

## Monitoring and Evaluation of Clinker Quality Using $T^2$ Hotelling-Generalized Variance Control Chart

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### Abstract

The cement industry is an important sector in infrastructure development, where the quality of clinker determines the final quality of the product. This study evaluates the application of  $T^2$  Hotelling's and Generalized Variance (GV) multivariate control charts to clinker data based on three main variables:  $FCaO$ ,  $C_3S$ , and  $C_3A$  at PT XYZ. The results show that  $C_3S$  has the highest variance in phase I and II (2.61 and 2.53), while  $FCaO$  has the lowest variance (0.10 and 0.06). All three variables had mean values within the specification limits, although there were still extreme values outside the limits. Assumption tests showed that the data was not multivariate normally distributed, but it was still assumed to be normal for control analysis purposes. In the wet season, the standard deviation decreased from 1.552 to 1.252, and in the dry season from 1.170 to 1.029, indicating a decrease in variability although the process is not yet fully under statistical control. Capability analysis shows that the dry season process is more stable, with most parameters having multivariate values that exceed the threshold. Compared to the wet season, the dry season process showed more consistent performance and was able to meet production quality standards.

**Keywords:** Clinker, generalized variance, process capability,  $T^2$  hotelling's

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## 1. INTRODUCTION

The cement industry is one of the key sectors that supports a country's infrastructure development [1]. One of the main factors that determine the quality of cement is the quality of clinker, a semi-finished material that is the result of burning a mixture of limestone, clay, and iron sand in a rotary kiln [2]. The clinker production process is complex and involves many interrelated variables, such as the chemical composition of the raw materials, combustion temperature, and residence time in the kiln. Therefore, the composition and characteristics of clinker must be carefully analyzed and controlled to optimize process efficiency and ensure consistent product quality [3].

Conventional quality monitoring methods such as the Shewhart chart in Statistical Process Control (SPC) are often insufficient for complex industrial processes because they only monitor one variable at a time [4]. Clinker production is a multivariate process that involves several interrelated variables which must be controlled simultaneously [5]. Multivariate control charts offer a more effective approach as they can detect changes in both the process mean and variability [6]. Methods like Hotelling's  $T^2$  chart and the Generalized Variance (GV) control are more suitable for maintaining the stability and quality of the clinker production process [7].

The application of multivariate control charts has been widely adopted across various industrial processes. For example, the  $T^2$  Hotelling chart has been employed to monitor the quality control of polyester fabric products [8], while the effectiveness of the Hotelling's  $T^2$  and Generalized Variance charts has been compared in monitoring production processes in the fertilizer industry [9].

The objective of this study is to evaluate the application of  $T^2$  Hotelling and Generalized Variance (GV) control charts on clinker data. Specifically, this study aims to obtain the results of quality control of clinker products through  $FCaO$ ,  $C_3S$ , and  $C_3A$  variables at PT XYZ. In addition, this research seeks to measure the performance of the clinker production process through process capability analysis and to examine the impact of seasonal variation on clinker production outcomes.

## 2. RESEARCH METHODOLOGY

### 2.1. Literature Review

#### 2.1.1 Clinker

According to the Ministry of Trade of the Republic of Indonesia, clinker is a semi-finished product made by burning raw materials in a rotary kiln at 1,400–1,500°C, consisting of calcium silica, aluminium oxide, and other oxides used for cement production [10]. Clinker production involves calcination, where limestone decomposes into calcium oxide and  $CO_2$ , followed by the kiln process where clinker is formed. The main components of clinker are  $FCaO$ ,  $C_3S$ , and  $C_3A$ . The  $C_3S$  content is calculated from oxide composition and, if within the required range, can be used as an independent variable in analysis [11].  $C_3A$  releases significant heat, accelerates setting time, and increases early strength but reduces sulphate resistance [3]. And the three main compositions have predetermined specification limits as tabulated in Table 1.

**Table 1.** Specification limits of three main compositions of clinker

Composition	Lower Bound (%)	Upper Bound (%)
FCaO	0.5	2
C3S	60	66
C3A	8	10

### 2.1.2 Independency Test

If controlling for multiple variables, it must be ensured that there is a relationship between the variables. Therefore, to consider the size of the dependency, we consider an independence test, with the null hypothesis (H0) states that X and Y are stochastically independent, meaning there is no statistical relationship between them. And the alternative hypothesis (H1) states that X and Y are not independent [12]. This independence can be tested using Barlett's test with the following test statistics and critical regions.

$$\chi^2 = (ln10)\{B - \sum(n-1)log s_i^2\} \quad (1)$$

Critical region,

If  $\chi^2 \geq \chi^2_{(1-\alpha)(k-1)}$  then reject H<sub>0</sub>

If  $\chi^2 < \chi^2_{(1-\alpha)(k-1)}$  then fail to reject H<sub>0</sub> / accept H<sub>1</sub>

Where  $\chi^2_{(1-\alpha)(k-1)}$  is obtained from the chi-squared distribution table with confidence level (1-α) and degrees of freedom df=(k-1).

