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LOCAL INSTRUCTIONAL THEORY OF DERIVATIVE TOPICS BASED ON REALISTIC MATHEMATICS EDUCATION FOR GRADE XI SENIOR HIGH SCHOOL STUDENTS

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Abstract

This study discusses the development of learning design for derivative topics based on Realistics Mathematics Education (RME). The design developed is different from the designs that have been developed by previous researchers, where in this study, the derivative concept was introduced through the flying fox trajectory, which refers to students' understanding of limits and average velocity. This development aims to obtain a valid, practical, and effective learning design in derivative topics to develop students' mathematical problem-solving abilities. The development model combines the Plomp and Gravemeijer & Cobb models, which are divided into the preliminary research stage, the development or prototyping stage, and the assessment stage. In the preliminary stage, needs analysis, curriculum analysis, concept analysis, and literature review are carried out. Product prototype development is carried out at the development stage, and formative evaluation consists of self-evaluation, expert validation, one-to-one, and small group phases. In this article, the study focuses on findings from one-to-one activities at the development stage. Through one-to-one activities, the resulting design was tested on three students with different abilities to find out whether this product could later be implemented in the classroom. After students learn to use the design, at the end of the activity, students are given a problem-solving ability test to determine the impact of the design on students' problem-solving abilities. The final product obtained is Local Instructional Theory (LIT) of derivative topics, valid, practical, and impact students' problem-solving abilities of RME-based teacher and student books.

Keywords: problem-solving ability, realistic mathematics education, derivative topics

1. Introduction

Derivatives are a topic studied by high school students. However, many students still have difficulty solving problems related to derivatives (Constantinou, 2014; Denbel, 2015; Hashemi et al., 2014; Ngilawajan, 2013; Saepuzaman et al., 2017; Salingkat, 2017; Widada et al., 2019). Whereas derivatives are important to study because they are often used by other fields of science such as economics. One of the uses of derivatives in the economic field is to be able to analyze the maximum profit in a printing business (Hignasari, 2019). Students' difficulty in solving derivative problems is suspected because the learning process is still mechanistic/ procedural. This fact was revealed by (Arumsari et al., 2019) that the learning process that emphasizes procedural understanding rather than conceptual understanding is one of the difficulties in understanding derivative concepts. It was further stated that learning should start from simple to complex problems to assist students' mathematical processes in finding derivative concepts.

The reality in the school is that most teachers only use textbooks to teach derivatives. Students do not own the books used by the teacher, so students only learn from the teacher's explanations in class. The theoretical learning topic presented in textbooks does not provide opportunities for students to be involved in discovering concepts. Thus, students' mathematical abilities are less trained.

In addition, the implementation of learning is also still mechanistic. This condition will have an impact on the following: 1) students are accustomed to imitating and recording problem-solving from the teacher, 2) mathematics learning approaches are less attractive and boring for students, and 3) teachers have difficulty in compiling teaching materials with new innovative approaches (Sumarmo, 2012) . For this reason, it is necessary to design teaching materials that can guide students and teachers to develop innovative learning. According to Prastowo, teaching materials are all tools (both information, tools, and texts) that are systematically arranged to display a complete figure of a competency that will be used and must be mastered by students in the learning process. Teaching material aims to plan and study learning implementation (Marika et al., 2020). Previous researchers have conducted several studies on the development of derivative teaching materials.

Previous research found learning barriers based on the results of test questions on derivative topic related to mathematical critical thinking skills (Arumsari et al., 2019; Brookfield, 1997; Halpern, 1996) One of the learning barriers in the indicators of analyzing and evaluating arguments and evidence is that there are some students who directly derive each function without operating the function first. In addition, students have difficulty in analyzing a statement that must be proven; students make a solution without seeing what arguments are contained in the problem. Previous researchers have provided didactic anticipation, namely students are given similar questions regarding the derivative of the multiplication operation of two functions, as well as pedagogical anticipation in the form of teachers creating groups so that students work together in solving problems in the module. It is recommended that in understanding derivative topics, students are expected to be trained in analyzing a problem. The problems given are in the form of contextual problems that are close to students in order to find derivative concepts.

