

Feasible Generalized Least Squares Based on Minimum Variance Quadratic Unbiased Estimation for an Unbalanced One-Way Error-Component Panel Model

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Abstract

Panel data combine cross-sectional and time-series information, but incomplete firm-year observations produce an unbalanced structure that requires an estimation procedure accommodating unequal observation counts. This study applies a one-way random-effects error-component model to 42 observations from 10 consumer-goods manufacturing firms listed on the Indonesia Stock Exchange during 2017–2021. The firm-specific and idiosyncratic variance components are estimated using Minimum Variance Quadratic Unbiased Estimation (MIVQUE), and the resulting covariance matrix is used to estimate the regression coefficients through feasible generalized least squares (FGLS). The estimated variance components are 0.0063 for the firm-specific effect and 0.0805 for the idiosyncratic disturbance, indicating that approximately 7.26% of the composite disturbance variance is attributable to persistent differences among firms. The fitted model is $Y_{it} = 0.0657 - 0.0229DER_{it} + 0.2089NPM_{it}$. The debt-to-equity ratio is negatively associated with stock returns, whereas the net profit margin is positively associated with stock returns. However, neither coefficient is statistically significant at the 5% level, with p-values of 0.0938 and 0.5165, respectively. Therefore, the estimated relationships are interpreted as conditional associations under the assumed random-effects specification rather than as causal effects. This study provides an empirical implementation of MIVQUE-based covariance estimation combined with FGLS for an unbalanced one-way error-component panel model.

Keywords: Business cycle, singular spectrum analysis, baxter-king filter, GDP, Indonesia.

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1. INTRODUCTION

The Indonesian Stock Exchange (IDX) facilitates capital allocation across sectors, including manufacturing. Manufacturing firms are economically important because of their production scale, capital intensity, and contribution to national output. For investors, stock returns summarize investment performance and compensation for risk [1] [2]. Firm-level characteristics commonly used to explain variation in returns include the debt-to-equity ratio (DER), which represents financial leverage, and net profit margin (NPM), which represents profitability. Changes in leverage may alter financial risk, while profitability reflects operational efficiency and income-generating capacity; both may therefore be associated with market valuation and return behavior [3].

Panel regression jointly uses cross-sectional and time-series information, allowing firm heterogeneity and temporal variation to be analyzed in one framework [4][5]. In practice, firm-year records are often incomplete because financial statements or required variables are not available for every firm in every year. The resulting unequal numbers of observations across firms form an unbalanced panel. Ignoring this structure can produce an incorrectly specified covariance matrix and inefficient coefficient estimates, particularly when persistent firm-specific heterogeneity is present [6][7]. One approach to address unobserved heterogeneity in unbalanced panel data is the error component model, in which individual-specific effects are incorporated into the disturbance structure. In the one-way error component specification, heterogeneity across firms is captured through time-invariant individual effects, whereas the remaining variation is attributed to idiosyncratic disturbances [8]. Several methods have been proposed for variance component estimation in such models, including Maximum Likelihood (ML), Restricted Maximum Likelihood (REML), Analysis of Variance (ANOVA), and Minimum Variance Quadratic Unbiased Estimation (MIVQUE) [9].

Previous studies have explored the application of these methods in various unbalanced panel data contexts. Study [10] applied MIVQUE to estimate variance components in two-way error-component models, whereas study [11] employed REML for unbalanced panel regression with two-way effects. More recently, study [12] combined ANOVA-based variance-component estimation with Feasible Generalized Least Squares (FGLS) to analyze unbalanced manufacturing-firm data. In contrast, the present study focuses on a one-way random-effects specification and uses MIVQUE to estimate the variance components required to construct the covariance matrix for FGLS estimation. Thus, the contribution of this study lies in the empirical application of MIVQUE-based FGLS to an unbalanced panel dataset with unequal numbers of observations across firms, rather than in the development of a new estimator or a full maximum likelihood procedure.

This study examines the stock returns of consumer-goods manufacturing firms listed on the Indonesia Stock Exchange from 2017 to 2021 using an unbalanced one-way random-effects panel model. Unobserved firm-specific heterogeneity is represented by a time-invariant random component. MIVQUE is used to estimate the variance components, from which the estimated covariance matrix is constructed and subsequently incorporated into the FGLS estimation of the regression coefficients. The analysis evaluates the conditional associations of the debt-to-equity ratio (DER) and net profit margin (NPM) with stock returns while explicitly accommodating unequal numbers of observations across firms.