### 2.1.3 Normality Test

This distribution is used on a group of data that has a correlation. If  $X \sim N_p(\mu, \Sigma)$  is a multivariate p-variate normal with mean  $\mu$  and variance-covariance matrix  $\Sigma$ , shown in the following equations.

$$\vec{x} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_p \end{bmatrix}, \vec{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \dots & \sigma_{pp} \end{bmatrix} \quad (2)$$

Comparison of four statistical tests to test the null hypothesis (H<sub>0</sub>) that the data follow a multivariate normal distribution, based on measures of skewness, kurtosis, and the Henze-Zirkler test. The Henze-Zirkler test was the most stable and accurate in maintaining the set significance level [13].

### 2.1.4 Hotelling

The Hotelling T<sup>2</sup> control chart is the most used procedure for multivariate process monitoring and control, specifically for tracking the mean vector of a process. In certain industries, such as the chemical and process industries, the subgroup size is naturally n=1, which means only one observation is collected per sample. In such cases, m samples of size n = 1 are typically available, with p quality characteristics measured in each sample [14]. The calculation of the covariance matrix with successive differences is shown in the following equation.

$$S = \frac{1}{2} \frac{V'V}{(m-1)} \quad (3)$$

with:

$$V = \begin{bmatrix} V'_1 \\ V'_2 \\ \vdots \\ V'_{m-1} \end{bmatrix} = \begin{bmatrix} (x_2 - x_1)' \\ (x_3 - x_2)' \\ \vdots \\ (x_{i+1} - x_i)' \end{bmatrix}, i = 1, 2, \dots, m-1 \quad (4)$$

Which means:

$x_{ij}$  : observation vector for the  $i$ -th sample of the  $j$ -th quality variable.

$\bar{x}_j$  : average observation vector of the  $j$ -th quality variable.

**Equation (5)** below shows the calculation of the test statistic used in the Hotelling  $T^2$  control chart:

$$T_i^2 = (x_{ij} - \bar{x}_j)' S^{-1} (x_{ij} - \bar{x}_j) \quad (5)$$

With:

$S^{-1}$ : Inverse of the covariance matrix

The control limits for the Hotelling  $T^2$  control chart for individual observations are given as follows

$$UCL = \frac{(m-1)^2}{m} \beta_{\alpha, \frac{p}{2}, (m-p-1)/2} \quad (6)$$

$$LCL = 0 \quad (7)$$

When the sample size is large ( $m > 100$ ), the control limit can be calculated using the following equation:

$$UCL = \frac{p(m-1)}{m-p} F_{\alpha, p, m-p} \quad (8)$$

$$LCL = 0 \quad (9)$$

Which means:

- UCL : Upper Control Limit
- LCL: Lower Control Limit
- $p$ : Number of quality characteristics ( $j = 1, 2, \dots, p$ )
- $m$ : Number of observations ( $i = 1, 2, \dots, m$ )

A process is said to be in control if the  $T^2$  value falls within the control limits.

### 2.1.5 Generalized Variance

Generalized Variance control diagram is a multivariate control diagram used to monitor variance of a process. The process variant is described from the covariance matrix a measuring  $p \times p$  where the main diagonal element is the variant and the other element is the covariance [14].

### 2.1.6 Capability Process

Process capability analysis is a statistical method used in quality control to evaluate whether a process can consistently produce outputs that meet specification limits. According to the 3-sigma standard, a process is considered capable if it has a capability index greater than 1.33. The process capability index, denoted as  $C_p$ , measures the ability of a process to produce within specified tolerance limits. The following formula is used:

$$C_p = \frac{UCL - LCL}{6\sigma} \quad (10)$$

Here,  $\sigma$  represents the sample standard deviation, while UCL and LCL denote the upper and lower specification limits, respectively. The  $C_{pk}$  index is an extension of the  $C_p$  index that accounts for the location of the process mean, thus capturing both the precision and accuracy of the process. For one-sided specifications, the capability indices are calculated as  $C_{pu}$  and  $C_{pl}$ , using the respective formulas. Furthermore, the process capability indices  $C_p$  and  $C_{pk}$  for multivariate data can be computed using the following equations:

$$MCp = \sum_{i=1}^p W_i C_{pi} \tag{11}$$

$$MCpk = \sum_{i=1}^p W_i C_{pki} \tag{12}$$

Where MCp and MCpk are the multivariate forms of Cp and Cpk.  $W_i$  is the weighting factor based on importance and satisfies the condition  $\sum_{i=1}^p W_i = 1$ . The weights  $W_i$  are assigned according to the importance of each quality characteristic set by the company. If no weighting is specified, equal weights are assumed [15].

