)^2 - 3(-3) \right) \right]$$

$$= \left(\frac{-1}{3} + 1 - 3 \right) - \left(-\frac{27}{3} + 9 + 9 \right)$$

$$= \frac{-1}{3} - 2 - 9$$

$$= \frac{-1}{3} - 11$$

Figure 3. M.S' Solution for Question 3

On the interview, M.S realized the mistakes made in the making of the mathematical model of this problem. Mistakes were made because (1) y_1 and y_2 were determined based on the order of the functions in the problem, not on the graph position, and (2) there was also an operating error. The following is an excerpt of an interview with M.S.

P : What to look for in question number 3?
 M.S : The area between the two curves
 P : How to determine the width of the area
 M.S : Integral with a lower bound of -3 and an upper bound of 1 of $(y_1 - y_2)$
 P : Why did you write here that $y_1 = x^2 + 4x + 3$ and $y_2 = 2x + 6$?
 M.S : Just like the question, sir.
 P : What do you mean?
 M.S : in the question it is said that the area is bounded by the curve $y = x^2 + 4x + 3$ and $y = 2x + 6$, so $y_1 = x^2 + 4x + 3$ and $y_2 = 2x + 6$
 P : From these equations, what is the answer you got?
 M.S : $\frac{32}{3}$ Sir
 P : How did you come to that conclusion?
 M.S : (explains as what is written on his/her paper)
 P : Do you think it is correct?
 M.S : (Think quickly) Yes, sir.
 P : Pay attention, please. $\frac{1}{3} - 2 - 9$ (While showing M.S' work) What is the result?
 M.S : (Recalculate) It is $-\frac{32}{3}$, sir.
 P : Can the width of an area be negative?
 M.S : (Think quickly) No, sir.
 P : Then something must be wrong.
 M.S : (Look at his/her paper) Perhaps the y_1 and y_2 are swapped.
 P : Then what must you do instead?
 M.S : then it must be $y_1 = 2x + 6$ and $y_2 = x^2 + 4x + 3$

From the description above, it can be concluded that M.S has a relatively good mathematical representation ability. More specifically, M.S's visual/pictorial representation ability was in the very good category, the symbol's representation ability was in the good category, and the verbal representation ability was in the moderate category.

3.2 Subject O.K

In question 1, O.K did not use the absolute value definition in picture making. O.K also did not make use of t values substitution at the $0 \leq t \leq 20$ interval. O.K was only seek for the $f(20) = 40$ value, then made a graph. As a result, the function $f(t) = -2|t - 10| + 20$ was incomplete. The picture referred to was as follows:

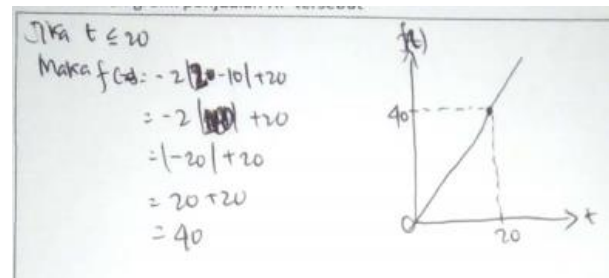


Figure 4. O.K's Solution for Question 1

The mistake made by O.K was due to O.K's lack of understanding of the absolute value concept. From the interview, O.K admitted that O.K forgot the absolute value concept. In creating the function graph, O.K viewed this absolute value function as a linear function. Since $t \leq 20$, then O.K substituted $t = 0$ and $t = 20$, so that the coordinates of $(0,0)$ and $(20,40)$ were obtained, the two points were then connected by a straight line.

To determine the total sales in 15 weeks, O.K also made a mistake in representing the symbol, O.K immediately looked for the $f(15)$ value. O.K misinterpreted the asked component in the questions. This was also shown in the results of the following interview:

P : Try to look back at question number 1 b. What was asked from the question?
 O.K : Calculate the total sales in 15 weeks
 P : The total sales in 15 weeks
 O.K : Yes, Ma'am
 P : Why did you get the result 10?
 O.K : Yes, ma'am, because I immediately replaced the t value with 15 and then looked for the result
 P : It asked for total sales in 15 weeks
 OK : Yes, Ma'am
 P : So, are you sure your answer is correct?
 OK : No, Ma'am
 P : Then, can you redo your answer again?
 OK : Yes, Ma'am. Here's the new answer

O.K then sought for the value of $f(1)$, $f(2)$, $f(3)$, $f(4)$, ..., $f(15)$ and added up all those values.