Other research designed Student Activity Sheets (LAM) to overcome student difficulties in derivative material (Ningsih, 2017). However, in this study, the problems were close to and often found by students. Because in understanding derivative topics, students are expected to be trained in analyzing a problem. The problems given are in the form of contextual problems that are close to students to find derivative concepts (Haciomeroglu et al., 2010).

Derivatives are part of the calculus material at higher education levels (Rasmussen et al., 2014; Tall, 2009). However, this derivative topic does not maximize students' problem-solving behavior (Harisman & Khairani, 2021). Problem-solving abilities that have not been maximized also occur among pre-service teachers (Ginta. O & Saleh. H, 2018). The process of learning mathematics should prioritize the process of student mathematization to improve students' problem-solving skills. This condition will only occur if the learning involves students finding concepts, namely by utilizing contextual problems that are close to students (Apriliyanto, 2019; Armiati & La'ia, 2020; Ngilawajan, 2013).

One of them is the Realistic Mathematics Education (RME) approach, which supports the mathematization process of finding concepts using real-life contexts. RME is a theory of learning mathematics that provides real problems related to students' daily lives (Dahlan, 2019; Sari et al., 2021; Syafriandi et al., 2020). This concept is under research conducted by (Armiati & Sutiaharni, 2021; Syafriandi et al., 2021) who have applied the Realistic Mathematics Education (RME) approach to the mathematics learning process.

This theory departs from Freudenthal's opinion that mathematics is a human activity and must be related to reality. Referring to Freudenthal's opinion, "Mathematics should not be given to students as a ready-made finished product, but as a form of activity in constructing mathematical concepts" (Wijaya, 2012). This statement means that learning mathematics must be related to problems relevant to students' daily lives in rediscovering derivative concepts.

Another study that applies RME in learning derivative topics has improved student learning outcomes (Siwi, 2010). His research focuses on the learning outcomes of students who have been able to reach the “Standard mark” for Indonesian people. In addition, the problem given in this study is to calculate the change in velocity of an object moving in a straight line. The context has not been devoted to the speed experienced by someone when playing a flying fox, so the researcher uses the context of speed when playing a flying fox in finding derivative concepts. Then learning trajectory calculus based on RME (Arnellis, 2019).

This research produces HLT for calculus, namely the concept of real functions, limits, and function derivatives. The results showed that higher-order mathematical thinking skills improved overall after using the developed product. However, only real function material could be seen for its effectiveness because its implementation in class could not be implemented. After all, this calculus topic was studied in the second semester.

Based on previous research that discusses derivative learning designs, it has had a positive impact on students. However, some weaknesses become the reason for developing new learning designs. The learning design then focuses on derivative topics with different sub-materials, contexts, and mathematical abilities than before.

This article presents the research results on the one-to-one evaluation stage of the mathematics learning design for the subtopic of derivative concepts using limits. The novelty in this research is that it has developed a learning design for RME-based derivative topics with the context of the flying fox game to improve the mathematical problem-solving abilities of Grade XI senior high school students.

2. Method

The development of this RME-based derivative topic learning design uses a combination of two development designs, namely the Plomp and Gravemeijer and Cobb development designs. The merging of these two types of design research is because, in the Gravemeijer and Cobb model, the development of the early-stage learning path only leads to a literature review. Also, the products developed do not consider validation. Meanwhile, in implementing the learning flow, a product is needed in the form of teaching materials (teacher books and student books) in the Plomp model. So using this model will reinforce each other at a particular stage suitable to be combined. The merging of these two designs also aims to produce Local Instruction Theory (LIT), a valid, practical, and effective teacher and student book. The Plomp model consists of 3 stages, namely: (1) preliminary research, (2) development or prototyping stage (product design, self-evaluation, expert review, one-to-one evaluation, small group evaluation, and field test), and (3) assessment stage (Misdalina et al., 2013; Syelfia & Armiati, 2020; Zulkardi, 2002). The prototype phase/learning flow is combined with Gravemeijer & Cobb, which consists of three phases: preparing for the experiment, experimenting, and retrospective analysis (Fauzan & Sari, 2017). This article will discuss the results of the one-to-one evaluation stage.

