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ADAPTIVE EXPONENTIALLY WEIGHTED MOVING AVERAGE WITH MEASUREMENT ERROR (COVARIATE) WITH AUXILIARY INFORMATION MAXIMUM FOR CEMENT QUALITY CONTROL

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Abstract
The Shewhart control chart exhibits limitations in detecting small process shifts and monitors the mean and variance separately. To address these shortcomings, this study introduces the Adaptive EWMA with Measurement Error (Covariate Method) and Auxiliary Information Max (AEWMA ME C AI Max) control chart. This novel approach integrates memory-based monitoring, joint mean-variance detection, measurement error correction through the covariate method, utilization of auxiliary variables, and adaptive adjustment mechanisms to enhance sensitivity across various shift magnitudes. The AEWMA ME C AI Max chart was applied to cement production data from PT XYZ, using Blaine fineness as an auxiliary variable for monitoring compressive strength. Comparative analysis demonstrates that the adaptive chart consistently produces control statistics closer to the upper control limit compared to the non-adaptive Max-EWMA ME C AI chart, validating its superior sensitivity in shift detection. Furthermore, the cement production process at PT XYZ was found to be statistically capable, with a lower capability index (Ppl) and process performance index (Ppk) of 1.45, indicating consistent compliance with lower specification limits and centered process performance. These results affirm the practical effectiveness of the AEWMA ME C AI Max chart in enhancing process monitoring and capability assessment in industrial applications.

Keywords: *Adaptive, Auxiliary Variable, Control Chart, Covariate, EWMA, Measurement Error*

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1. INTRODUCTION

A process is considered to exhibit good quality when its variability is reduced and the process outputs approach the target value, as quality is inversely related to variability [1]. Consequently, quality improvement is achieved through the reduction and control of process variability, which forms the foundation of Statistical Quality Control (SQC) or Statistical Process Control (SPC), with control charts serving as one of the principal tools [2]. Conventional control charts, first introduced by Shewhart, are memoryless, meaning they do not incorporate information from past observations [3]. To address this limitation, Page (1954) proposed the Cumulative Sum (CUSUM) control chart, and Roberts (1959) introduced the Exponentially Weighted Moving Average (EWMA) control chart, both designed to enhance sensitivity to small shifts in the process (Roberts, 1959). Subsequently, EWMA was extended to monitor process variability by Crowder and Hamilton (1992) and by MacGregor and Harris (1993), leading to the development of the Exponentially Weighted Moving Variance (EWMV) chart [4], [5].

Initially, monitoring of the process mean and variance was conducted separately. Quensenberry (1995) and Chen & Cheng (1998) introduced the Max chart for the simultaneous monitoring of mean and variance, which was further extended by Xie (1999) into the Max-EWMA chart [6]. However, the Max-EWMA chart assumes the absence of measurement error—an assumption that is often violated in practice and can significantly impair chart performance [7], [8], [9], [10], [11]. To mitigate this issue, Maravelakis et al. (2004) recommended incorporating covariate adjustment and multiple measurement methods. Auxiliary variables are those highly correlated with the quality characteristic of interest but do not directly define or constitute it, and their use can enhance measurement precision [12], [13]. Several studies have demonstrated that leveraging auxiliary information can substantially improve the efficiency of control charts [3], [7], [14]. As a consequence, the Max-EWMA chart has been further developed to account for measurement errors and auxiliary information, resulting in the Max-EWMA ME AI chart [3], [15], [16], [17]. Given that the EWMA chart is highly sensitive to small shifts, Capizzi & Masarotto (2003) developed the Adaptive EWMA (AEWMA) chart, which incorporates adaptive weighting mechanisms to improve detection of both small and large process shifts. Building upon these developments, this study proposes a further extension: the Adaptive Exponentially Weighted Moving Average with Measurement Error (Covariate Method) and Auxiliary Information Max (AEWMA ME C AI Max) control chart, which integrates measurement error adjustments, auxiliary information, and adaptive mechanisms.

The objective of this study is to evaluate the application of the AEWMA ME (Covariate) AI Max control chart on cement data. Specifically, the study aims to obtain the control results for cement compressive strength by utilizing blaine as an auxiliary variable in the cement production process at PT XYZ. This is done by simultaneously monitoring process mean and variability using the AEWMA ME (Covariate) AI Max chart, comparing its sensitivity to the Max-EWMA ME (Covariate) AI chart, and determining whether PT XYZ's cement production process is statistically capable.

2. RESEARCH METHODOLOGY

2.1. Literature Review

Correlations Valuep

According to Karl Pearson (1990), the correlation coefficient quantifies the strength and direction of the linear relationship between two continuous variables. This metric, commonly referred to as the Pearson correlation coefficient, is computed using the following formula [18]:

$$r = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \cdot \sum_{i=1}^{n} Y_i}{\sqrt{\left[n\sum_{i=1}^{n} {X_i}^2 - \left(\sum_{i=1}^{n} X_i\right)^2\right]\left[n\sum_{i=1}^{n} {Y_i}^2 - \left(\sum_{i=1}^{n} Y_i\right)^2\right]}} \quad \textbf{(1)}$$

The Pearson correlation coefficient ranges from -1 to 1, reflecting the degree and direction of a linear association between two continuous variables. A coefficient with an absolute value of $|r| = 1$ indicates a perfect linear relationship, whereas a value of $r = 0$ signifies the absence of any linear correlation [18]. A negative correlation implies that an increase in one variable corresponds with a decrease in the other, while a positive correlation denotes that both variables tend to increase together. The statistical significance of the observed correlation can be evaluated through a *t-test*, based on the null hypothesis is no significant correlation between variables ($\rho = 0$) and the alternatif hypothesis is there is a significant correlation between variables ($\rho \neq 0$) [19]. The t-test statistic for assessing the significance of the Pearson correlation coefficient can be computed using the following equation [19].

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \quad \textbf{(2)}$$

Statistics t denotes the test statistic for the significance of the Pearson correlation, r represents the observed correlation coefficient, and n refers to the sample size. The null hypothesis (H_0) is rejected if the absolute value of the calculated t-statistic exceeds the critical value $t_{\alpha;\, n-2}$, or equivalently, if the $p-value$ is less than the significance level α [19].