### 2.1.7 Ishikawa Diagram

Ishikawa diagram, also known as the cause-and-effect diagram, is one of the tools commonly used to improve quality. This diagram serves to systematically identify, organize, and analyze potential causes of a particular problem. Visually, the diagram resembles a fishbone structure, where the head of the fish represents the main problem (effect) being analyzed, and the bones extending from the spine illustrate the categories of contributing factors. Typically, these categories are grouped into six main factors, known as the 5M+1E framework: material, man, method, machine, measurement, and environment [14].

### 2.2. Data Structure

The dataset consists of three main parameters that become Critical Operating Parameters. The clinker quality variables taken are the levels of FCaO (%), C3S (%), and C3A (%) for the period October 2023 to September 2024. the data structure can be seen as follows.

**Table 2.** Data structure of clinker main parameters

No.	FCaO	C3S	C3A
1	$x_{11}$	$x_{21}$	$x_{31}$
2	$x_{12}$	$x_{22}$	$x_{32}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
313	$x_{1\ 313}$	$x_{2\ 313}$	$x_{3\ 313}$

Based on Table 2, there are 313 observations where the observations will be divided into two phases. Phase 1 from October 2023 to March 2024 because the rainy season is 181 observations. then for phase 2 from May through September 2024 as many as 132 observations which is the dry season. differentiated by season because the treatment given is different so that the analysis should be different.

### 2.3. Analysis Step

The analytical procedure employed in this study is outlined as follows.

1. Obtain clinker quality data from October 2023 until September 2024
2. Pre-processing data by detecting outliers.
3. Calculating the daily average for each variable.
4. Conducting independence tests to determine whether the variables FCaO, C<sub>3</sub>S, and C<sub>3</sub>A are correlated.
5. Checking the multivariate normality assumption to verify whether the variables FCaO, C<sub>3</sub>S, and C<sub>3</sub>A satisfy multivariate normal distribution assumptions.
6. Monitoring clinker production variability using Generalized Variance control chart based on Rainy Season Phase I data (October 2023 to January 2024)

7. Applying  $T^2$  Hotelling control chart if the Generalized Variance chart is statistically in control; if the process is out of control, identifying causes of points outside control limits and making corrections using the  $T^2$  Hotelling control chart
8. Monitoring clinker production using Generalized Variance and  $T^2$  Hotelling control charts based on Rainy Season Phase II data (February to March 2024).
9. Repeating the monitoring procedures for data from May until August 2024 (Dry Season Phase I) and September 2024 (Dry Season Phase II).
10. Analyzing process capability of clinker production during rainy (Phases I and II) and dry seasons (Phases I and II).
11. Constructing an Ishikawa diagram to identify the root causes of variables that exceed control limits.
12. Interpreting the data analysis results.
13. Drawing conclusions and providing recommendations

### 3. RESULTS AND DISCUSSION

#### 3.1. Descriptive Statistics of Clinker Quality Characteristics

Descriptive analysis is used to give initial information about the data before further analysis. The following are the results of descriptive statistics of the quality characteristics variables used.

**Table 3. Descriptive Statistics of Clinker Quality Characteristics**

Fase	Variable	Specification (%)	N	Mean (%)	Variance	Min (%)	Max (%)
I	FCaO	0.5-2	181	1.38	0.10	0.78	2.52
	C <sub>3</sub> S	60-66	181	61.80	2.61	57.20	65.50
	C <sub>3</sub> A	8-10	181	8.86	0.14	8.03	9.93
II	FCaO	0.5-2	132	1.31	0.06	0.92	2.28
	C <sub>3</sub> S	60-66	132	62.18	2.53	55.33	67.74
	C <sub>3</sub> A	8-10	132	9.52	0.12	8.84	10.78

Based on [Table 3](#), it can be seen the average, variance, minimum and maximum of each main composition of clinker. C<sub>3</sub>S is the variable with the largest variance in both phase I and phase II, which is 2.61 and 2.53. This variant shows how far the data is spread from its average value. Then, FCaO is the variable with the smallest variant value of 0.10 for phase I and 0.06 for phase II. The three quality characteristics have average values that are still within the specification limits, although there are still minimum and maximum.