The preliminary stage is to obtain information related to the form of learning carried out by the teacher on derivative topics, teaching materials used, and student conditions. This activity was carried out through observation and unstructured interviews with teachers and students. The instruments used were observation sheets, interview guidelines, and field notes. In the prototype phase/learning flow, the Plomp model is combined with Gravemeijer & Cobb, which consists of three phases: preparing for the experiment, conducting the experiment, and retrospective analysis (Fauzan & Sari, 2017).

Activities can be seen as a cycle of an experimental process which can be seen in Figure 2.

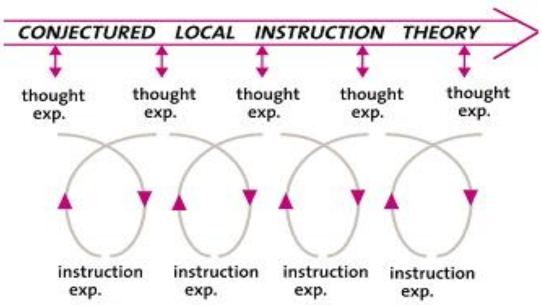


Figure 2. Thought & Experiment Cyclic Process

The activity begins with a thought experiment, namely thinking about the learning trajectory that will be passed by students in the form of HLT and will be implemented in products designed in the form of teacher books and student books. In the early stages, a self-evaluation of the product was carried out. Then it was validated by experts consisting of mathematicians, education experts, language experts, and experts in graphics design. Products that have been declared valid are then tested to see their feasibility of the product. Use a validation sheet that includes content and constructs validation for validation activities. Based on the responses given by the validator, the average score is calculated using the following formula (Sugiyono, 2015).

$$\bar{x}_j = \frac{\sum_{i=1}^{n} x_i}{n} \quad (1)$$

Information:
$\bar{x}_j$ =average of each $-j$ item
x_i =the score is given by the $-i$ validator
n =the number of validators

Furthermore, the validity of the product is determined by the formula:

$$R = \frac{\sum_{j=1}^{m} \bar{x}_j}{m} \quad (2)$$

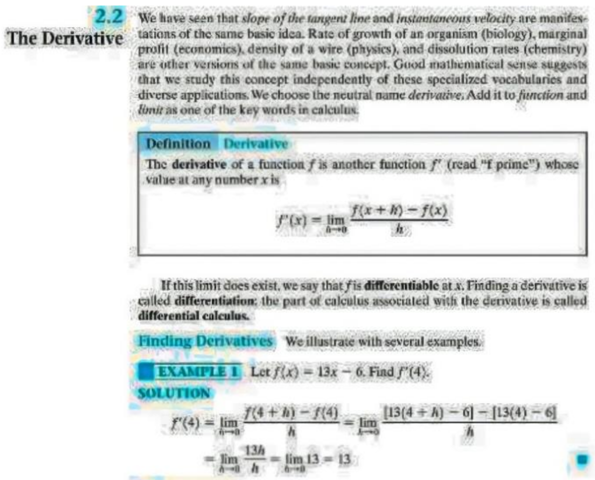
Description :
R = validity of learning tools
$\bar{x}_j$ = the average of the -j assessment result
m = the number of items

The validation results from experts were reflected and continued with the following thought experiment, namely the one-to-one evaluation stage. The validated product was tested on three students with different ability levels in the one-to-one evaluation stage. At this stage, each student is given a student book to work on. Students were observed when they used the learning material for the problems; if students' experienced difficulty, the researcher gave directions and noted the obstacles to be considered for product improvement later. After studying the book, the next step is to give students a mathematical problem-solving ability to see the product's effectiveness. The product is said to be effective if students score above the "Standard mark" for Indonesian people ≥ 60%, which is in the effective category.

3. Result and Discussion

3.1 Research result

Based on the needs analysis results obtained through interviews with mathematics teachers, the books teachers use are not owned by students. This fact causes students only to accept explanations from the teacher. In this book, derivative material is given theoretically. The following is an example of presenting derivative topics in several teachers' textbooks.


2.2 The Derivative We have seen that *slope of the tangent line* and *instantaneous velocity* are manifestations of the same basic idea. Rate of growth of an organism (biology), marginal profit (economics), density of a wire (physics), and dissolution rates (chemistry) are other versions of the same basic concept. Good mathematical sense suggests that we study this concept independently of these specialized vocabularies and diverse applications. We choose the neutral name *derivative*. Add it to *function* and *limit* as one of the key words in calculus.