Estimations of Indivisual Variance Parameters (σ^2)

One of the estimating variance method is to use moving average value $\overline{MR} = \frac{\sum_{i=1}^{m-1} MR_i}{m-1} = \frac{\sum_{i=1}^{m-1} (x_{i+1} - x_i)}{m-1}$ and the adjustment factor $d_2 = 1{,}128$ for normally distributed individual data. Estimated variance calculated by this following equation [20].

$$\hat{\sigma}^2_{\overline{MR}} = \left(\frac{\overline{MR}}{d_2}\right)^2 \quad \textbf{(3)}$$

Normality Test

One test used to assess bivariate normality is the Shapiro-Wilk Multivariate Normality Test, which evaluates whether a set of bivariate data follows a multivariate normal distribution. Random vector comprising p variables is said to follow a miltoivariate nornal distribution, denoted by $\mathbf{X} \sim \mathbf{N_p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, if it possesses the following probability density function $\mathrm{f}(\mathbf{X}) = \frac{1}{(2\pi)^{p/2}\,|\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu})}$ where $\mathbf{X} = [\mathbf{X}_1\ \mathbf{X}_1 \ldots \mathbf{X}_p]^{\mathrm{T}}$ represents the vector of observed variables, $\boldsymbol{\mu}$ denotes the mean vector and $\boldsymbol{\Sigma}$ is the covariance matrix. To assess the assumption of multivariate normality, one may employ the Shapiro-Wilk test, adapted for multivariate data. The null hypothesis is data follow a multivariate normal distribution and the alternatif hypothesis is data do not follow a multivariate normal distribution [21]. Test statistics W^*following the equation [21]

$$W^* = \frac{1}{p}\sum_{i=1}^{p}\frac{[\sum_{j=1}^{n} a_{ij}x_{ij}]^2}{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2} \quad \textbf{(4)}$$

Where $W^*$is a mutivariate normal test statistic, p is the number of variables, n is the number of observations, a_{ij} is the eigenvalue ke $-$ j of the variable ke $-$ i, x_{ij} which is the observation value ke $-$ j variable ke $-$ i, and $\bar{x}_i$ is the average of the variable ke $-$ i. The decision rule for the test is based on the test statistic W^* where the null hypothesis H_0 is rejected if $W^* < C_{\alpha;n;p}$, the $C_{\alpha;n;p}$ denoting the critical value at significance level α for a sample of size n and dimension p Alternatively, the null hypothesis may be rejected if the corresponding p-value is less than the predefined significance threshold α, i.e., $p-value < \alpha$ [21].

Max-EWMA ME (Covariate) AI

Covariate variable is defined as the true value of quality characteristics, denoted by X that has a distribution $X\sim N(\mu_x, \sigma^2)$ and $\varepsilon\sim N(0, \sigma_m^2)$. Within the covariate model $Y = AX + B + \varepsilon$, cuality characteristic (denoted by Y) is thus normally distributed with expectation $E(Y) = \mu_y = A + B\mu_x$ and variance $Var(Y) = \sigma_y^2 = B^2\sigma^2 + \sigma_m^2$ [16]. Futhermore, let W be an auxiliary variable that exhibits a correlations ρ_{YW} with quality characteristics Y. Those variables assumed that the joint distribution of the pair (Y_j, W_j) follows bivariate normal distribution with mean vector (μ_Y, μ_W) and varians components (σ_Y^2, σ_W^2) [22]. Based on these properties, define ${M_{YW}}_j^{(1)}$ as a differentiation estimator for mean and ${V_j}^{(1)}$ as a differentiation estimator for varians by [3].

$$ {M_{YW}}_j^{(1)} = \bar{Y}_j + \rho\left(\frac{\sqrt{B^2\sigma^2 + \sigma_m^2}}{\sigma_W}\right)(\mu_W - \overline{W}_j) \quad \textbf{(5)}$$

$$ {V_j}^{(1)} = \Phi^{-1}\left[H\left\{\frac{(n-1)S_{Y,j}^2}{B^2\sigma^2 + \sigma_m^2}, (n-1)\right\}\right] - \rho^*\,\Phi^{-1}\left[H\left\{\frac{(n-1)S_{W,j}^2}{\sigma_W^2}, (n-1)\right\}\right] \quad \textbf{(6)}$$

Given the expression for expectation of the mean differentiation estimator is $E\left({M_{YW}}_j^{(1)}\right) = \mu_Y$ and expectation of the mean differentiation estimator is $Var\left({M_{YW}}_j^{(1)}\right) = \frac{1}{n}(B^2\sigma^2 + \sigma_m^2)(1-\rho_{YW}^2)$. the expectation and variance of the variance differentiation estimator are $E({V_j}^{(1)}) = 0$ and $Var({V_j}^{(1)}) = 1 - \rho^{*2}$. The samples means and variance express by $\bar{Y}_j = \sum_{i=1}^{n} y_{ij}/n$, $\overline{W}_j = \sum_{i=1}^{n} W_{ij}/n$, $S_{Y,j}^2 = \frac{\sum_{i=1}^{n}(Y_{ij}-\bar{Y}_j)^2}{n-1}$ and $S_{W,j}^2 = \frac{\sum_{i=1}^{n}(W_{ij}-\overline{W}_j)^2}{n-1}$. Furthermore, the function $H(\xi, v)$ denotes the chi-square distribution with v degrees of freedom, and $\Phi^{-1}(.)$ represents the quantile function (inverse cumulative distribution function) of the standard normal distribution [3].

Based on these properties, the transformed estimators for the population mean $({M_{je}}^{(1)})$ and variance $({V_{je}}^{(1)})$ are formulated accordingly

$$ {M_{je}}^{(1)} = \frac{M_{YW_j} - (A + B\mu_x)}{\sqrt{\frac{1}{n}(B^2\sigma^2 + \sigma_m^2)(1-\rho_{YW}^2)}} \quad \textbf{(7)}$$

$$ {V_{je}}^{(1)} = \frac{V_j}{\sqrt{1-\rho^{*2}}} \quad \textbf{(8)}$$

The EWMA statistics for monitoring process mean ($Pi1$) and process variance ($Qi1$) are computed according to the values of ${M_{je}}^{(1)}$ and ${V_{je}}^{(1)}$ according to the following formulation

$$P_i^{(1)} = \lambda M_{je} + (1-\lambda)P_{i-1};\ i = 1,2,\ldots,m \quad \textbf{(9)}$$

$$Q_i^{(1)} = \lambda V_{je} + (1-\lambda)Q_{i-1};\ i = 1,2,\ldots,m \quad \textbf{(10)}$$

Futhermore, *Max-EWMAMEAI* statistics M_i express by

$$\mathrm{M_i} = \max\{|\mathrm{P_i}|, |\mathrm{Q_i}|\} \quad \textbf{(11)}$$

Given that the EWMA statistic M_i is inherently non-negative, the lower control limit (LCL) is fixed at zero. Consequently, the upper control limit (UCL) for the Max-EWMA statistic can be expressed analytically as follows [6].

$$\mathrm{UCL_{MAX-EWMA}} = 1{,}128379 + 0{,}602810 \cdot \mathrm{L}\sqrt{\frac{\lambda}{2-\lambda}} \quad \textbf{(12)}$$