#### 3.2. Assumption Testing

Before analyzing the multivariate control map, there are several assumptions that must be met. The main assumptions tested include independence between FCaO, C<sub>3</sub>S, and C<sub>3</sub>A variables, and the fulfilment of multivariate normal distribution. These assumptions were tested on the clinker production process data, with the following results.

##### 3.2.1 Independency Test

The first assumption test carried out is testing the independence of variables with the Bartlett method. This test is conducted to determine whether the three are interconnected or not. The hypothesis and test results are as follows.

$H_0$  = FCaO, C<sub>3</sub>S, and C<sub>3</sub>A are not correlated

$H_1$  = FCaO, C3S, and C3A are correlated

Using SPSS software, the test results are obtained in the following table.

**Table 4. Testing Variable Independence**

<b>Kaiser-Meyer-Olkin Measure of Sampling Adequacy.</b>		<b>.496</b>
Bartlett's Test of Sphericity	Approx. Chi-Square	1384.440
	Df	3
	Sig.	.000

Based on **Table 4**, it is known that the p-value (Sig.) is 0.00 and if a significance level ( $\alpha$ ) of 0.05 is used, the decision is to reject  $H_0$  because  $p\text{-value} < \alpha$ . It can be concluded that the FCaO, C3S, and C3A variables are correlated and the analysis can be continued.

### 3.2.2 Normality Test

The second test is to determine if the FCaO, C3S, and C3A variables follow a multivariate normal distribution pattern or not. The hypotheses and test results are as follows.

$H_0$  = The three variables are multivariate normally distributed

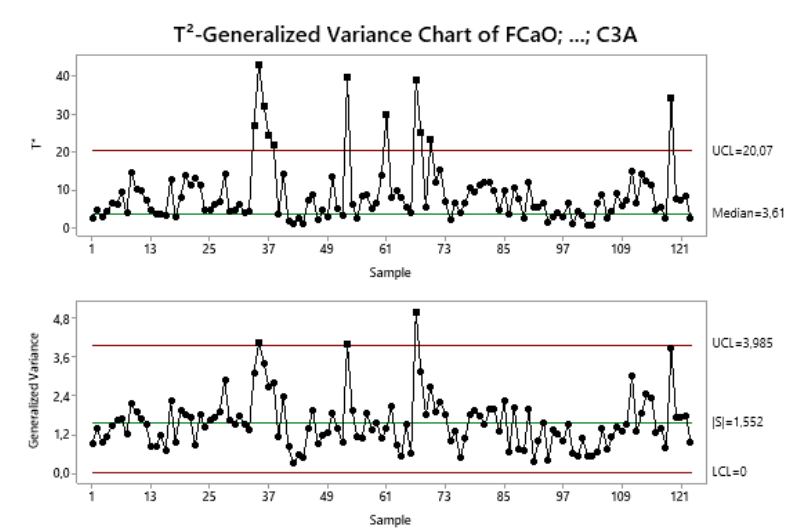
$H_1$  = The three variables are not multivariate normally distributed.

Then the test is made with Python software by looking at the p-value of the output. A p-value of  $3.16E^{-5}$  is obtained with a significance level ( $\alpha$ ) of 0.05, so the decision is to reject  $H_0$  because the  $p\text{-value} < \alpha$ . It can be concluded that the FCaO, C3S, and C3A variables are not multivariate normally distributed. When the sample size is large and the techniques used depend on the properties of  $\bar{X}$ , the assumption of normality is not very important. Also, in real data, clinker data is often not normal because it is affected by fluctuations in temperature, pressure, and inconsistent variations in raw materials. So the three variables were assumed to be multivariate normally distributed to continue the process control analysis. And distribution identification was carried out using EasyFit software. The results show that FCaO follows the Pearson Type V distribution, C3S follows the Log-Pearson Type III distribution, and C3A follows the Weibull distribution.