Definition Derivative
The **derivative** of a function f is another function f' (read "f prime") whose value at any number x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If this limit does exist, we say that f is **differentiable** at x. Finding a derivative is called **differentiation**; the part of calculus associated with the derivative is called **differential calculus**.

Finding Derivatives We illustrate with several examples.

EXAMPLE 1 Let $f(x) = 13x - 6$. Find $f'(4)$.

SOLUTION

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h \to 0} \frac{[13(4+h) - 6] - [13(4) - 6]}{h}$$
$$= \lim_{h \to 0} \frac{13h}{h} = \lim_{h \to 0} 13 = 13$$

Figure 1. Presentation of Derivative Concepts Given Directly

Based on Figure 1, it can be seen that the book does not fully contain information that directs students to solve problems because they are not directed to find concepts with problem-solving steps. The questions also presented only aim to train using formulas. This finding is contrary to the expectation of learning mathematics, where students are expected to construct their understanding. Students have not been directed to construct their knowledge with contextual questions that are close to students' real lives.

So, the available books only lead to teacher-centered learning because, in learning, students still need explanations from the teacher. Books focus more on students' attention to memorizing concepts and formulas without leading students to understand derived concepts and rediscover these concepts. Finally, students only accept the results of the conclusions and formulas to be memorized without understanding their meaning. This presentation technique causes students to be unfamiliar with using thinking skills in solving a problem and not contributing to the development of student learning, especially in the development of mathematical problem-solving abilities. This fact resulted in the problem-solving ability of students being low. The previous research also found that students' problem-solving abilities were still low (Mauk et al., 2021).

Information was also obtained through interviews; the teacher had never applied the learning flow with the RME approach to a derivative topic. In the implementation of learning, the teacher directly gives the general derivative form, the properties of the derivatives, and how to solve them by applying the properties of the derivatives. Students follow the steps in the examples given by the teacher and memorize each completion procedure. The teacher revealed that students were lazy to think and rarely worked on problem-solving questions because they tended to prefer working on questions directly related to questions and applying formulas that the teacher had given. After getting the results of the needs analysis then, the product is designed.

In this study, HLT was designed to find the concept of derivatives using limits and average velocity. Learning mathematics by applying the RME approach contains activities in which contextual problems are presented to explore students' thinking abilities to achieve the goals set.

These problems are designed from simple problems to more complex problems. This activity is expected to be able to develop horizontal mathematization skills towards vertical mathematization in finding a concept. In this lesson, two activities are given using the same problem context: the flying fox ride. The activities designed are as follows.

Activity 1.1: determine the slope of several flying fox trajectories with different starting tower heights and distances between the start and finish towers.

This activity aims to guide students to determine the slope of the flying fox trajectory. The activity starts with simple things first; students can design the existing trajectory through the graph and determine the slope. This stage is to show that the three designs have the same slope even though the spire is different. This fact is because the three designs of the flying fox trajectory form a straight line, so the slope from any point will be the same.

Activity 1.2: determine the speed of playing flying fox in the specified time interval

Activity 1.2 is given to direct students to find the concept of derivative based on the instantaneous velocity, which applies the concept of limit. Students are asked to be able to determine the instantaneous speed in the specified time interval. At first, the students determine in an interval of 1 to 2 seconds. Then students determine their average speed when the time change is getting closer to zero, which is getting closer to 1 second. Students can estimate the velocity in the time interval through tables and graphs and the concept of limits that have been studied with each student's ways and thought processes. However, the teacher must anticipate that students will use the limit concept to achieve the expected formal concept, namely the derivative concept.

The designed HLT was further validated by three mathematicians, one language expert, and one educational technology expert. Researchers carried out the one-to-one evaluation activity directly to guide students when they had difficulties in finding answers and observing students' thinking processes. This one-to-one evaluation activity was implemented on 27 February – 4 March 2020 at SMAN 4 Padang. The teacher assisted in selecting students at the one to one evaluation stage to select three students with low, medium, and high abilities at SMAN 4 Padang. This evaluation aims to review the student books that have been designed, whether the instructions are not clear for students, sentences that are difficult to understand, and student responses in solving contextual problems in the given student books. One of the problems designed in this research is as follows.