Adaptive EWMA

The EWMA method is well-suited for detecting small process shifts but is overly sensitive to large shifts [2]. To address this limitation, Capizzi and Mosarotto developed a control chart that modifies the smoothing parameter to adapt effectively to both small and large shifts [23]. The deviation between the current observation and the previous target mean is referred to as the *error*, which is then considered for determining the weight. A smaller error leads to a larger selected weight to enhance sensitivity to small shifts, and vice versa [24]. An independent sample of the quality characteristic y_i is taken, consisting of n samples m subgroups, and is assumed to follow a Normal distribution with mean (μ_y) and variance (σ_y^2). The AEWMA control chart statistic is formulated as follows [25]:

$$AEWMA_i = \phi(e_i) + AEWMA_{i-1}\ ,\quad i = 1,2,\ldots,m \quad \textbf{(13)}$$

denotes $e_i = y_i - AEWMA_{i-1}$, $AEWMA_0 = \mu_0$ is the initial target mean value, and $\phi(e_i)$ is the score function. The score function [$\phi(e_i)$] possesses the following characteristics [23]:

a. $\phi(e_i)$ is a monotonically increasing function in e_i
b. The score function satisfies the condition $\phi(e_i) = -\phi(-e_i)$
c. The score function takes the value $\phi(e_i) \approx \lambda e$ when $|e|$ is small, with $0 < \lambda \leq 1$. It can be shown that for small shifts, the AEWMA statistic behaves similarly to the standard EWMA statistic as follows:

$$AEWMA_i = \phi(e_i) + AEWMA_{i-1}$$
$$AEWMA_i = \lambda e + AEWMA_{i-1}$$
$$AEWMA_i = \lambda[y_i - AEWMA_{i-1}] + AEWMA_{i-1}$$
$$AEWMA_i = \lambda \cdot y_i - \lambda \cdot AEWMA_{i-1} + AEWMA_{i-1}$$
$$AEWMA_i = \lambda y_i + (1-\lambda)\cdot AEWMA_{i-1}$$

d. The score function takes the value $\phi(e_i) \approx 1$ when $|e|$ is large. In this case, the AEWMA statistic behaves similarly to the Shewhart statistic for large shifts.

Based on these four criteria, Capizzi and Mosarotto proposed three score functions: the Huber function, Tukey's Bisquare function, and the Cubic Polynomial function. Among the three, the Huber function provides the best performance for various shift magnitudes [23]. The Huber score function is defined by the following equation:

$$\phi_{hubber}(e_i) = \begin{cases} e_i + (1-\lambda)r & ,when\quad e_i < -r \\ \lambda r & ,when -k \leq e_i \leq r \\ e_i - (1-\lambda)r & ,when\quad e_i > r \end{cases} \quad \textbf{(14)}$$

Where λ is the EWMA weighting parameter, taking a value of $0 < \lambda \leq 1$, r is a positive constant. It is obtained that $\lim_{n\to\infty} \phi_{hubber}(e_i)/e_i = 1$, but $\phi_{hubber}(e_i) \neq e_i$ for any nonzero e_i, the function implies that the Huber function does not completely, but almost entirely, disregard past observations for large measurement errors. In contrast, the Tukey's Bisquare and Cubic Polynomial functions fully ignore past observations.

The upper control limit (UCL) and lower control limit (LCL) for the AEWMA control chart using the Huber function are given as follows [13], [23]

$$UCL = \mu_0 + h\,\sigma_0 \quad \textbf{(15)}$$

$$UCL = \mu_0 + h\,\sigma_0 \quad \textbf{(16)}$$

Where h is a parameter chosen to meet desired in-control performance criteria, μ_0 is equal to 1.128379, and σ_0 equals 0.60281. The UCL and LCL are derived from the EWMA control chart formulation with the consideration that the unit variance σ_0 no longer contains $\sqrt{\frac{\lambda}{(2-\lambda)}}$, as the AEWMA control chart has been adaptively smoothed using the corresponding score function [13], [26].

Process Capability

Process capability analysis is a statistical method used in quality control to assess whether a process can consistently produce within specification limits. A process is deemed capable if its capability index exceeds 1.33, indicating sufficient precision [1]. For processes in statistical control, indices C_p and C_{pk} are applied, while for out-of-control processes, performance indices P_p and P_{pk} are used to evaluate overall process behavior [1]. The process capability index C_p can be computed using the following formula [1].

$$C_p = \frac{USL - LSL}{6\sigma} \quad \textbf{(17)}$$

Where σ denotes is the sample standard deviation, USL and LSL represent the upper and lower specification limits, respectively. The Index C_{pk} refines of the C_p index by incorporating process centering, thus capturing both precision and accuracy. It is computed using the following expression[1].

$$C_{pk} = min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right) \quad \textbf{(18)}$$

For quality characteristics constrained by a one-sided specification limit, the C_{pk} index remains computable, despite the inability to define C_p In such cases, one-sided capability indices are applied $C_{pl} = \left(\frac{\mu - LSL}{3\sigma}\right)$ when a lower specification limit exists, or $C_{pu} = \left(\frac{USL - \mu}{3\sigma}\right)$ when only an upper specification limit is defined [1].

Cement Quality

This study utilizes two primary quality characteristics of cement, each of which serves as a critical indicator of performance:

1) Compressive Strength
   Compressive strength quantifies the capacity of a material to resist compressive loading, predominantly influenced by the cement's mineralogical composition. Standard practice involves measuring compressive strength at designated curing intervals—specifically on the 3rd, 7th, and 28th days. The test is conducted using a calibrated compressive strength testing machine by applying axial pressure to mortar specimens (cured for a defined age and pre-dried for 24 hours), prepared using 740 grams of cement, 2,035 grams of Ottawa standard sand, and 260 milliliters of water [27].

2) Blaine (Fineness of cement)

Blaine fineness, denoted as specific surface area (in cm^2/g), is an indirect measure of the cement's particle size distribution and is empirically associated with early-age compressive strength. Finer particles accelerate the hydration reaction, thereby enhancing reactivity and strength development [28]. The test involves measuring 113.2 grams of cement and determining air permeability through a packed bed of the sample under controlled conditions [29]. The apparatus operates on the principle of flow resistance, where air is drawn through the compacted cement bed and the time taken is used to estimate fineness based on grain size characteristics [30].

The quality specification limits for both compressive strength and Blaine fineness are governed by the Indonesian National Standard (SNI), as issued by the National Standardization Agency (BSN) [30].

Table 1 . Quality Spesification Limit by BSN

Quality Characteristics	Unit	Type Requirements	
		IP-U	IP-K
Blaine	m^2/kg	Min. 280	Min. 280
Compressive strength at 3 days	Kg/cm^2	Min. 130	Min. 110

2.2. Data Structure

The dataset comprises compressive strength values (Y in kg/cm^2) for 3-day-old cement and corresponding blaine measurements (W in m^2/kg) from samples produced between 1 January and 30 November 2023, structured as paired observations for analysis.

Table 2. Research Data Structure

Subgroup	Subgroup Units	Quality Characteristics	
		Y	W
1	1	Y_{11}	W_{11}
	2	Y_{22}	W_{22}
	…	…	…
	n	Y_{n1}	W_{n1}
…	…	…	…
i	1	Y_{1m}	W_{1m}
	2	Y_{2m}	W_{2m}
	…	…	…
	n	Y_{nm}	W_{nm}

2.3. Analysis Steps

The analytical procedure employed in this study is outlined as follows.