### 3.3. Rainy Season Control Chart

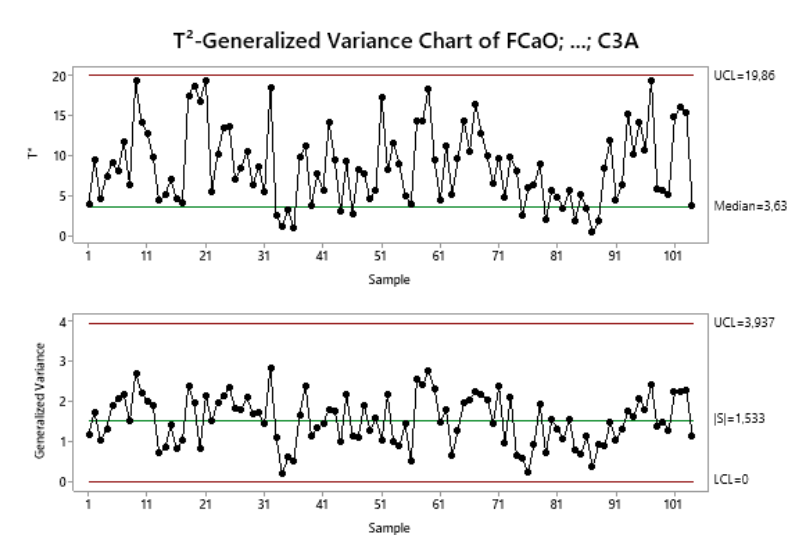
To get more accurate results from the control chart, the data should be separated based on specific conditions, such as rainy and dry phases. This separation is important because the different characteristics of each phase can affect process variations, making the analysis more precise and representative. In the rainy phase, the data used is data from October to March with 181 observations as phase 1. Phase I to establish control limits based on historical data that is in control uses data from October to January. while phase II monitors new data to detect deviations using data from February to March with the mean and covariance of phase 1. If the rainy season data is considered a single phase, large internal variations can cause the control chart signal to be biased. So, dividing the rainy phase data into two subphases can be more effective than simply distinguishing between rainy and dry phases. This is because conditions within a rainy season are often not homogeneous. The following is the T2 hotelling and Generalized Variance control chart of the rainy season clinker product phase 1.





**Figure 1.** Diagram of Phase I Rainy Season

Based on **Figure 1**, the quality control of clinker production in phase I hasn't been statistically controlled both from the process average and the process variability. This is indicated by the presence of observation points that are above the USL. There are 11 samples that are out of the upper control limit, namely samples in the 34th, 35th, 36th, 37th, 38th, 53rd, 61st, 67th, 68th, 70th, and 119th observations. After analysing by looking at the p-value of the variable that causes out of control, it is obtained that the variable that causes the most out of control process is the C3A variable. When the variables that cause the observation points to be out of control are known, then proceed to make the T2-Hotelling and Generalized Variance control charts of phase I improvement by eliminating the observations that are furthest from the upper control limit point. If after removal there are still points that are out of control, then the process of removing out of control data is repeated for several iterations until all points are within all control limits. The following is a T2-Hotelling and Generalized Variance control diagram of phase I of the improvement results.



**Figure 2.** Diagram of Phase I of the Rainy Season In Control



Figure 2 shows that the quality control of the clinker production process in the rainy season phase I of the improvement results is statistically controlled. This is indicated by there are no observation points that come out of the Upper Control Limit (UCL) of 19.86 and the Lower Control Limit (LCL) of 0. Then the control limits will be used to control the average and variability of the clinker production process in phase II. Then, quality control in phase II uses the mean and covariance matrix from phase I that is already in control. The mean parameter used to monitor quality in Phase II consists of the values 1.2723, 62.3173, and 9.8539. The vector above is the mean value of each variable, namely the FCaO, C3S, and C3A in phase 1 that are in control. Then the data used in phase 2 analysis are 58 observations. Using Minitab, the T2 Hotelling and Generalized Variance phase II control charts are obtained as shown in the following figure.