A company will build an outbound playground, namely flying fox. The team that design the flying fox is currently holding a meeting to discuss the design of the flying fox to be built. The team consists of 3 people, namely Anna, Beno, and Coki. They are still arguing about the design of the flying fox.

Anna plans to build two towers, namely the start tower (the place where the flying fox starts) and the finish tower (the place where the flying fox finish). Anna planned for the height of the start tower and finish tower are 50m and 10m. While the distance between the two towers is 80m. However, Beno did not agree because he thought the slope of the track was not safe, so Beno planned to add another 10m for the height of the start tower and another 20m for the distance between the two towers. Coki believes that Anna and Beno's slope design is the same. However, Coki wants to add the height of the starting tower by 30m and the distance between the two towers is 60m from Anna's design, with the aim of making the track longer but the slope of the track is still the same as Anna's design. Beno doesn't trust Coki. If you play Anna, whose opinion is correct? (assuming both towers are built on flat ground)

Figure 3. Problems with Activity 1.1

Based on the activities, students can understand the context of the contextual problems in Figure 1. These problems are a *starting point* in the discussion *to* find derivative concepts. Next, students write down the information they read in activity problem 1.1, which aims to determine the slope of the *flying fox trajectory* designed by team members. At this stage, horizontal mathematization occurs, where students describe the trajectory of the *flying fox* in their way. The student's answers can be seen as follows.

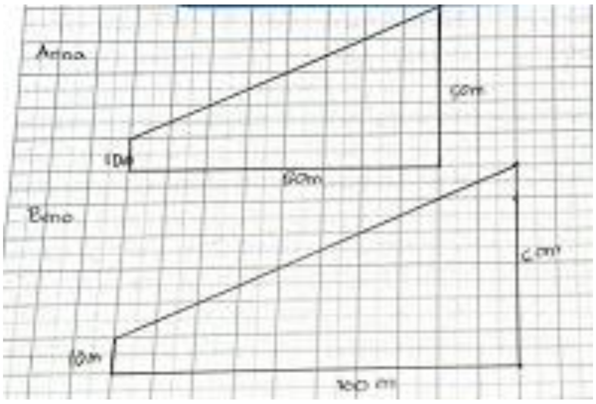


Figure 4. Students' Initial Answers

In Figure 4, it can be seen that the students initially described the design of Anna's *flying fox trajectory, then they described* Beno's design. Researchers do not blame students' answers but direct students to draw pictures, not separately. This instruction is helpful so students can know if the three designs form a straight line. Then the students combined the three designs of Anna, Beno, and Coki in one picture, but the student's drawings had not reached the expected goal. The researcher gave a *probing question* as follows.

P : take a look at your drawing, which is the height of the finish tower and the start tower of each team's design.
S : this one, ma'am (pointing to each height correctly)
Q : So, is the height of the finish tower different for each of Anna, Beno, and Coki's designs?
S : no bang, still 10m.
Q : If so, can the finish tower position be placed in one place for each of the designs of anna, beno and coki?
S : you can.
Q : If you can, can you describe the combination of different designs of anna, beno, and coki?
S : that means starting from the finishing tower, right?
P : make the picture according to what you think.

After a question and answer session, students describe the design of the *flying fox trajectory* that is different from the previous picture, as shown below.