Motivated by these considerations, this study investigates the stock returns of manufacturing firms listed on the Indonesia Stock Exchange from 2017 to 2021 using an unbalanced panel data framework. A one-way error-component model is employed to account for unobserved firm-specific heterogeneity. The variance components are estimated using Minimum Variance Quadratic Unbiased Estimation (MIVQUE), and the resulting covariance matrix is subsequently used to obtain the regression coefficient estimates through Feasible Generalized Least Squares (FGLS). This approach explicitly accommodates unequal numbers of observations across firms and provides a framework for examining the conditional associations of financial leverage and profitability with stock returns.

2. METHOD

2.1. Data and Research Variable

This study uses secondary financial-statement data obtained from the official IDX database (www.idx.co.id). The analytic sample comprises 10 consumer-goods manufacturing firms that remained listed during 2017–2021 and had sufficient observations on stock return, DER, and NPM for model estimation. The choice of this sector provides firms with comparable business classification while retaining variation in leverage and profitability. Because the availability of the required firm-year variables differed across companies, the final dataset contains 42 of 50 possible firm-year observations. The source dataset recorded the availability pattern but did not retain a verified company-specific reason for every missing year; therefore, Table 2 reports the observation counts rather than attributing unverified causes. Stock return is the dependent variable, while DER and NPM are the explanatory variables.

2.2. Multiple Linear Regression Analysis

Multiple linear regression was employed as an initial analytical framework to examine the relationship between a response variable and more than one explanatory variable. This approach enables the assessment of the effect of each independent variable while controlling for the influence of the other variables included in the model [13]. Formally, the multiple linear regression model with K explanatory variables can be expressed as Equation (1):

$$Y_i = \beta_0 + \sum_{k=1}^K \beta_k x_{ik} + \varepsilon_i, i = 1, 2, \dots, n \quad (1)$$

For notational convenience and computational efficiency, Equation (1) can be rewritten in the matrix form, as shown in Equation (2).

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (2)$$

where $\mathbf{Y}$ is an $n \times 1$ vector of observations of the dependent variable, $\boldsymbol{\beta}$ denotes the $(K + 1) \times 1$ vector of regression parameters, $\boldsymbol{\varepsilon}$ represents the vector of random disturbances, and $\mathbf{X}$ is the corresponding design matrix containing the intercept and explanatory variable [14].

2.3. The Concept of Analysis of Variance

In the context of panel data models, Analysis of Variance (ANOVA) is used to estimate variance components by decomposing the total variability of the disturbance term into within-individual and between-individual components. The ANOVA framework derives estimators by equating the observed quadratic forms of the residuals with their expected values, resulting in closed-form expressions for the variance

components. Under the assumption of normally distributed disturbances, the ANOVA estimator belongs to the class of quadratic unbiased estimators, and satisfies the minimum variance criterion within this class. Owing to this property, ANOVA-based estimators are frequently used as benchmark estimators in variance-component models. Importantly, the ANOVA approach remains applicable in unbalanced panel settings because it allows unequal numbers of observations across cross-sectional units. For one-way error component models, variance decomposition corresponds to an incomplete one-way classification structure, in which unobserved heterogeneity arises solely from individual-specific effects. In this setting, variance component estimators are obtained by separating the variation attributable to individual effects from the idiosyncratic error variation. The resulting estimators are given by [Equation \(3\)](#) and [\(4\)](#).

$$\sigma_{av}^2 = \frac{SSE}{\sum_{i=1}^N (T_i - 1)}, \quad SSE = \sum_{i=1}^N \sum_{t=1}^{T_i} \varepsilon_{it}^2 - \sum_{i=1}^N \frac{\varepsilon_{io}^2}{T_i} \quad (3)$$

$$\sigma_{a\mu}^2 = \frac{SST_r}{N - 1}, \quad SST_r = \sum_{i=1}^N \frac{\varepsilon_i^2}{T_i} - CF \quad (4)$$

where SSE denotes the sum of the squared residuals, SST_r represents the adjusted between-unit sum of squares, CF is the correction factor, N is the number of cross-sectional units, and T_i indicates the number of time observations for unit i .

2.4. Regression Analysis Using Panel Data

Panel data regression explicitly incorporates observations indexed by both cross-sectional units and periods, enabling the joint analysis of individual heterogeneity and temporal variation within a unified modeling framework[\[15\]](#). Cross-sectional observations capture differences across units at a given point in time, whereas time-series observations describe the evolution of each unit over successive periods[\[16\]](#). The panel structures used in this study are summarized in [Table 1](#).