1. Obtain cement quality data from January 1st 2023 until November 30th 2023,
2. Perform exploratory data analysis on compressive strength and blaine,
3. Segment the dataset into phase I data, that is data produced January 1st until August 31th , and phase II data , that is data produced September 1st until November 30th 2023.
4. Compute Pearson's correlations coefficient (ρ_{YW}) using **Equation 1** and test its statistical significace by **Equation 2**,

5. Testing the bivariate normal asumption for Cement Compressive strangth and the auxiliary blainee using the shapiro-Wilk normal multivariate test for phase I dataset using **Equation 4**,
6. Monitoring quality of cement production using Maximum Exponentially Weighted Moving Average with Measurement Error (Covariate Method) with Axiliary Variable (Max-EWMA ME C AI) based on Phase I and Phase II data.
7. Monitoring quality of cement production using Adaptive Exponentially Weighted Moving Average with Measurement Error (Covariate Method) with Auxiliary Variable Maximum (AEWMA ME C AI Max) based on Phase I data with steps:
   1) Defines covariate model parameters A and B in the model $Y = A + BC + \varepsilon$,
   2) Estimates the populations variance of the true value X and its covariates model error using **Equation 3** and calculate $\sigma_m^2 \div \sigma_x^2$,
   3) Considering the desired $ARL_0 \cong 370$ for given combinations of parameter $\rho_{YW}, A, B,$ and $\sigma_m^2 \div \sigma_x^2$, the parameters values are determined to be combinations of λ and r,
   4) Calculate $U_{ME\ AI\ C_{ie}}$ with the following equation and its components below
   $$U_{ME\ AI\ C_{ie}} = \frac{U_{YW\ ME\ AI\ C\ i} - (A + B\mu_x)}{\sqrt{\frac{1}{n}(B^2\sigma^2 + \sigma_m^2)(1 - \rho_{YW}^2)}}$$
   Where
   $$U_{YW\ ME\ AI\ C\ i} = \bar{Y}_i + \rho_{YW}\left(\frac{\sqrt{B^2\sigma_x^2 + \sigma_m^2}}{\sigma_W}\right)(\mu_W - \bar{W}_i)$$
   $$\bar{Y}_j = \sum_{i=1}^{n} y_{ij}/n \quad ; \quad \bar{W}_j = \sum_{i=1}^{n} W_{ij}/n$$
   5) Calculate $U_{ME\ AI\ C_{ie}}$ with the following equation and its components below
   $$V_{ME\ AI\ C\ ie} = \frac{V_{ME\ AI\ C\ i}}{\sqrt{1 - \rho^{*2}}}$$
   Where
   $$V_{ME\ AI\ C\ i} = V_{Y,i\ \ ME\ AI\ C} - \rho^*\, V_{W,i\ \ ME\ AI\ C}$$
   $$V_{Y,i\ \ ME\ AI\ C} = \Phi^{-1}\left[H\left\{\frac{(n-1)S_{Y,i}^2}{B^2\sigma_x^2 + \sigma_m^2}, (n-1)\right\}\right]$$
   $$V_{W,i\ \ ME\ AI\ C} = \Phi^{-1}\left[H\left\{\frac{(n-1)S_{W,i}^2}{\sigma_W^2}, (n-1)\right\}\right]$$
   $$S_{Y,i}^2 = \frac{\sum_{j=1}^{n}(Y_{ij} - \bar{Y}_i)^2}{n-1} \quad ; \quad S_{W,i}^2 = \frac{\sum_{j=1}^{n}(W_{ij} - \bar{W}_i)^2}{n-1}$$
   6) Calculate Max- Statistics $M_{ME\ AI\ C_{ie}}$ for simultaneous control chart with the following equation
   $$M_{ME\ AI\ C\ ie} = \max\left(|U_{ME\ AI\ C\ ie}|, |V_{ME\ AI\ C\ ie}|\right)$$
   7) Calculate statistic $AEWMA\ ME\ AI\ Max\ C_0 = \mu_M$, the average of characteristics quality observations that becomes target value
   8) Calculate error value with this equation
   $$e_{ME\ AI\ C_i} = M_{ME\ AI\ C\ ie} - AEWMA\ ME\ AI\ Max\ C_{i-1}$$
   $$AEWMA\ ME\ AI\ Max\ C_0 = \mu_M$$
   9) Determined hubber scor function as the adaptive scor function for calculating statistics $AEWMA\ ME\ AI\ Max\ C_i$
   $$\phi_{hubber}(e_i) = \begin{cases} e_i + (1-\lambda)r & ,ketika \quad e_i < -r \\ \lambda r & ,ketika -r \le e_i \le r \\ e_i - (1-\lambda)r & ,ketika \quad e_i > r \end{cases}$$
   10) Calculate statistics $AEWMA\ ME\ AI\ Max\ C_i$ with this equation
   $$AEWMA\ ME\ AI\ Max\ C_i = \phi(e_i) + AEWMA\ ME\ AI\ Max\ C_{i-1}$$
   for $i = 1,2,3, \dots, m$
   11) Calculate the Upper Control Limit (UCL) with previously determined λ and r parameter values by this equation

$$UCL = \mu_M + h\,\sigma_M = \mu_M + r\sqrt{\frac{\lambda}{2-\lambda}}\,\sigma_M$$

12) If the overall test statistic is within the control limits, the process is considered statistically in control. If the test statistic falls outside the control limits, the cause of the out-of-control point is identified. If the cause is assignable, the observation is removed, and Steps 1) until 10) are repeated until the control limits reflect a statistically in-control process. Then, Calculate the control limits for the Max-EWMA ME (Covariate) control chart, ensuring Phase I data is statistically in control.
13) Apply the Max-EWMA ME (Covariate) control chart to the Phase II data.
14) Plot the test statistics for Phase II data using the control limits obtained from Phase I, ensuring the process remains in control.

8. Perform process capability analysis to evaluate the ability of the process to meet specification limits.
9. Derive conclusions based on the analysis of the research results.

3. RESULT AND DISCUSSION

3.1. Exploratory Data Analysis for Cement Quality Characteristics

Compressive Strength Exploratory Data Analysis

The descriptive statistics of the compressive strength quality characteristic for each phase are summarized in **Table 3**.

Table 3. Statistics Descriptive Compressive Strength

Periode	N	Mean	Variance	Stdev	Min	Max	Skewness	Kurtosis
Phase I	100	252.582	737.6583	27.16	180.6	335.6	0.461	0.481
Phase II	56	256.354	1015.712	31.870	194.4	320.186	0.290	-0.554

Based on **Table 3**, the mean compressive strength in Phase I is 252.5823 kg/cm², increasing to 256.3538 kg/cm² in Phase II. The variance of compressive strength in Phase I is lower than in Phase II, indicating that Phase I data are more homogeneous. Referring to Table 2.4, the minimum specification limit for compressive strength is 130 kg/cm², confirming that both Phase I and Phase II data comply with BSN standards. Both datasets exhibit low skewness values, suggesting distributions concentrated around their respective means. The descriptive statistics are visualized through the raincloud plot in **Figure 1**.