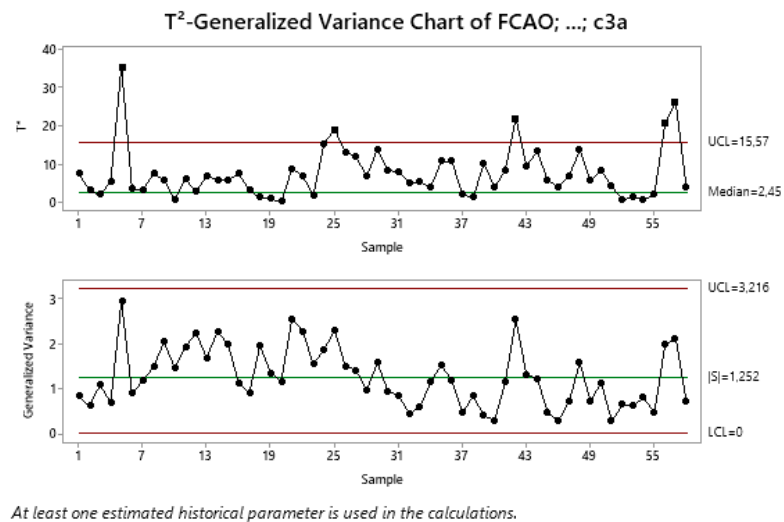
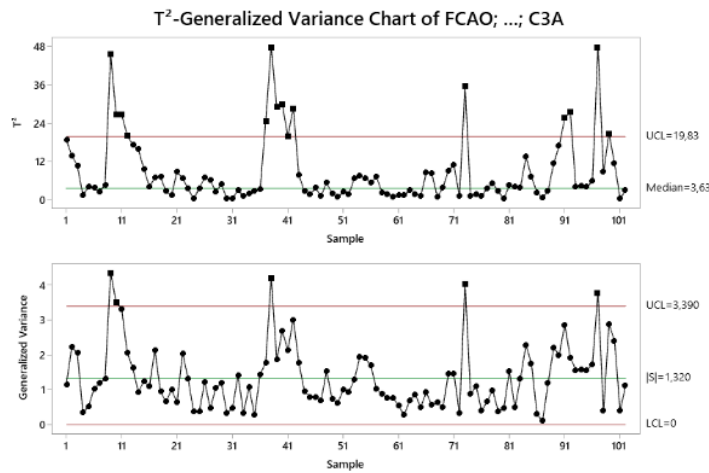


Figure 3. Diagram of Phase II Rainy Season

Figure 3 shows that the quality control of the clinker production process in the rainy season phase II is still not statistically controlled. There are still points that are above the USL. However, the number of points that are outside the isolated limits is reduced. The standard deviation value in phase I of 1,552 is smaller than phase II of 1,252. This shows that the variability is decreasing, which means that the process is more in control.

### 3.4. Dry Season Control Chart

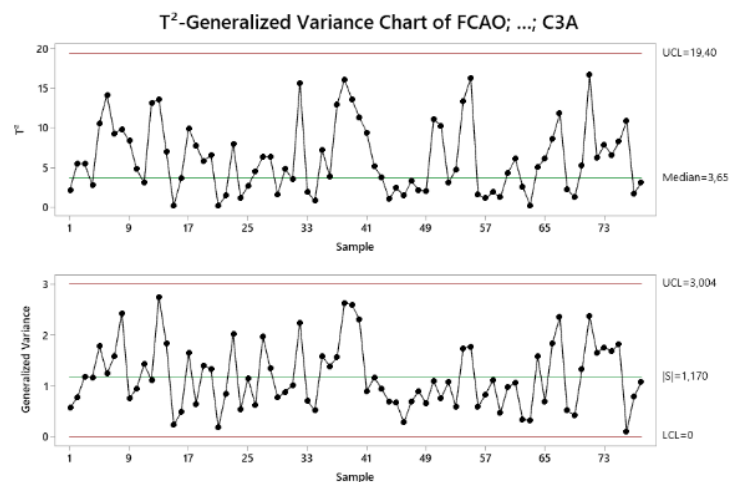
This section analyzes 132 observations collected from May to September 2024. The data for the dry season are divided into two phases. Phase I is used to establish control limits based on a subset of stable historical data, where Phase II is employed to monitor subsequent observations to detect any potential process deviations or irregularities. In dry season phase I, the analysis is conducted using 102 observations. Utilizing Minitab software, the T<sup>2</sup> Hotelling control chart and the Generalized Variance control chart can be directly generated, as shown in Figure 4.



**Figure 4.** Diagram of Phase I Dry Season

Based on **Figure 4**, the clinker production quality control in phase I has not been statistically controlled. There were that 15 samples exceeded the upper control limit, specifically at observations 9th, 10th, 11th, 12th, 37th, 38th, 39th, 40th, 41st, 42nd, 73rd, 91st, 92nd, 97th, and 99th. Upon further analysis using the p-values of each variable contributing to the out-of-control condition, it was found that the variable C3A most frequently contributed to the process instability during phase I.