In question 2, O.K understood the purpose of the problem, but he could not make a visual/pictorial representation correctly. O.K understood the outer alliance tangent of two circles, but O.K's drawing was not conscientious, O.K did not pay attention to the specified radius.

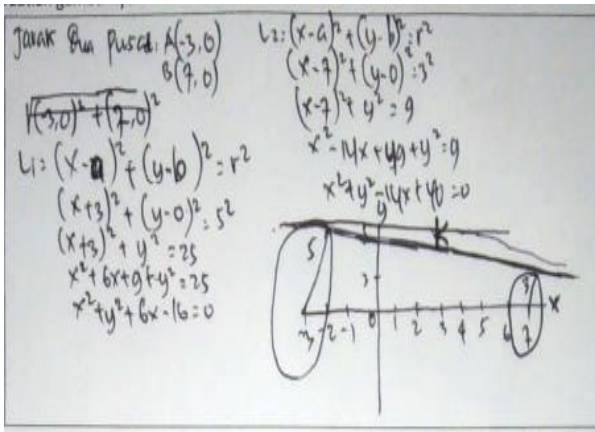
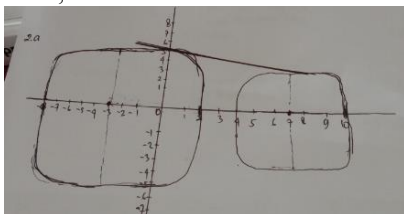


Figure 5. O.K's Solution for Question 2

From the solution above, it was clear that O.K could interpret the known and asked components. O.K made the first circle centered at A (-3,0) with radius 5 and the second circle centered at B (7,0) with radius 3. O.K also drew an alliance tangent of the two circles. O.K could make the visual representation of the given problem. It was just his/her picture that was less accurate. O.K did not notice that the first circle is supposed to intersect the x-axis at (-8,0) and (2,0), likewise the second circle intersects the x-axis at (4,0) and (10,0). During the interview, O.K realized his/her mistake. Below is the interview result:

- P : Was your picture correct?
 OK : (While looking at his/her picture) I don't know, ma'am.
 P : What's the question?
 OK : (Read the question)
 P : The circle l_1 is centered at and the radius is ... while the circle l_2 is centered at ... dan the radius is ...
 OK : A (-3,0) and the radius is 5 units, and B (7,0) and the radius is 7 units
 P : So, the picture you drew is correct?
 OK : No, ma'am.
 P : Why did you get it wrong?
 OK : I didn't pay attention to the radius of the circle, ma'am.
 P : What's the radius?
 OK : circle 1's radius is 5 units and circle 2's radius is 3 units. So the picture I drew is wrong.
 P : You can draw it again.
 OK : Yes, ma'am.



- P : Vina your picture is not a circle.
 OK : Yes ma'am, I don't have a math compass.
 P : Why did you draw like that?
 OK : Since circle 1 is centered at point A(-3,0) and has a radius of 5 units, then its circle is 5 units away to the left, 5 units to the right, 5 units up and 5 units down. While circle 2 is centered at point B(7,0) and has a radius of 3 units, so circle 2's distance is 3 units to the left, 3 units to the right, 3 units up and 3 units down.

In calculating the alliance tangent length of the two circles, O.K could write the formula correctly, but in the process, O.K made a mistake in substituting the radius of the circles.

In question 3, O.K understood well the known and asked components. O.K understood how to draw linear and quadratic functions. O.K could draw the graph correctly and define the area delimited by the two curves. O.K could also understand the use of definite integrals to determine the area between the two curves; therefore, O.K could make mathematical models correctly. O.K could also substitute the y_1 and y_1 functions correctly. But, O.K made a mistake in the operation, so the end result was wrong.

From the results of the above analysis, it can be concluded that O.K's mathematical representation ability was in the moderate category. Specifically, O.K's visual/pictorial, symbolic, and verbal representation abilities was in the moderate category.