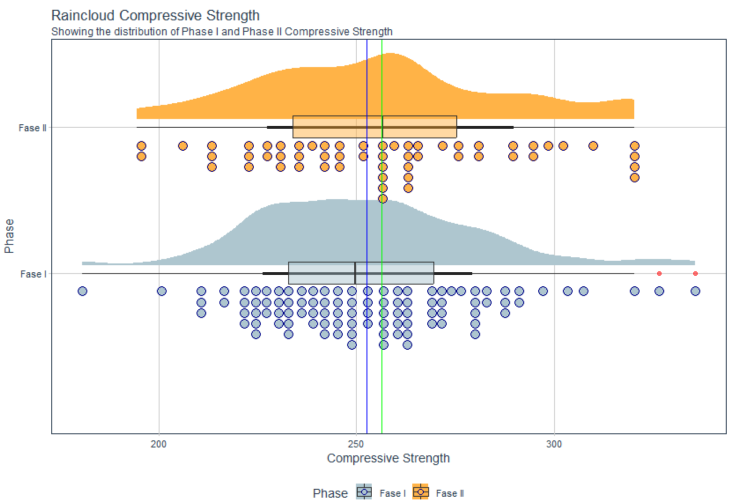

Figure 1. Raincloud Compressive Strength

Blaine Exploratory Data Analysis

The descriptive statistics of auxiliary variable, blaine, for each phase are summarized in Table 4.

Table 4. Statistics Descriptive Blaine

Periode	N	Mean	Variance	Stdev	Min	Max	Skewness	Kurtosis
Phase I	100	341.047	140.27	11.844	302.992	390.409	0.355	1.41
Phase II	56	344.642	139.751	11.822	323.737	369.906	0.061	-0.688

Table 4 shows that the mean Blaine fineness in Phase I is 341.0365 m^2/kg, increasing to 344.6416 m^2/kg in Phase II. The variance in Phase I is greater than in Phase II, indicating that Phase I data are more dispersed (heterogeneous) compared to the more homogeneous Phase II data. Referring to Table 4, the minimum specification limit for Blaine fineness is 280 m^2/kg, confirming that both Phase I and Phase II data meet BSN standards. Both datasets exhibit low skewness values, suggesting that observations are symmetrically distributed around the mean. The descriptive statistics are further visualized using the raincloud plot in Figure 2.

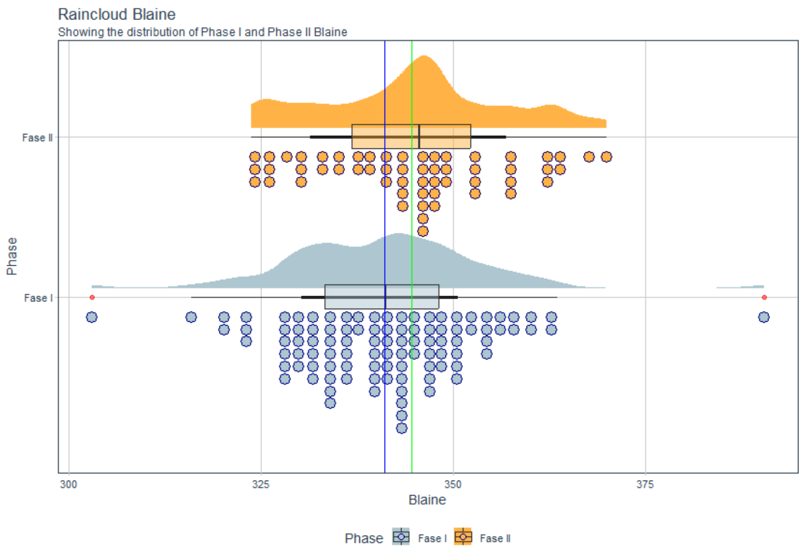

Figure 2. Raincloud Compressive Strength

3.2. Correlations

The results of the correlation test are presented in Table 3.

Table 5. Correlation Test Result

Phase	Notation	Correlation Value	*t*-Statistics	$t_{\frac{\alpha}{2}=0.025;n-2}$	Test decision	*Information*
I	ϱ	0.2551	2.6115	1.984	Reject H_0	Significant
	ϱ^*	-0.1449	-0.7022	2.005	Fail to reject H_0	Not significant
II	ϱ	0.5541	4.8919	1.984	Reject H_0	Significant
	ϱ^*	0.1207	0.4213	2.005	Fail to reject H_0	Not significant

Based on Table 5 the correlation between compressive strength and Blaine values for both Phase I and Phase II data is significantly positive, indicating that an increase in compressive strength is associated with an increase in Blaine. Meanwhile, the correlation between the variances of compressive strength and Blaine is not statistically significant, suggesting that changes in compressive strength variability are not necessarily accompanied by changes in Blaine variability. However, since the correlation is not exactly zero, a portion of the variability can still be explained, supporting the use of Blaine as an auxiliary variable in the Max-EWMAMEAI control chart for compressive strength monitoring.

3.3. Normality Asumption Test

The results of the Bivariate Normal test are presented in Table 6.

Table 6. Shapiro-Wilk Multivariate Normality Test

*W**	*p-value*
0.97825	0.05427

Using a significance level of α = 0.05, the critical region for rejecting H_0 occurs when the p-value is less than α. Based on Table 4.5, the multivariate normality test yields a p-value of 0.05427, which exceeds α, leading to a failure to reject H_0. Thus, it can be concluded that the data follow a bivariate normal distribution, validating the appropriateness of applying control charts based on this dataset.

3.4. Max- EWMA ME (Covariate) AI Control Chart

Max-EWMA ME (Covariate) AI Control Chart Phase I

Max-EWMA ME (*Covariate*) AI control chart was formed using phase I data and severals parameters, among others $\mu_x = 251.490902$, $\mu_w = 341.0465041$, $\sigma_x^2 = 975.8091$, $\sigma_w^2 = 163.0266$, $\sigma_m^2 = 917.798$, $A = 198.1430044$, $B = 0.2164664$, $\rho = 0.2550732$, $\rho^* = -0.1448688$, $\lambda = 0.05$, and $L = 2.709$ for a number of $m = 25$ subgrubs and $n = 4$ number of observations. A summary of the results of statistics M_{ie}, V_{ie}, P_i, Q_i, and $Max - EWMA_i$ are obtaines as follows:

Table 7. Max-EWMA ME C AI Statistics Phase I

Subgrub	M_{yw}	V_i	M_{ie}	V_{ie}	P_i	Q_i	$Max - EWMA_i$
1	255.8386	-1.61164	0.216985	-1.62882	0.010849	-0.08144	0.081441
2	241.3177	-1.92128	-0.75063	-1.94176	-0.02722	-0.17446	0.174457
3	226.1275	-0.68147	-1.76284	-0.68874	-0.11401	-0.20017	0.200171
4	248.6839	-1.22522	-0.25977	-1.23828	-0.12129	-0.25208	0.252077
5	234.8089	-2.00906	-1.18435	-2.03048	-0.17445	-0.341	0.340997