Table 1. Panel Data Structure

Individual (i)	Time (t)	Y_{it}	X_{it1}	X_{it2}	...	X_{itK}
1	1	Y_{11}	X_{111}	X_{112}	...	X_{11K}
	2	Y_{12}	X_{121}	X_{122}	...	X_{12K}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	T	Y_{1T}	X_{1T1}	X_{1T2}	...	X_{1TK}
2	1	Y_{21}	X_{211}	X_{212}	...	X_{21K}
	2	Y_{22}	X_{221}	X_{222}	...	X_{22K}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	T	Y_{2T}	X_{2T1}	X_{2T2}	...	X_{2TK}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N	1	Y_{N1}	X_{N11}	X_{N12}	...	X_{N1K}
	2	Y_{N2}	X_{N21}	X_{N22}	...	X_{N2K}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	T	Y_{NT}	X_{NT1}	X_{NT2}	...	X_{NTK}

Source: [\[18\]](#)

Depending on the completeness of the observations, the panel datasets may be classified as balanced or unbalanced. In a balanced panel, each unit is observed for the same number of periods, whereas in an unbalanced panel, the number of time observations varies across units. The modeling framework adopted in this study was designed to accommodate unbalanced observation patterns commonly encountered in empirical applications [\[17\]](#). Let $i = 1, 2, 3, \dots, N$ index individuals and $t = 1, 2, 3, \dots, T_i$ index periods. The general panel regression model is expressed by [Equation \(5\)](#) [\[19\]](#).

$$Y_{it} = \beta_0 + \sum_{k=1}^K \beta_{ik} X_{itk} + \varepsilon_{it}, \quad i = 1, 2, 3, \dots, N; t = 1, 2, 3, \dots, T, \text{ and } k = 1, 2, 3, \dots, K \quad (5)$$

In Equation (5), Y_{it} denotes the dependent variable for individual i at time t , β_0 is the intercept, X_{itk} represents the k -th explanatory variable, and β_k is the corresponding regression coefficient. The term ε_{it} captures the composite disturbance in the panel regression model. To represent unobserved heterogeneity explicitly, the composite disturbance in Equation (5) can be decomposed using an error component structure. Under the one-way specification, the error term is expressed as in Equation (6)

$$\varepsilon_{it} = \mu_i + v_{it} \quad (6)$$

Alternatively, when time effects are included explicitly, the decomposition can be extended, as in Equation (7).

$$\varepsilon_{it} = \mu_i + \lambda_t + v_{it} \quad (7)$$

In Equation (6) and Equation (7), μ_i denotes the unobserved individual-specific effect that is time-invariant, λ_t represents the time-specific effect common to all individuals in period t , and v_{it} is the idiosyncratic disturbance that captures the remaining random variation. In this study, emphasis is placed on the one-way component structure error in Equation (6), where heterogeneity is primarily attributed to individual effects. [20]

2.5. One-Way Error Component Unbalanced Panel Data Regression Model

The one-way random-effects model assumes that unobserved heterogeneity is captured by a time-invariant firm-specific component, while the remaining temporal variation is represented by the idiosyncratic disturbance. A key identifying assumption is $E(\mu_i | X_i) = 0$: the unobserved firm effect must be uncorrelated with DER and NPM in all observed periods. The idiosyncratic disturbance is also assumed to have zero conditional mean, constant variance, and no serial correlation, and to be independent of the firm effect. Under these assumptions, the random-effects covariance structure can be used for efficient estimation in an unbalanced panel [9]. In this study, the specification in Equation (8) is adopted as the assumed model because the analysis focuses on parameter estimation for an unbalanced one-way error-component panel model.

$$y_{it} = X_{it}\beta + u_{it}, u_{it} = \mu_i + v_{it}, \quad i = 1, 2, \dots, N, \quad t = 1, 2, \dots, T_i \quad (8)$$

where X_{it} is a $1 \times (K + 1)$ vector of explanatory variables including the intercept, and β is the corresponding parameter vector. The composite disturbance term u_{it} is decomposed as $u_{it} = \mu_i + v_{it}$, where $\mu_i \sim \text{IID } N(0, \sigma_\mu^2)$ represents the unobserved individual-specific effect, and $v_{it} \sim \text{IID } N(0, \sigma_v^2)$ denotes the idiosyncratic error term, assumed to be independent of μ_i . Because individuals are observed over different time periods in an unbalanced panel, with T_i varying across $i = 1, 2, \dots, N$, Equation (8) can be rewritten in a stacked vector form, as shown in Equation (9).