3.3 Subject J.M

In question 1, to draw the function $f(t) = -2|t - 10| + 20$, J.M did not use the absolute value definition, but directly used the substitution method:

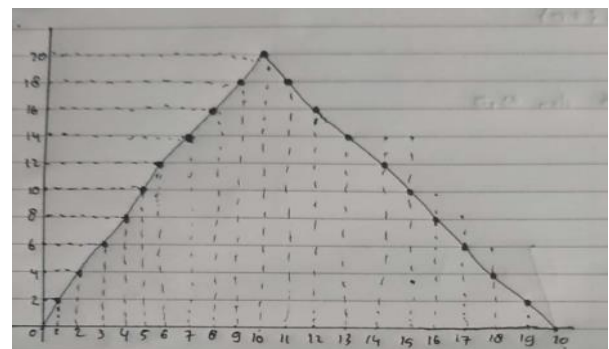


Figure 6. J.M's Solution for Question 1

From the interview, it was known that J.M understood the purpose of the questions, the known and asked components. But J.M did not understand well the concept of absolute value, so to draw a graph, J.M used the substitution of the t value to $f(t)$ to determine the points in the function $f(t)$, then

the points were connected to a curve function $f(t) = -2|t-10|+20$.

In determining total sales in 15 weeks, J.M did not use the arithmetic series formula, but instead, directly added $f(1)+ f(2)+ f(3)+ \dots + f(15)$. In question 2, J.M understood how to draw a circle centered at A (-3,0) with the radius of 5 units, as well as a circle centered at B (7,0) and with a radius of 3 units. Even so, the graph was not smooth, as in the following image.

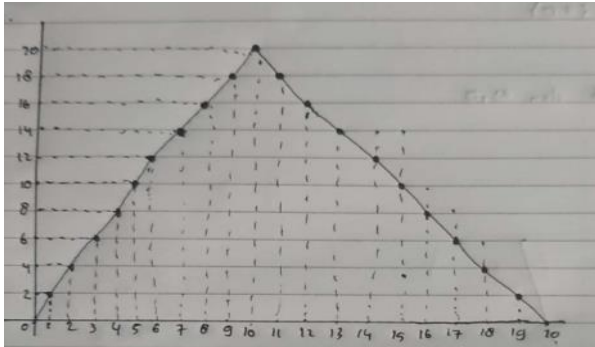


Figure 7. J.M's Solution for Question 2

In that figure, several weaknesses could also be identified, namely (1) the circle was less smooth, so it looks more like an ellipse, and (2) there was no tangent, both the outer and inner tangents. From the interview, J.M did not understand well the concept of tangents, J.M forgot how to calculate the length of the outer and inner tangents.

In question 3, J.M understood that the $y = x^2 + 4x + 3$ and $y = 2x + 6$ curves had to be drawn. He understood that to graph a linear function, the points of intersection of the x-axis and y-axis could be determined, then a straight line connecting the two points was made. However, J.M made a mistake in determining the x-intercept, he got (3, 0), it should be (-3, 0), so J.M drew the graph of $y = 2x + 6$ incorrectly. Likewise in graph of $y = x^2 + 4x + 3$, J.M was wrong in determining the x-intercept, JM obtained the x-intercept, namely (1, 0) and (3,0), it should be (-1, 0) and (-3, 0). Consequently, J.M also erred in graphing the function $y = x^2 + 4x + 3$.

In determining the area bounded by the two curves, J.M was able to determine y_1 and y_2 correctly, but in determining the upper and lower limits, J.M made a mistake. This happened because of errors in graphing the two functions. From the results of J.M's work, it could also be identified that J.M understood the integral rule of $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ and $\int_a^b f(x)dx = F(b) - F(a)$. However, because J.M made a mistake in determining the upper and lower integral limits, the area obtained was also wrong.

From the results of the above analysis, it could be concluded that J.M's mathematical representation ability is in the low category. Specifically, J.M's visual/pictorial representation ability was in the medium category, J.M's symbolic representation ability was in the very low category, and J.M's verbal representation ability was in a low category.

The results of this study indicated that most new students have relatively low mathematical representation ability. From the tests and interviews results that were enriched with the results of classroom observations at high schools (Sekolah Menengah Atas; SMA).