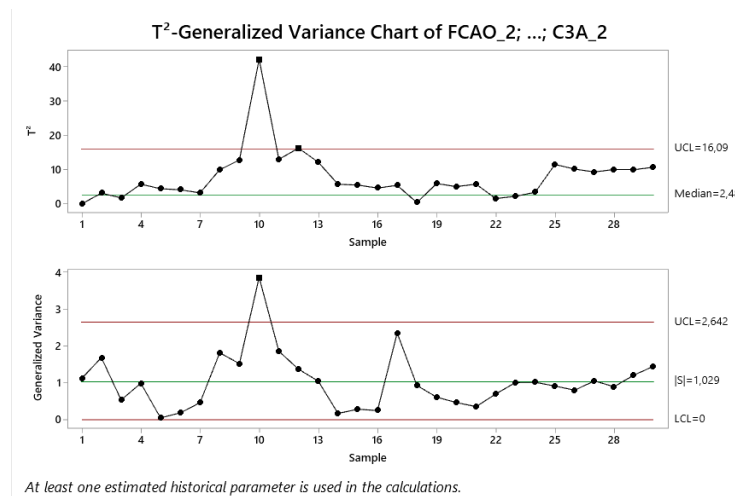
Once the contributing variable for the out-of-control points was identified, then proceed to make a  $T^2$  Hotelling control diagram and Generalized Variance phase I improvement by eliminating the observation that is furthest from the upper control boundary point. If after being eliminated there are still points that are out of control, then the process of deleting data that is out of control is repeated until several iterations until all points are within the control limit. The following is a control diagram of  $T^2$  Hotelling and Generalized Variance phase I of the improved results.



**Figure 5.** Diagram of Phase I of the Dry Season In Control

**Figure 5** shows that the quality control of the clinker production process in phase I of the improvement results has been statistically controlled. This is shown by the absence of observation points that come out of the Upper Control Limit (UCL) of 19.40 and the Lower Control Limit (LCL) of 0. The control limit will be used to control the average and variability of the clinker production process in phase II. Then, quality control in phase II uses the  $T^2$  Hotelling and Generalized Variance control diagrams with mean and covariance matrix from phase I that have been controlled. The mean vector used to

monitor process quality in Phase II is 1.22925, 62.4970, 9.53066. This vector represents the mean values of the FCaO, C3S, and C3A variables, respectively. The data analyzed in phase II consists of 30 observations. Using Minitab software, a control diagram of  $T^2$  Hotelling and Generalized Variance phase II was obtained as shown in [Figure 6](#).



**Figure 6.** Diagram of Phase II Dry Season

[Figure 6](#). shows that the quality control of the clinker production process in phase II is still statistically uncontrollable, as some data points still fall above the upper control limit (UCL). However, the number of out-of-control points has decreased compared to phase I. The standard deviation in phase II is lower than in phase I. The standard deviation value in phase I is 1.170 while in phase II is 1.029. This shows that the variability decreases, this reduction in variability suggests an improvement in process stability, even though fluctuations are still present.

### 3.5. Capability Process

The determination of process capability indices can be conducted either manually or with statistical software. In this study, process capability analysis is performed both univariately and multivariate to provide a comprehensive overview of the process's ability to meet quality specifications. [Table 5](#) presents the results of process capability calculations for phase I and phase II during the rainy season.

**Table 5.** Comparison Table of Process Capabilities in Rainy Season

Phase	Index	Quality Characteristic			
		FCaO	C3S	C3A	Multivariate
I	Cp	1,32	1,11	1,82	1,42
	Cpk	1,24	0,79	1,63	1,22
	Cpl	1,41	0,79	1,63	1,26
	Cpu	1,24	1,43	2,01	1,56
II	Cp	1,17	1,11	1,48	1,25
	Cpk	0,69	0,40	1,18	0,76
	Cpl	1,66	1,82	1,18	1,55
	Cpu	0,69	0,40	1,77	0,95

Given the absence of a special weight determination from the company for the three observed quality characteristics, it is assumed that each variable has an equal level of importance. Therefore, equal weights of 0.333 are assigned to each variable: FCaO, C3S, and C3A. As an example, the following presents a manual calculation of the multivariate

capability index MCp. Similar calculations are also conducted for the indices MCpk, MCpl, and MCpu.

$$MCp = (0.333 \times 1.27) + (0.333 \times 1.11) + (0.333 \times 1.70) = 1.36$$

In the first phase of the rainy season, the production process showed quite good capabilities. The C3A parameter has the highest capability, indicating a stable process that conformed to specifications, while C3S shows the lowest capability. However, in phase II there was a general decline in capability. The multivariate Cpk value decreased to 0.76, indicating the process no longer meets the capability standard. C3S remains the worst performing parameter, while C3A is still relatively stable despite the decline. This condition indicates the need for process evaluation during the rainy season to ensure consistency and compliance with product specifications.