$$y = X\beta + u, \quad (9)$$

where y and u are vectors of dimensions $n \times 1$, X is an $n \times (K + 1)$ design matrix, and the total number of observations is $n = \sum_{i=1}^N T_i$. The vectors of the individual effects and idiosyncratic errors are defined as $\mu = (\mu_1, \mu_2, \dots, \mu_N)'$ and $v = (v_{11}, v_{12}, \dots, v_{NT_N})'$, respectively. Following the ordering scheme proposed by Wansbeek and Kapteyn (1989), observations are stacked sequentially by individual and time period such that all observations for the first individual precede those for the second individual [21]. Under this arrangement, the disturbance vector in Equation (9) can be expressed as Equation (10).

$$u = Z\mu + v \quad (10)$$

where $\mathbf{Z} = (\mathbf{D}_1, \mathbf{D}_2, \dots, \mathbf{D}_N)'$ is an $n \times N$ incidence matrix constructed from the identity matrix by deleting the rows corresponding to unobserved individual time combinations. Each submatrix $\mathbf{D}_i$ has dimension $T_i \times 1$, and $\mathbf{1}_{T_i}$ denotes a $T_i \times 1$ vector of one. Based on the distributional assumptions imposed on $\boldsymbol{\mu}$ and $\mathbf{v}$, the variance–covariance matrix of the composite disturbance vector $\mathbf{u}$ in Equation (10) is given by Equation (11).

$$\boldsymbol{\Omega} = E(\mathbf{u}\mathbf{u}') = \sigma_\mu^2 \mathbf{Z}\mathbf{Z}' + \sigma_v^2 \mathbf{I}_n = \sigma_v^2 (\mathbf{I}_n + \phi \mathbf{Z}\mathbf{Z}') = \sigma_v^2 \boldsymbol{\Sigma} \quad (11)$$

where $\mathbf{I}_n$ is an $n \times n$ identity matrix. The ratio of the variance components, which plays a central role in the generalized least squares estimation, is defined in Equation (12).

$$\phi = \frac{\sigma_{a\mu}^2}{\sigma_{av}^2} \quad (12)$$

In Equation (12), ϕ represents the relative magnitude of individual-specific variation compared with idiosyncratic variation, and its value depends on the underlying variance component estimates.

2.6. Minimum Variance Quadratic Unbiased Estimation

Following Rao (1971), the variance components in the one-way error component model were estimated using the Minimum Variance Quadratic Unbiased Estimation (MIVQUE) approach. In this study, the MIVQUE formulation was constructed by combining the disturbance covariance structure in Equation (11) and the variance ratio definition in Equation (12), which were then incorporated into the MIVQUE estimating system given in Equation (13).

$$\begin{bmatrix} \hat{\sigma}_\mu^2 \\ \hat{\sigma}_v^2 \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{bmatrix}^{-1} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \quad (13)$$

with $\gamma_{11} = \text{tr}(\mathbf{Z}\mathbf{Z}'\mathbf{R}\mathbf{Z}\mathbf{Z}'\mathbf{R})$, $\gamma_{12} = \text{tr}(\mathbf{Z}\mathbf{Z}'\mathbf{R}\mathbf{R})$, $\gamma_{22} = \text{tr}(\mathbf{R}\mathbf{R})$, $\delta_1 = \mathbf{y}'\mathbf{R}\mathbf{Z}\mathbf{Z}'\mathbf{R}\mathbf{y}$, $\delta_2 = \mathbf{y}'\mathbf{R}\mathbf{R}\mathbf{y}$, $\mathbf{R} = \frac{\boldsymbol{\Omega}^{-1} - \boldsymbol{\Omega}^{-1}\mathbf{X}(\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-1}}{\sigma_{av}^2}$

Equation (13) produces the MIVQUE estimates $\hat{\sigma}_\mu^2$ and $\hat{\sigma}_v^2$. These estimates are substituted into the covariance structure in Equation (11) to obtain the estimated covariance matrix in Equation (14).

$$\hat{\boldsymbol{\Omega}} = \hat{\sigma}_\mu^2 \mathbf{Z}\mathbf{Z}' + \hat{\sigma}_v^2 \mathbf{I}_n = \hat{\sigma}_v^2 (\mathbf{I}_n + \phi \mathbf{Z}\mathbf{Z}') = \hat{\sigma}_v^2 \hat{\boldsymbol{\Sigma}} \quad (14)$$

The estimated covariance matrix $\hat{\boldsymbol{\Omega}}$ in Equation (14) accounts for the unbalanced one-way error-component structure. It is subsequently used as the covariance matrix in the FGLS coefficient estimation.