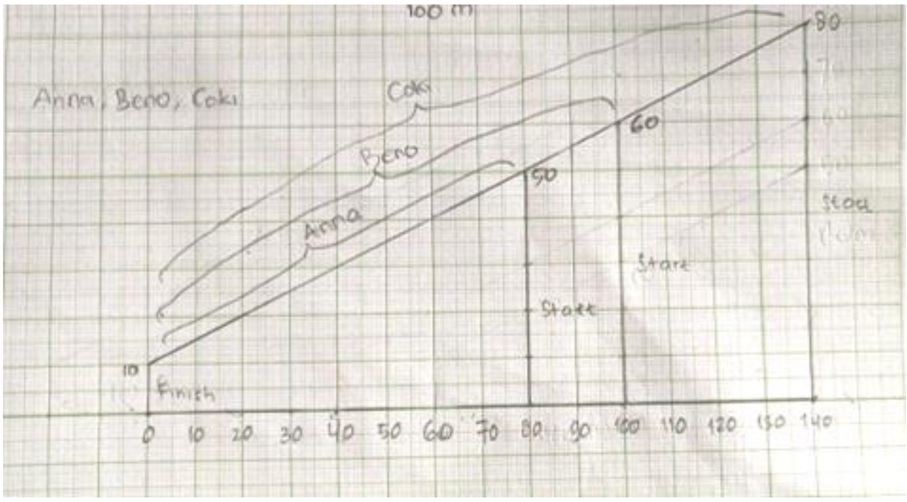


Figure 5. Students' Answers After Being Anticipated

Students can describe the design of the *flying fox trajectory* after being guided by the researcher. After students describe the design, then students determine the slope of each track. However, students are confused about determining the slope of each path even though they have been reminded about gradients. After being asked and answered, information was obtained that students were confused about which point was meant. The researcher finally gave an example as follows.

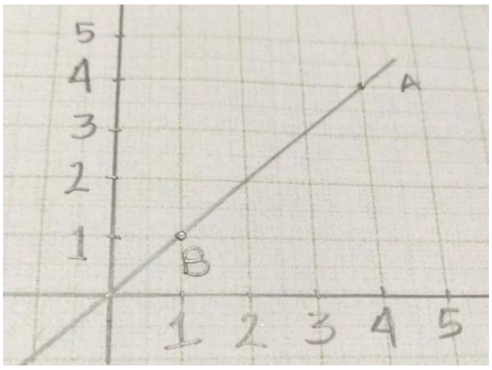


Figure 6. Example of a Line Through Two Points

Q *: if mom asks what the coordinates of point A and point B are, can you explain to mom?*
S *: point A (4 ,4) and point B (1,1)*
P *: Now look at your drawing (back to the previous page). Which are the top point of the start tower and finish tower? Just look at Anna's design trajectory.*
S *: (student shows the top of the tower that is asked correctly)*
P *: from the picture, try to determine the coordinates.*
S *: start point (50,80) and finish point (0,10)*
Q *: Are you sure? Looking back, what is the x-point for the starting tower?*
S *: upside down. The x point is 80, the y is 50.*
Q *: right. Now proceed to determine the slope.*

Students must be guided and given supporting examples to find their answers. The researcher does not directly show the coordinates of the existing trajectory so that students can construct their knowledge. After the student has correctly obtained the slope of the track, he has not answered the question "whose opinion is correct and what is the slope of each track". The researcher reminds students to reread and answer questions that students have not answered.

After completing activity 1.1, students continue with activity 1.2. Problems in activity 1.2 can be seen as follows.

Finally, the design ofthe outbound have been completed. The team conducted to test the speed when playing flying fox as the first experiment. Anna is willing to try flying fox and prepares for the start tower by wearing safety precautions so she doesn't fall. Anna is also getting ready to go down by passing the track that connects the start tower to the finish tower as shown in Figure 2. Anna slides $4t^2$ meter in t seconds, so that in the first second Anna has moved 4m, in the second, she has moved 16m from starting point of the track.

a. Can you help the team to find the average speed in the interval $t_1 = 1$ second to $t_2 = 2$ second? What happens if the time difference between the two gets smaller?
b. If the team wants to know the speed when the time is c seconds and the change in time is close to zero, can you help the team calculate the speed?

Figure 7. Problem with Activity 1.2

In this activity, vertical mathematization begins, which directs students toward the formal mathematical concept to be achieved, namely the derivative concept. In activity 1.2, the researcher checked the students' prior knowledge about the average speed; after the students answered correctly, they worked on activity 1.2 by constructing their knowledge. Students work on activity 1.2 by working on the question point first. Students can determine the average speed in 1 to 2 seconds, so there is no difficulty determining the average speed requested.

Next, students determine the average speed with a time change close to zero. Students have not been able to determine the average speed when the change in time approaches zero precisely. The initial answers given by students can be seen in the following picture.

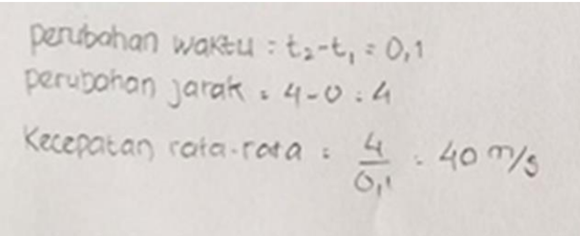


Figure 8. Students' Initial Answers

At first, the researcher gave the idea that the meaning of approaching zero was the numbers greater than zero but smaller than 1. The student then answered Figure 8. The student had not been able to determine the change in distance correctly, so the average speed obtained was wrong. Next, the researcher asked the following questions.