The first observation was carried out under the theme of relation and function material which was managed by R. The core activities in this study were as follows:

- a. The teacher explained the types of relations and how to express them
- b. The teacher drew 4 relations in the form of arrow diagrams and directed the students to distinguish between relations and functions.
- c. The teacher explained how to write or state relations with pictures and a set of consecutive pairs.
- d. The teacher gave questions to work on in groups, students work in groups
- e. The teacher directed the class discussion to discuss the students' work.

The second observation was carried out under the theme vector material learning managed by E.T. The core activities in this study were as follows:

- a. The teacher briefly explained the material about vectors
- b. The teacher explained while writing the properties of vector algebra operations on the blackboard
- c. The teacher gave some examples of vector algebra operations.
- d. The teacher asked the students to work on the questions selected in the textbook.
- e. Students discuss solving problems in groups; the teacher went around each group to monitor their discussions.
- f. Groups presented their work and were received responses from other groups.

In grade 1, the teacher pays attention to visual representations by describing relations and functions in the form of arrow diagrams. Also the symbolic representation, by presenting a way of presenting functions in the form of a set of ordered pairs. However, attention to this representation had

not been optimal. In learning, the teacher did not pay attention to variations in presentation from one type of representation to another. For example, a function expressed in the form of an arrow diagram was not converted to a set of consecutive pairs, nor is a function in the form of a set of consecutive pairs to be converted into an arrow diagram or a Cartesian diagram.

In the second lesson, when explaining the concept of vector algebra operations, the teacher did not display a visual representation. The teacher only wrote the algebraic properties of vectors, without making visual representations.

From the description above, it can be identified that two factors caused students' mathematical representation ability as low, namely:

- a. Weak prerequisite knowledge. The results of this study showed that 31 students (77.5%) were not able to take pictures correctly. This was due to the relatively low or very low mastery of the absolute value concept, the circle and alliance tangent concept, as well as linear and quadratic functions. In terms of symbolic representation, there are 33 students (82.5%) students whose abilities are low or very low. This was due to the relatively low mastery of absolute value definition, alliance tangent, and the use of integrals in determining the area under the curve. Furthermore, there are 32 students (80%) who have relatively low or very low verbal representation skills. This was due to the inadequate ability of number operations, algebraic operations, and interpretation.
- b. Weak learning process. Even though high school high schools/madrasah aliyah/vocational high schools (SMA/MA/SMK) has implemented a 2013 curriculum that uses a scientific approach to learning, the facts showed that mathematics learning is dominated by teachers using an expository approach. Learning that involves a mathematical representation process has not received attention. The research results of (Ratumanan & Ayal, 2018; Ratumanan & Tetelepta, 2019) showed that there are many weaknesses in learning, namely (1) There is still a tendency for teachers to transfer knowledge, (2) Teachers have not been able to properly facilitate students in observing activities. There has not been any presentation of interesting materials or materials to encourage the observation process to take place properly, (3) The teacher's ability to

provoke or motivate students to ask questions is still low; students tend to be silent, are less active in asking questions or expressing opinions, and (4) Reasoning activities (associating) are not going well, teachers are less able to facilitate students in these activities.

4. Conclusion

The results of this study indicated that the mathematics education study program's new students' mathematical representation ability is relatively low. Only 10% of students had the mathematical representation ability in the good category, 12.5% in the moderate category (enough); the largest percentage was in the very poor category, namely 67.5%, and 10% in the poor category. In detail, for verbal skills, there is 2.5% in the very good category, 7.5% in the good category, 17.5% in the moderate category, 15% in the poor category, and 55% in the very poor category. For the ability of visual/pictorial representation, there is 2.5% in the very good category, 7.5% in the good category, 12.5% in the moderate category, 7.5% in the poor category, and 70% in the very poor category. Furthermore, for the symbolic representation ability, 10% is in a good category, 10% in the moderate category, 7.5% in the poor category, and 72.5% in the very poor category.

The low mathematical representation ability is caused by two main things, namely (1) relatively low prerequisite knowledge and (2) mathematics learning that is mostly dominated by teachers; learning does not provide opportunities for students to develop mathematical representation ability.

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