2.7. MIVQUE Based Feasible Generalized Least Squares Estimation

Based on the one-way error-component model specified in Equation (8), the stacked response vector is assumed to follow the multivariate normal distribution[23].

$$\mathbf{y} \sim N(\mathbf{X}\boldsymbol{\beta}, \boldsymbol{\Omega}) \quad (15)$$

where $\boldsymbol{\Omega}$ denotes the variance–covariance matrix of the composite error term. Under the distributional assumption in Equation (15), the component of the Gaussian log-likelihood that depends on $\boldsymbol{\beta}$, for a given $\boldsymbol{\Omega}$, can be written as

$$\ell(\boldsymbol{\beta}|\boldsymbol{\Omega}) = -\frac{1}{2}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Omega}^{-1}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) + \text{C} \quad (16)$$

where C contains terms that do not depend on $\boldsymbol{\beta}$. Differentiating Equation (16) with respect to $\boldsymbol{\beta}$ produces the score equation

Differentiating Equation (15) with respect to β yields the score equation

$$\frac{\partial \ell(\beta | \Omega)}{\partial \beta} = X' \Omega^{-1} (y - X\beta) = 0$$

Solving Equation (17) for β yields the generalized least squares estimator

$$\hat{\beta}_{GLS} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \quad (17)$$

Because Ω is unknown, it is replaced by $\hat{\Omega}_{MIVQUE}$, which is constructed from the variance-component estimates obtained using MIVQUE. Substituting $\hat{\Omega}_{MIVQUE}$ for Ω in Equation (18) gives the feasible generalized least squares estimator

$$\hat{\beta}_{FGLS} = (X' \hat{\Omega}_{MIVQUE}^{-1} X)^{-1} X' \hat{\Omega}_{MIVQUE}^{-1} Y \quad (18)$$

2.8. Stock Return

Stock returns represent the compensation investors receive for allocating funds to financial assets and bearing the associated risk over a given investment horizon[24]. From an investment perspective, returns serve as a primary indicator of performance and reflect the market's assessment of both profitability and risk[25]. Variations in stock returns are influenced by firm-specific financial conditions and market factors. Among firm-level indicators, financial leverage and profitability ratios are commonly used to explain the differences in stock returns. The debt-to-equity ratio (DER) measures the proportion of total liabilities relative to shareholder equity and reflects a firm's capital structure[26]. A higher DER indicates a greater reliance on debt financing, which increases the financial risk and potential volatility of returns. Conversely, a lower DER suggests a more conservative capital structure, implying stronger capacity to absorb financial obligations and mitigate risk exposure. As a result, investors often perceive firms with relatively high DER values as riskier, leading to more cautious investment behavior[27]. Profitability is frequently proxied by net profit margin (NPM), which captures a firm's ability to generate net income from its sales activities. A higher NPM indicates greater operational efficiency, reflecting a firm's effectiveness in controlling costs and converting revenue into profit. In contrast, lower NPM signals weaker profitability and limited cost efficiency. Firms exhibiting higher NPM values tend to attract investor interest because strong profitability prospects are generally associated with favorable expectations regarding future performance. Increased investor demand for such stocks may contribute to upward pressure on share prices, thereby influencing the observed stock returns[26].

3. RESULTS AND DISCUSSION

3.1. Unbalanced Panel Data Regression with One-Way Error Component on Stock Return Data

The unbalanced panel data regression model with a one-way error component, as defined in Equations (5) and (6), is applied to analyze the stock return data of manufacturing firms listed on the Indonesia Stock Exchange (IDX). The dataset consists of ten firms observed over a maximum of five years, resulting in an unbalanced panel due to incomplete observations for several cross-sectional units. The regression specification for stock returns can be written as Equation (19).

$$Y_{it} = \beta_0 + \sum_{k=1}^2 \beta_{ik} X_{itk} + \mu_i + v_{it}, i = 1, 2, \dots, 10, t = 1, 2, \dots, 5 \quad (19)$$

where Y_{it} denotes the stock return for firm i at time t , β_0 is the intercept, X_{itk} represents the explanatory variables, β_{ik} is the corresponding regression coefficient, μ_i is the unobserved individual-specific effect, and v_{it} is the idiosyncratic disturbance term. In this application, firm-specific effects denote unobserved, time-invariant differences among companies, including managerial characteristics, business structure, and internal policies. These differences are considered random deviations and are presumed to be uncorrelated with the observed DER and NPM values. The assumption of normality is justified as each effect may represent the combined influence of multiple unobserved factors, while mutual independence differentiates persistent firm heterogeneity from temporary firm-year shocks. These assumptions facilitate the covariance structure necessary for FGLS estimation. To facilitate estimation, Equation (19) is expressed in a stacked matrix form, as given in Equation (20).