$Subgrub$	M_{yw}	V_i	M_{ie}	V_{ie}	P_i	Q_i	$Max - EWMA_i$
6	279.4645	1.04872	1.791316	1.059901	-0.07616	-0.27095	0.270952
7	253.9941	-1.16662	0.094076	-1.17906	-0.06765	-0.31636	0.316357
8	251.0153	1.850722	-0.10442	1.870453	-0.06949	-0.20702	0.207017
9	276.5034	-1.79588	1.594	-1.81502	0.013689	-0.28742	0.287417
10	252.4919	-0.74066	-0.00603	-0.74856	0.012703	-0.31047	0.310474
11	250.5965	-0.04449	-0.13233	-0.04496	0.005451	-0.2972	0.297199
12	253.0245	0.033268	0.029463	0.033623	0.006652	-0.28066	0.280658
13	263.7124	0.374208	0.74166	0.378197	0.043402	-0.24771	0.247715
14	288.7264	0.410734	2.40849	0.415113	0.161656	-0.21457	0.214573
15	253.3684	-0.67076	0.052382	-0.67791	0.156193	-0.23774	0.23774
16	260.1155	-1.56782	0.501976	-1.58454	0.173482	-0.30508	0.30508
17	255.824	-0.53634	0.216014	-0.54206	0.175609	-0.31693	0.316929
18	230.4171	-1.82143	-1.477	-1.84084	0.092978	-0.39312	0.393125
19	242.7622	-1.84738	-0.65437	-1.86707	0.055611	-0.46682	0.466822
20	261.9919	0.103009	0.627014	0.104107	0.084181	-0.43828	0.438276
21	249.7719	-1.05937	-0.18728	-1.07066	0.070608	-0.4699	0.469895
22	236.1984	-1.04396	-1.09175	-1.05509	0.01249	-0.49915	0.499155
23	261.7444	0.521848	0.610519	0.527411	0.042391	-0.44783	0.447827
24	243.3709	0.127161	-0.61381	0.128516	0.009581	-0.41901	0.41901
25	242.6881	-1.10377	-0.65931	-1.11553	-0.02386	-0.45384	0.453836

Please Max-EWMA ME (Covariate) AI with decision parameters $L = 2{,}709$ and $\lambda = 0.05$ yields an UCL of 1.503018, which can be computed as follows (using **Equation 12**)

$$UCL_{MAX-EWMA} = 1{,}128379 + 0{,}602810 \cdot L\sqrt{\frac{\lambda}{2-\lambda}} = 1{,}128379 + 0{,}602810 \cdot 2{,}709\sqrt{\frac{0{,}05}{2-0{,}05}} = 1.503018$$

Based on **Table 7** and the UCL, **Figure 3** illustrates the Max-EWMA control chart for Phase 1 data.

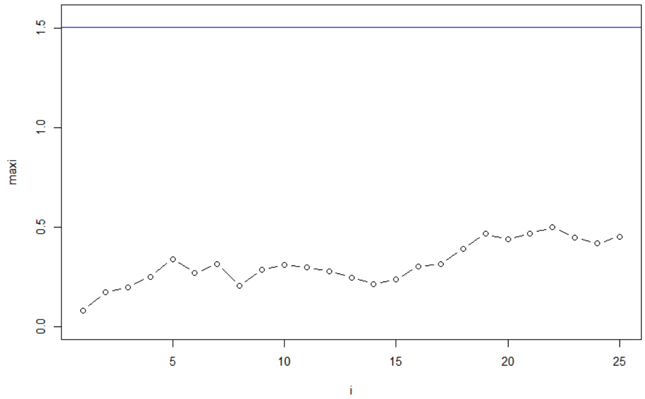

Figure 3. **Max-EWMA ME (Covariate) AI Control Chart Phase I**

As observed in **Figure 3**, all plotted points lie within the established control limits. This indicates that both the mean and variance of the cement production process are jointly monitored and remain under statistical control when assessed using the Max-EWMA ME (Covariate) AI control chart

Max-EWMA ME (Covariate) AI Control Chart Phase II

Using the same set of parameters as those applied in the Phase I control chart, the results for the statistics U_i, V_i, P_i, Q_i, and $Max - EWMA_i$ for the Phase II data, consisting of of $m = 14$ subgrubs and $n = 4$ number of observations, are obtained as follows:

Table 8. **Max-EWMA ME C AI Statistics Phase II**

Subgrub	M_{yw}	V_i	M_{ie}	V_{ie}	P_i	Q_i	$Max - EWMA_i$
26	229.2963	-0.78465	-1.55168	-0.79302	0.010309	-0.52208	0.522079
27	213.4805	-1.00965	-2.60558	-1.02042	-0.12049	-0.547	0.546996
28	254.2661	-0.9025	0.112199	-0.91213	-0.10885	-0.56525	0.565253
29	260.5101	1.134513	0.528276	1.146609	-0.077	-0.47966	0.47966
30	239.0567	-2.8587	-0.90129	-2.88918	-0.11821	-0.60014	0.600135
31	246.7692	-0.79454	-0.38737	-0.80301	-0.13167	-0.61028	0.610279
32	255.2952	1.166118	0.180771	1.178551	-0.11605	-0.52084	0.520838
33	239.4217	-1.2825	-0.87697	-1.29617	-0.15409	-0.5596	0.559604
34	239.2227	-0.59474	-0.89023	-0.60108	-0.1909	-0.56168	0.561678
35	279.2152	-0.50845	1.774704	-0.51387	-0.09262	-0.55929	0.559288
36	276.2304	-0.68452	1.57581	-0.69182	-0.0092	-0.56591	0.565914
37	284.4274	-0.91436	2.122022	-0.92411	0.097363	-0.58382	0.583824
38	278.0709	0.552237	1.698453	0.558125	0.177418	-0.52673	0.526727
39	262.4805	-1.79952	0.659574	-1.81871	0.201526	-0.59133	0.591326

Using UCL from Phase I data, **Figure 4** illustrates the Max-EWMA control chart for Phase 2 data.

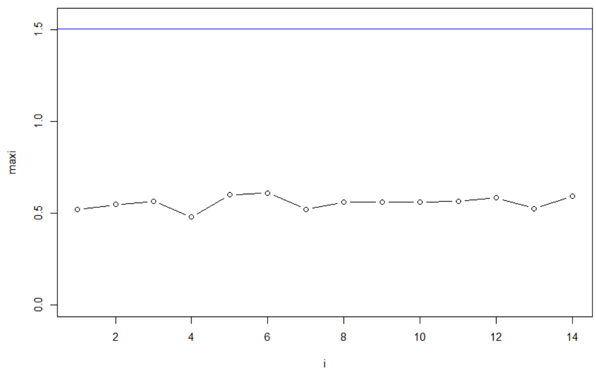

Figure 4. **Max-EWMA ME (Covariate) AI Control Chart Phase II**

Based on **Table 8** and **Figure 4**, all Phase II observation points fall within the control limits. Furthermore, its shows that the Max-EWMA statistics for each subgroup do not exceed the upper control limit of 1.503018. Thus, the joint monitoring of the process mean and variance for cement production using the Max-EWMA ME (Covariate) AI control chart indicates that the process is statistically in control.