Process capability analysis was also conducted for production during the dry season. The results of the capability analysis for both Phase I and Phase II are presented in the [Table 6](#), which includes univariate indices (Cp, Cpk, CPL, and CPU) for each quality parameter, as well as multivariate indices representing the overall performance of the process.

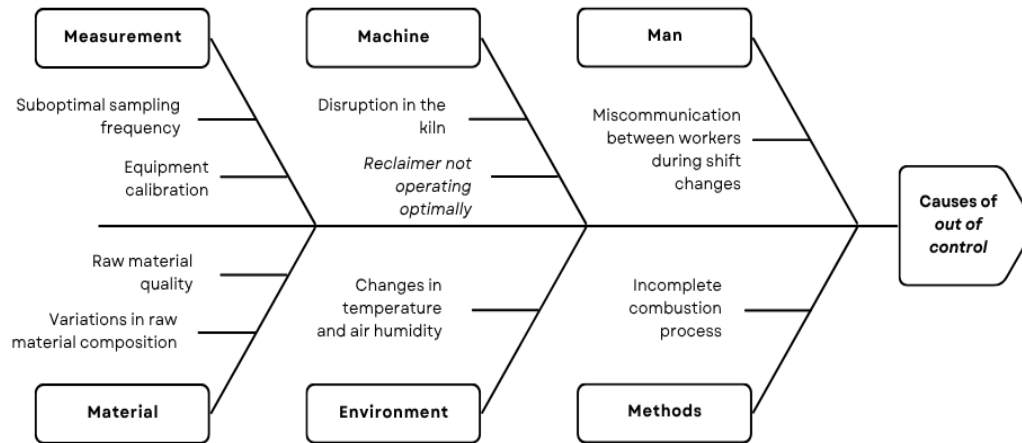
**Table 6. Comparison Table of Process Capabilities in Dry Season**

Phase	Index	Quality Characteristic			
		FCaO	C3S	C3A	Multivariate
I	Cp	1,64	1,13	2,18	1,65
	Cpk	1,60	0,94	1,63	1,19
	Cpl	1,60	0,94	2,83	1,79
	Cpu	1,69	1,32	1,02	1,34
II	Cp	1,62	0,88	1,72	1,41
	Cpk	1,24	0,46	1,13	0,94
	Cpl	1,99	0,46	2,30	1,58
	Cpu	1,24	1,29	1,13	1,22

Based on the calculation results, the process capability in the dry season showed relatively good performance, most of which exceeded the threshold that indicates that the process is classified as capable. The multivariate index also indicates that the overall process is within an acceptable range, especially in phase I. Compared to the rainy season, the process capability in the dry season is generally better and more stable. This is evidenced by the increase in Cpk values for most parameters and higher multivariate Cp values, indicating that the production process during the dry season exhibits lower variability and a greater ability to meet the specified quality standards.

### 3.6. Ishikawa Diagram

Based on the quality control of the clinker production process, especially in phase II, it is known that the process has not been statistically controlled. Therefore, the identification of factors that have the possibility of causing out of control will be carried out using the Ishikawa diagram as shown in [Figure 7](#).



**Figure 7.** Ishikawa diagram of clinker quality control

#### 4. CONCLUSION

Based on the results and discussion that has been carried out, the following conclusions are obtained.

1. Quality control of the variability and average of the clinker production process in the rainy season and dry season phase I has not been statistically controlled. After removing the out-of-control observation points, the Generalized Variance and T2 Hotelling control diagrams of phase I were obtained. Using the mean and covariance of phase 1 that is in control, phase II monitoring is carried out in both seasons. Quality control of the clinker production process in phase II in the variability and average process is also not statistically controlled. This is evidenced by the presence of several observation points that are out of control on the control diagram. The most influential variable that causes out of control in clinker quality control is C3A due to fluctuations in kiln temperature.
2. Overall, the clinker production process has improved after quality control, especially in stabilizing variations in the wet season. The process capability in the dry season shows relatively good performance because it has exceeded the threshold compared to the rainy season capability,
3. Process capability analysis shows that the dry season provides better and more stable production performance than the wet season. In the wet season, especially phase II, there is a decrease in capability with multivariate process capability index values that do not meet the standard, especially in the C3S parameter. In contrast, during the dry season, most parameters showed high capability with lower variation, indicating a more consistent and compliant process. Process evaluation and improvement are required during the rainy season to maintain production quality.

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