$$Y = X\beta + \Delta_1\mu + v \quad (20)$$

where Δ_1 denotes an $n \times N$ matrix with $N_t \leq N$, and represents the total number of individuals observed during the study period. Furthermore, $\mu = (\mu_1, \mu_2, \dots, \mu_N)'$, and $v = (v_{11}, v_{12}, \dots, v_{1T}, \dots, v_{N1}, \dots, v_{NT})'$. The empirical application involves two explanatory variables: the debt-to-equity ratio (X_1) and net profit margin (X_2), with stock returns as the dependent variable. Owing to incomplete observations over time for several firms, the resulting dataset constitutes an unbalanced panel. A summary of the observation pattern for each firm is presented in Table 2, which reports the number of observed and unobserved years for manufacturing companies included in the sample.

Based on Equation (19), the estimated regression model for the unbalanced panel with a one-way error component is expressed as Equation (21):

$$Y_i = \beta_0 + \beta_1(X_1)_{it} + \beta_2(X_2)_{it} + \mu_i + v_{it}, i = 1, 2, \dots, N_t, \text{ and } t = 1, 2, \dots, 5 \quad (21)$$

Table 2. Observed and Unobserved Company Data

No	Company	Number of Years Observed	Number of Unobserved Years
1	AISA	4	1
2	CINT	2	3
3	DLTA	5	0
4	DVLA	5	0
5	INDF	3	2
6	KICI	5	0
7	KINO	5	0
8	LMPI	4	1
9	MBTO	5	0
10	PYFA	4	1

Source: Data processed

To represent the one-way error component structure explicitly, the vector of individual effects was incorporated through an incidence matrix. Following the stacking convention adopted in this study, the disturbance term can be written in a vector form using Equation (10), where the corresponding incidence matrix Z is constructed as described in Equation (22).

$$\mathbf{Z}_{(42 \times 5)} = \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \\ \mathbf{D}_3 \\ \mathbf{D}_4 \\ \mathbf{D}_5 \\ \mathbf{D}_6 \\ \mathbf{D}_7 \\ \mathbf{D}_8 \\ \mathbf{D}_9 \\ \mathbf{D}_{10} \end{bmatrix} \quad (22)$$

where matrix $\mathbf{D}$ is an identity matrix of size 5×5 with matrix structure, as follows:

$$\begin{aligned} \mathbf{D} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \mathbf{D}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \mathbf{D}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}; \\ \mathbf{D}_3 = \mathbf{D}_4 = \mathbf{D}_6 = \mathbf{D}_7 = \mathbf{D}_9 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \mathbf{D}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \\ \mathbf{D}_8 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \mathbf{D}_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

3.2. Estimation of Error Variance Components

The error-variance components σ_μ^2 and σ_v^2 were first summarized using the unequal-replication one-way classification associated with the unbalanced panel. Table 3 reports the degrees of freedom and sums of squares used to describe between-firm and within-firm variation. These quantities provide initial variance-component values for constructing the covariance structure used in the MIVQUE stage. The ANOVA results corresponding to the unbalanced panel structure are summarized in Table 3, which reports the degrees of freedom and associated sums of squares for the treatment and error components.

Table 3. Analysis of Variance Calculation Results

Source of Variance	Db	JK
Treatment	9	0.7184
Error	32	2.5970
Total	41	3.3155

Source: Data processed

Using the expressions derived for the one-way classification model, the variance component estimators were obtained as follows:

$$\begin{aligned} \sigma_{av}^2 &= \frac{SSE}{\sum_{i=1}^N (T_i - 1)} = \frac{2.5970}{32} = 0.0811 \\ \sigma_{a\mu}^2 &= \frac{SST}{N - 1} = \frac{0.7184}{10 - 1} = 0.0798 \end{aligned}$$

The estimates $\hat{\sigma}_\mu^2 = 0.0798$ and $\hat{\sigma}_v^2 = 0.0811$ are similar in magnitude. The implied share of disturbance variance attributable to persistent between-firm heterogeneity is approximately $0.0798 / (0.0798 + 0.0811) = 0.496$. Thus, nearly half of the modeled disturbance variation is associated with time-invariant differences among firms,

supporting the practical relevance of an error-component structure. These ANOVA quantities are used as descriptive starting values; the final MIVQUE estimates should satisfy the non-negativity constraints required for a valid covariance matrix.