Q : what is the initial value of t?
S : 1, ma'am.
P : if the change is getting closer to zero. You take the example of 0.1, then for the final t, how much? If t is initially one, then the change in the initial and final t times is 0.1, so what is the final value of t?
S : 1.1 ma'am.
P : if the initial t is 1, then the distance is 4 $(1)^2 = 4$. *Replace the value of t on* $4t^2$ *with 1 so that you get 4. So, if t ends at 1.1, then how far is it?*
S : input this 1.1 value into this distance $4t^2$, *ma'am?*
Q : yes. Try to finish the average speed. Then look for time changes that are smaller than 0.1. Find as many as 2 examples.

After students complete the average speed for three, the time changes are close to zero. Students get the answer that the average speed value is getting closer to 8, and then the researcher asks what he knows about limits. Students answer infinity limit. The researcher asked the students to write down the infinite limit they knew and then asked the students to explain the meaning of x→∞. Students explain that it is read as x approaches infinity.

P : If x approaches a point, the result of the function will approach a specific natural number. Do you remember what math concept it was?
S : limit, ma'am?
Q : right. Take a look at the answers you have made, the longer the result is closer to 8, and the time change is getting closer to zero. Can we use the concept of limit?
S : yes, ma'am.
Q : then, can you determine which is x and its function?
S : x is the change in time, the function is the average speed, ma'am.

The researcher guides students to relate the concept of the limit with the results of the answer so that they can use the concept of limit for the next question. After being given anticipation, students complete the question using the concept of limit, so an answer is obtained as shown in Figure 9 below.

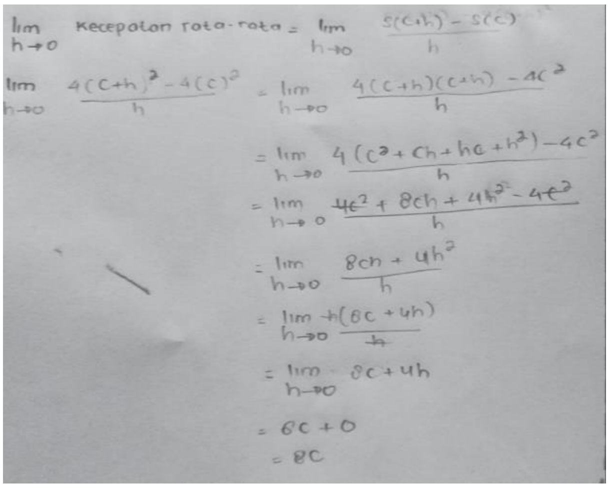


Figure 9. Students' Answers After Being Anticipated

Students complete activity 1.2 with the guidance of the researcher. Then the researcher asked the following question, "after you find the average velocity with a time change close to zero by using a limit. This concept is called a derivative. Instantaneous speed of one of the derivative applications." Students conclude that the limit of the change in average velocity as the time change approaches zero is a derivative. Furthermore, students can use the average velocity limit as an instantaneous velocity.

After the *one-to-one activity* was followed by the three students, the researchers obtained the results that students could solve problems well; students could understand the material presented and follow all the activities contained in the book.

The context of the problems in the student book is also interesting to students.

3.2 Assessment Phase Results

The assessment phase was carried out on a small scale to determine the effectiveness of RME-based learning designs through teacher and student books. The effectiveness of RME-based learning design on derived topics is measured by the results of students' problem-solving ability tests.

The mathematical problem-solving ability test given at the end of the experiment in one-to-one evaluation obtained scores for low-ability students 72.22, medium-ability students 86.11, and high-ability students 91.67. This result shows that 2 out of 3 students score more than the "Standard mark" for Indonesian people, which is 80. The students with low abilities have not yet reached the "Standard mark" for Indonesian people. This result happens because students do not complete the answers correctly, caused by the results of improper algebraic operations.