3.5. Adaptive EWMA ME (Covariate) AI Max Control Chart

Adaptive EWMA ME (Covariate) AI Max Control Chart Phase I

Adaptive EWMA ME (Covariate) AI Max control chart was formed using phase I data and several parameter among others $\rho_{YW} = 0.2550732$, $\sigma_m^2 \div \sigma_x^2 = 0.9405508$, $A = 198.1430044$, and $B = 0.2164664$. For given combinations of parameter $\rho_{YW}, A, B,$ and $\sigma_m^2 \div \sigma_x^2$, the parameters values are determined to be $\lambda = 0.1$ and $r = 2.712$. Additional parameter combinations are also considered, specifically $\mu_x = 251.4909020$, $\mu_w = 341.0465041$, $\sigma_x^2 = 975.8091000$, and $\sigma_w^2 = 163.0266000$ for a number of $m = 25$ subgrubs and $n = 4$ number of observations. A summary of the results of statistics $U_i, Ue_i, V_i, Ve_i, M_i$, and $AEWMA\ ME\ AI\ Max\ C_i$ are obtaines as follows:

Table 9. **Adaptive EWMA ME C AI Max Statistics Phase I**

Subgrub	U_i	Ue_i	V_i	Ve_i	M_i	$AEWMA\ ME\ AI\ Max\ C_i$
1	264.0534	0.764387	-1.61164	-1.62882	1.628819	1.178423
2	254.5658	0.132169	-1.92128	-1.94176	1.941762	1.254757
3	226.2501	-1.75467	-0.68147	-0.68874	1.754671	1.304748
4	254.6748	0.139434	-1.22522	-1.23828	1.238281	1.298102
5	243.0558	-0.63481	-2.00906	-2.03048	2.030484	1.37134
6	280.6749	1.871972	1.04872	1.059901	1.871972	1.421403
7	257.0721	0.299182	-1.16662	-1.17906	1.179059	1.397169
8	254.9184	0.155664	1.850722	1.870453	1.870453	1.444497
9	276.702	1.607233	-1.79588	-1.81502	1.815023	1.48155
10	253.4289	0.056415	-0.74066	-0.74856	0.748561	1.408251
11	265.3942	0.853727	-0.04449	-0.04496	0.853727	1.352798
12	260.742	0.543724	0.033268	0.033623	0.543724	1.271891
13	277.8229	1.681923	0.374208	0.378197	1.681923	1.312894
14	279.4271	1.78882	0.410734	0.415113	1.78882	1.360487
15	243.2206	-0.62383	-0.67076	-0.67791	0.677907	1.292229
16	253.0153	0.028854	-1.56782	-1.58454	1.584539	1.32146
17	249.2917	-0.21928	-0.53634	-0.54206	0.54206	1.24352
18	228.4652	-1.60706	-1.82143	-1.84084	1.840845	1.303252
19	239.615	-0.86409	-1.84738	-1.86707	1.867071	1.359634
20	254.9336	0.156681	0.103009	0.104107	0.156681	1.239339
21	245.2151	-0.49092	-1.05937	-1.07066	1.070661	1.222471
22	228.5521	-1.60127	-1.04396	-1.05509	1.601273	1.260351
23	253.1915	0.040595	0.521848	0.527411	0.527411	1.187057
24	232.163	-1.36066	0.127161	0.128516	1.36066	1.204418
25	238.1128	-0.96419	-1.10377	-1.11553	1.115535	1.195529

Adaptive EWMA ME (Covariate) AI with decision parameters $\lambda = 0.1$ and $r = 2.712$ yields an UCL of 1.503433, which can be computed as follows

$$UCL_{AEWMA\ ME\ C\ AI\ Max} = 1{,}128379 + 0{,}602810 \cdot r\sqrt{\frac{\lambda}{2-\lambda}} = 1{,}128379 + 0{,}602810 \cdot 2{,}712\sqrt{\frac{0{,}1}{2-0.1}} = 1.503433$$

Based on **Table 9** and the UCL, **Figure 5** illustrates the AEWMA ME (Covariate) AI Max control chart for Phase 1 data.

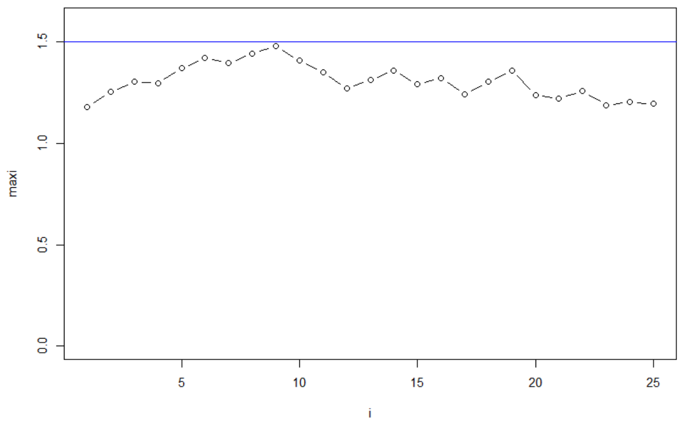

Figure 5. Adaptive EWMA ME (Covariate) AI Max Control Chart Phase I

Based on **Figure 5**, it can be observed that all observations in the Phase I AEWMA ME C AI Max control chart fall within the control limits. Therefore, it can be concluded that both the process mean and variance of the cement production process are statistically in control when jointly monitored using the AEWMA ME C AI Max control chart. Consequently, control chart monitoring may proceed with the Phase II data.

Adaptive EWMA ME (Covariate) AI Max Control Chart Phase II

Using the same set of parameters as those applied in the Phase I control chart, the results for the statistics $U_i, Ue_i, V_i, Ve_i, M_i$, and $AEWMA\ ME\ AI\ Max\ C_i$ for the Phase II data, consisting of of $m = 14$ subgrubs and $n = 4$ number of observations, are obtained as follows:

Table 10. **Adaptive EWMA ME C AI Max Statistics Phase I**

Subgrub	U_i	Ue_i	V_i	Ve_i	M_i	$AEWMA\ ME\ AI\ Max\ C_i$
26	234.8567	-1.18116	-0.77265	-0.77835	1.181164	1.133657
27	226.1511	-1.76126	-1.15699	-1.16552	1.761264	1.196418
28	263.5158	0.728558	-0.6696	-0.67454	0.728558	1.149632
29	259.1644	0.438599	0.669536	0.674471	0.674471	1.102116
30	232.5211	-1.3368	-2.30885	-2.32587	2.32587	1.224491
31	249.9383	-0.17619	-0.87008	-0.87649	0.876489	1.189691
32	267.8574	1.017868	1.758322	1.771281	1.771281	1.24785
33	246.0809	-0.43323	-0.99742	-1.00477	1.004771	1.223542
34	240.0023	-0.83828	0.028877	0.02909	0.838278	1.185016
35	278.1474	1.70355	0.098165	0.098888	1.70355	1.236869
36	268.0389	1.029962	-0.67185	-0.6768	1.029962	1.216179
37	283.1836	2.039139	-0.58301	-0.5873	2.039139	1.298475
38	275.5461	1.530207	0.686543	0.691603	1.530207	1.321648
39	263.9494	0.757457	-1.97298	-1.98752	1.987521	1.388235

Using UCL from Phase I data, **Figure 6** illustrates the Max-EWMA control chart for Phase 2 data.