3.3. Parameter Estimation Using MIVQUE-Based Feasible Generalized Least Squares Parameter Estimation Using Maximum Likelihood

Minimum Variance Quadratic Unbiased Estimation (MIVQUE) was applied to estimate the variance components of the one-way error-component model. Based on the variance-component estimates obtained in Subsection 3.2, the variance–covariance matrix of the composite disturbance was constructed according to Equation (11). The dataset contains 42 observations from 10 firms. Therefore, the incidence matrix $\mathbf{Z}$ has dimension 42×10 , while the resulting variance–covariance matrix $\hat{\mathbf{\Omega}}$ has dimension 42×42 . The variance-component estimation was performed using the system presented in Equation (13). After the incidence matrix was specified according to the 10 firms, the MIVQUE estimates were obtained as follows:

$$\begin{bmatrix} \hat{\sigma}_\mu^2 \\ \hat{\sigma}_v^2 \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{bmatrix}^{-1} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} 0.006301 \\ 0.080499 \end{bmatrix} \quad (21)$$

The previous negative estimate of $\hat{\sigma}_\mu^2$ resulted from specifying $\mathbf{Z}$ according to the five observation years rather than the 10 firms. In the one-way firm-specific error-component model, each column of $\mathbf{Z}$ must represent a firm. After correcting this specification, both variance-component estimates are nonnegative and therefore satisfy the admissibility condition for constructing a valid covariance matrix. The estimated proportion of the composite disturbance variance attributable to time-invariant differences among firms is:

$$\hat{\rho} = \frac{\hat{\sigma}_\mu^2}{\hat{\sigma}_\mu^2 + \hat{\sigma}_v^2} = 0.0726$$

Thus, approximately 7.26 % of the modeled disturbance variation is associated with persistent differences among firms, whereas the remaining variation is attributable to idiosyncratic disturbances. Substitution of the estimates in Equation (21) into the covariance structure specified in Equation (11) gives:

$$\hat{\mathbf{\Omega}} = 0.006301 \mathbf{Z}\mathbf{Z}' + 0.080499 \mathbf{I}_{42}.$$

Accordingly, the diagonal elements of $\hat{\mathbf{\Omega}}$ are 0.086800, the covariance between observations belonging to the same firm is 0.006301, and the covariance between observations from different firms is zero. In general, the estimated covariance matrix can be represented as:

$$\hat{\mathbf{\Omega}} = \begin{bmatrix} 0.080499 & 0.006301 & \cdots & 0 \\ 0.006301 & 0.080499 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0.080499 \end{bmatrix}_{42 \times 42}$$

where the off-diagonal value 0.006301 occurs only for observations belonging to the same firm. The estimated covariance matrix was subsequently used as the weighting matrix in the feasible generalized least squares estimator using Equation (18). Conditional on the estimated variance–covariance matrix, the covariance matrix of the coefficient estimator is given by:

$$\widehat{Var}(\hat{\beta}_{FGLS}) = (\mathbf{X}'\hat{\mathbf{\Omega}}\mathbf{X})^{-1}$$

The statistical significance of each coefficient was evaluated using

$$t_j = \frac{\hat{\beta}_j}{SE(\hat{\beta}_j)}$$

with $42 - 3 = 39$ degrees of freedom. The complete estimation results are presented in [Table 4](#).

Table 4. Parameter Estimates of the Unbalanced One-Way Error-Component Panel Regression Model

Parameters	Estimate	Standard Error	T-statistics	P – Value
β_0	0.0657	0.0552	1.1908	0.2409
β_1	-0.0229	0.0134	-1.7178	0.0938
β_2	0.2089	0.3191	0.6546	0.5165

Source: Data processed

Table 4 presents the estimated regression coefficients, showing a negative coefficient for the debt-to-equity ratio (DER) and a positive coefficient for the net profit margin (NPM). Based on these estimates, the fitted regression model is expressed in [Equation \(22\)](#).

$$\text{Return}_{it} = 0.0657 - 0.0229\text{DER}_{it} + 0.2089\text{NPM}_{it}$$

The intercept estimate of 0.0657 represents the expected stock return when DER and NPM are equal to zero. However, the intercept is not statistically significant at the 5% level $p = 0.2409$. The DER coefficient of -0.0229 indicates that, holding NPM constant, a one-unit increase in DER is associated with an average decrease of 0.0229 units in stock return. This negative association is not statistically significant at the 5% level ($p = 0.0938$), although it provides limited evidence of an association at the 10% significance level. In contrast, the NPM coefficient of 0.2089 indicates that, holding DER constant, a one-unit increase in NPM is associated with an average increase of 0.2089 units in stock return. Nevertheless, this association is not statistically significant ($p=0.5165$). Its 95% confidence interval also includes zero, indicating substantial uncertainty regarding the direction and magnitude of the association.