Table 1 follows an overview of the results of the *one-to-one evaluation stage of students' mathematical problem-solving ability tests* for each indicator.

Table 1. Percentage of Problem-Solving Ability Test Scores Per Indicator

No.	Troubleshooting Indicator	Percentage	Information
1	Understand the problem by identifying the known elements	100.00%	Very effective
2	Presenting the formulation of the problem in the form of a mathematical model.	91.67%	Very effective
3	Apply strategies to solve various math problems.	75.00%	Effective
4	Explain or interpret the results based on the original problem.	70.83%	Effective
	Average Percentage	84.38%	Very effective

Based on Table 1, the indicator that students must master is the first indicator, and the weakest indicator is the fourth indicator. The fourth indicator is explaining or interpreting the results based on the original problem. Still, students' answers are incorrect because indicator three does not get the highest score which ultimately affects indicator 4. Most students don't get the highest score for indicator four because students' answers did not correct in their algebraic operations when they answered indicator three. Students' mathematical problem-solving skills are suitable for the four problem-solving indicator.

3.3 Discussion

Based on the results of the research that has been done, the researchers found a novelty in the research on the development of RME-based derived topic learning designs that the researchers had done, namely by using the context of the flying fox game. This research positively impacts the mathematical problem-solving ability of XI high school students. The resulting product is in the form of Local Instructional Theory (LIT) which is implied in the teacher's book and student's book as a learning tool adapted to the 2013 Curriculum.

First, Siwi who applied RME in learning derivative topics. This study develops learning using the RME approach with derived topics. This study found that student learning outcomes have reached the "Standard mark" for Indonesian people. In addition, the problem given in this study is to calculate the change in speed of a straight-moving object where the given context has not been devoted to the speed experienced by a person (Gueudet-chartier, 2003; Siwi, 2010; Zulkardi & Putri, 2010; Zulkardi & Ilma, 2013). At the same time, the results of the research conducted by the researchers had a positive impact on students' problem-solving abilities with a context devoted to the speed when someone plays a flying fox.

Second, Kinasih developed worksheets on derived materials using the PMRI approach. This study is the same as previous research, namely developing learning designs on derivative topics (Kinasih, 2016). However, the difference in the research results is that previous studies have shown that the development of learning designs has a potential effect on learning outcomes. This result is in accordance with this research, which also positively impacts learning outcomes, especially problem-solving abilities.

Third, Arnellis developed a learning trajectory calculus based on RME. Previous research has developed a learning path, namely *Hypothetical Learning Trajectory* calculus, with material on Real Functions, Function Limits, and Function Derivatives. Research results from previous researchers showed that higher-order mathematical thinking skills were overall better after using the developed product (Arnellis, 2019). This research also positively impacts mathematical abilities, namely students' mathematical problem-solving abilities.

4. Conclusion

Through this research, an RME-based derivative topic learning design has been produced in the form of a learning flow that is implemented in the form of a teacher's book and a student's book. Based on trials in the one-to-one phase, the resulting designs are categorized as valid, practical, and effective. Using the context of the flying fox as the initial problem in this design has helped students find derivative concepts through the notion of limits. The choice of the flying fox context is very familiar to students because this ride is in the Lubuk Minturun area in the city of Padang, which is close to the students' environment. This research differs from previous research on derivatives (Arumsari et al., 2019) which developed modules for derivative topics based on mathematical critical thinking skills.

The choice of a context close to the student's environment is a feature of RME learning (Armiati & Sutiaharni, 2021; Arnellis, 2019; Fauzan et al., 2018; Syafriandi et al., 2020; Syelfia & Armiati, 2020). Learning with RME provides opportunities for students to solve problems based on the given context. In contrast to previous studies (Arumsari et al., 2019; Salingkat, 2017; Siwi, 2010), in this study, the context of flying fox was given. Students could imagine the movement of players from one point to another, which could be related to the concept of limits and the concept of velocity. This activity also provides opportunities for students to explore their problem-solving abilities, which begin with identifying problems, planning solutions, implementing strategies, and determining solutions. Through activities in RME, students, independently and in groups, can solve problems that help them build derivative concepts, starting from a simple way and proceeding to more complex ways that lead them to formulate derived concepts. The activities presented in this design have also developed students' problem-solving abilities.

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