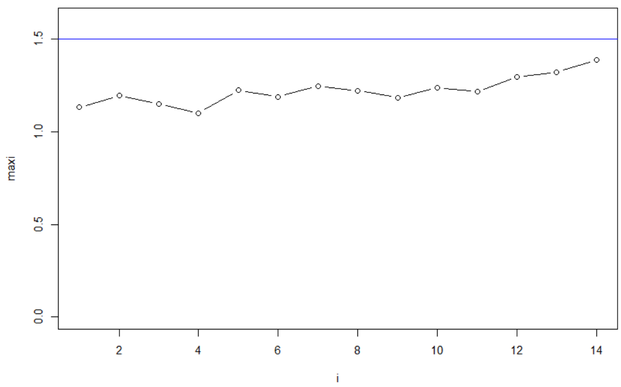

Figure 6. **Adaptive EWMA ME (Covariate) AI Max Control Chart Phase I**

According to **Table 10** and **Figure 6**, all Phase II observation points remain within the control limits, and the Max-EWMA statistics for each subgroup consistently fall below the upper control limit of 1.503433. This confirms that the Adaptive EWMA ME (Covariate) AI Max control chart effectively demonstrates the cement production process is statistically stable and under control.

3.6. Comparative Study on the Senstivity of Control Charts

Max EWMA ME (Covariate) AI Control Chart has $UCL = 1.503018$ and the Adaptive EWMA ME (Covariate) AI Max has the value of $UCL = 1.503433$. The statistics plotted to the control chart given by **Table 7-10**. The sensitivity of a control chart can be evaluated by the number of out-of-control signals identified, i.e., the number of subgroups that fall outside the control limits. Additionally, sensitivity can be inferred from the pattern exhibited by the plotted statistics.

Based on the quality control analysis using the Max-EWMA ME (Covariate) AI chart and the Adaptive EWMA ME (Covariate) AI Max chart, neither chart produced out-of-control signals, indicating that the cement production process, as monitored by both methods, is statistically in control. However, a closer examination of the plotted control statistics reveals that the statistics on the Adaptive EWMA ME (Covariate) AI Max chart tend to be nearer to the UCL compared to those on the Max-EWMA ME (Covariate) AI chart. This indicates a higher sensitivity of the Adaptive EWMA ME (Covariate) AI Max chart. A control chart with higher sensitivity is more capable of detecting shifts in the process mean and variance simultaneously, and does so more promptly. Therefore, based on the proximity of the control statistics to the control limits, it can be concluded that the Adaptive EWMA ME (Covariate) AI Max chart exhibits greater sensitivity than the non-adaptive Max-EWMA ME (Covariate) AI chart. This finding is consistent with the theoretical understanding that while the standard EWMA chart is particularly effective for detecting small shifts, the integration of an adaptive scoring function enhances its performance by allowing the control chart to dynamically adjust to both small and large process shifts.

3.7. Process Capability Analysis

The results of the process capability analysis for the compressive strength quality characteristic based on Phase I in-control data can be summarized as follows.

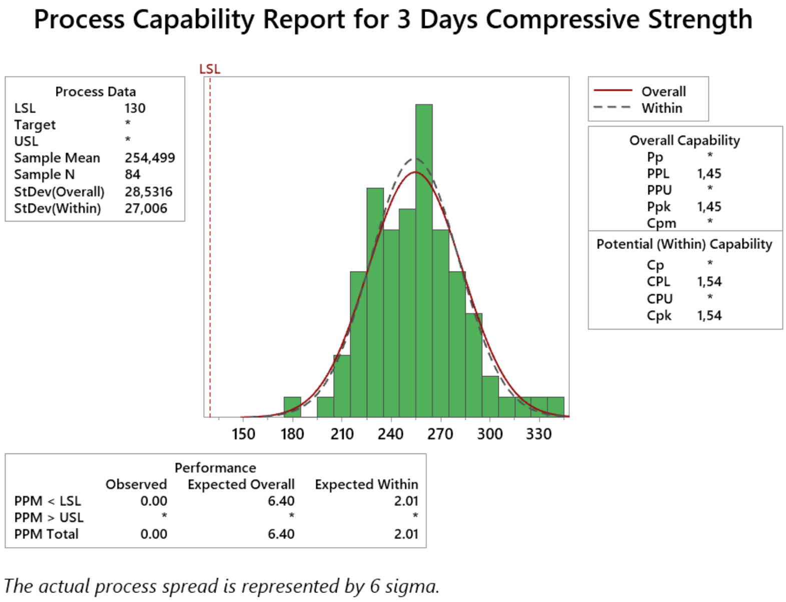

Figure 7. Adaptive EWMA ME (Covariate) AI Max Control Chart Phase I

Based on **Figure 7**, the process capability ratio (Ppl) for the compressive strength variable is 1.45. This indicates that the cement production process, in terms of the compressive strength quality characteristic, can be considered capable, as the Ppl value exceeds the critical threshold of 1.33. Additionally, the Ppk value is also reported as 1.45, which is greater than 1.33. This suggests that the production process is not only consistent but also exhibits a high level of accuracy, ensuring that the process is centered within specification limits and aligns with the desired target.

4. CONCLUSIONS

1. Joint monitoring of the process mean and variance in cement production using both the Max-EWMA ME (Covariate) AI and Adaptive EWMA ME (Covariate) AI Max control charts indicates that the process remains statistically in control
2. The control statistics from the Adaptive EWMA ME (Covariate) AI Max chart are consistently closer to the upper control limit (UCL) compared to those from the non-adaptive Max-EWMA ME chart. This reflects greater sensitivity of the adaptive chart in detecting shifts in both mean and variance. The adaptive mechanism enhances responsiveness to process changes, aligning with the theoretical premise that adaptive EWMA charts outperform standard EWMA charts by dynamically adjusting to varying shift magnitudes.
3. The cement production process at PT XYZ (Persero) Tbk demonstrates high capability, with a lower capability index (Ppl) of 1.45, indicating that the process consistently meets lower specification limits. A process performance index (Ppk) of 1.45 confirms that the process is accurate and centered, ensuring the compressive strength quality characteristic meets target specifications.

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