Overall, the estimated coefficient signs indicate an inverse association between leverage and stock returns and a positive association between profitability and stock returns. However, neither association is statistically significant at the 5% level. Therefore, the results do not provide sufficient statistical evidence that DER or NPM explains variations in stock returns within the observed sample. These findings should be interpreted as conditional associations under the assumed unbalanced one-way random-effects specification rather than as causal effects.

The directions of the estimated coefficients may be compared with previous empirical studies reporting a negative relationship between DER and stock-market performance [\[24\]](#) and a positive relationship between profitability and stock-market performance [\[26\]](#). However, differences in statistical significance, sampling periods, industrial sectors, variable definitions, and estimation methods should be considered when comparing the present results with those studies. Consequently, the evidence from previous research does not override the statistical uncertainty found in the present analysis.

4. CONCLUSION

This study applies an unbalanced one-way random-effects panel model to 42 firm-year observations from 10 consumer-goods manufacturing firms listed on the Indonesia Stock Exchange during 2017–2021. The variance components estimated using Minimum Variance Quadratic Unbiased Estimation were used to construct the covariance matrix

of the composite disturbance, after which the regression coefficients were estimated using feasible generalized least squares. Following the correction of the firm-incidence matrix, the estimated firm-specific and idiosyncratic variance components were 0.006301 and 0.080499, respectively. Both estimates are nonnegative and produce an admissible covariance matrix. The estimated intraclass correlation of 0.0726 indicates that approximately 7.26% of the disturbance variation is attributable to persistent differences among firms. The fitted regression model is

$$Y_{it} = 0.0657 - 0.0229DER_{it} + 0.2089 NPM_{it}$$

Conditional on the assumed random-effects specification, DER is negatively associated with stock returns, whereas NPM is positively associated with stock returns. However, neither coefficient is statistically significant at the 5% level. DER has a p – value of 0.0938, while NPM has a p – value of 0.5165. Thus, the results do not provide sufficient statistical evidence that DER or NPM is associated with stock returns within the observed sample. These coefficient estimates represent conditional associations and should not be interpreted as causal effects. The study provides an empirical implementation of MIVQUE-based feasible generalized least squares for an unbalanced one-way error-component panel model. Nevertheless, it does not establish that this estimator is superior to alternative approaches. The findings are limited by the relatively small sample, the short observation period, the unbalanced firm-year structure, and the absence of comparisons with alternative variance-component estimators. Future studies should employ larger datasets, document the firm-year missingness pattern and its underlying mechanism, assess the sensitivity of the results to alternative model specifications, and compare MIVQUE-based FGLS with maximum likelihood, restricted maximum likelihood, and other feasible generalized least squares procedures using common performance criteria.

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Author Contributions Statement

The authors contributed to this work according to Contributor Roles Taxonomy (CRediT):

- Refa Joyce Semida: Conceptualization, methodology, data curation, formal analysis, investigation, writing – original draft, visualization.
- Raupong Raupong: Conceptualization, methodology, supervision, validation, writing – review and editing, project administration, and correspondence.
- Siswanto: Resources, formal analysis, validation, writing, reviewing, and editing.
- A. Muthiah Nur Angriany: Data curation, visualization, writing, review, and editing.

Conflict Of Interest Statement

The authors declare that they have no known financial or personal relationships that could have influenced the work reported in this study.

Informed Consent

This study is based exclusively on secondary financial data obtained from publicly accessible company reports listed on the Indonesia Stock Exchange (IDX). No human participants, patients, or identifiable personal data were involved; therefore, informed consent was not required.

Ethical Approval

This study utilized secondary financial data sourced from publicly available databases provided by the Indonesian Stock Exchange (IDX). As the research did not involve human participants or experimental interventions, ethical approval from an institutional review board or ethics committee was not required in accordance with applicable institutional and national guidelines.

Data Availability

The data supporting the findings of this study are derived from publicly available sources, namely, the financial reports of manufacturing companies listed on the Indonesia Stock Exchange (IDX) for the period 2017–2021. This data can be accessed through the official IDX website (<https://www.idx.co.id>). The analytical scripts and additional materials used in this study are available from the corresponding author upon reasonable request.

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