

Modeling of the Electronic Cigarettes with Awareness Effect

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Abstract

One of the prevalent behaviors that are frequently observed within societal structures is the habit of smoking. Concurrently with traditional cigarette consumption, there has been a documented rise in the utilization of electronic cigarettes, commonly known as e-cigarettes or vapes. Electronic cigarettes function by vaporizing an e-liquid, typically composed of nicotine, propylene glycol, glycerin, and flavorings, into an inhalable aerosol, namely, potential new smokers (P_1), potential regular smokers (P_2), active conventional smokers (S), electronic cigarette users (E), and individuals who have quit smoking (Q). The analysis entails the calculation of the smoking-free and smoking-endemic equilibrium points via the Cardano method. Additionally, an analysis of the local stability of each equilibrium point is conducted using the eigenvalues of the Jacobian matrix via the Routh-Hurwitz criterion. The results indicate that when the basic reproduction number $R_0=0.1961<1$, the system converges to a stable, smoking-free equilibrium, suggesting that smoking behavior will eventually disappear from the population. Conversely, when $R_0=1.98>1$, the system tends towards an asymptotically stable smoking-endemic equilibrium. Sensitivity analysis indicates that an increase in the transition rate from conventional smokers to electronic cigarette users (α) has the most significant influence in reducing the value of R_0 . The numerical simulations are provided to confirm the results analysis.

Keywords: Awareness effect, cardano method, cigarettes model, sensitivity analysis, stability.



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1. INTRODUCTION

Smoking is a prevalent practice observed across all demographics within society. The detrimental impact of smoking on public health worldwide is widely recognized. Consequently, both active and passive smoking are capable of inducing illness or disability and can adversely affect virtually every organ within the human body. Moreover, nicotine addiction associated with smoking presents considerable challenges for individuals attempting to cease the habit. Broadly speaking, cigarettes are divided into two primary categories: conventional and electronic. The use of conventional cigarettes, cigars, or pipes entails the combustion of tobacco, resulting in the emission of toxic smoke that contains harmful chemicals such as tar and carbon monoxide [1]. In contrast, e-cigarettes, or vapes, function by heating a solution (e-liquid) composed of nicotine, propylene glycol, glycerin, and flavorings. This process bypasses combustion, unlike traditional cigarettes, and generates an aerosol or vapor for inhalation [2]. A significant global health concern is the persistent prevalence of traditional smoking, with more than 1.3 billion adults worldwide engaging in this habit in 2022, contributing to roughly 8 million deaths annually due to diseases linked to tobacco [3]. As a result, the government incentivizes involvement in programs aimed at quitting smoking by designating areas as smoke-free. However, e-cigarettes and vapes have seen a swift and substantial rise in popularity as alternatives since their emergence in the early 2000s, especially among smokers aiming to moderate or eliminate their smoking. E-cigarette use has significantly increased globally, rising from under 1% in 2010 to over 10% in some developed nations by 2022. Many individuals are using e-cigarettes as a way to quit traditional cigarettes [4].

The significance of this subject is increasing in conjunction with public health initiatives focused on decreasing the prevalence of smoking and mitigating its adverse health consequences. The establishment of apt regulations for e-cigarettes presents a significant hurdle for governmental bodies across numerous countries, such as Indonesia. Despite the implementation of e-cigarette regulations in 2018 via a Minister of Health Regulation, their availability and consumption are notably restricted, notwithstanding a growing prevalence of 5–7% among urban youth [5]. The significant prevalence of smoking in Indonesia imposes a considerable economic strain on the national healthcare system, with annual costs associated with tobacco-related illnesses amounting to as much as Rp 17.9 trillion [6]. On the one hand, it is imperative to safeguard the populace from prospective health hazards. Consequently, it is imperative to perform a stability and sensitivity assessment regarding the influence of e-cigarettes concerning awareness levels. This will furnish policymakers with a more profound, evidence-based comprehension. Conversely, individual consciousness is essential for modifying human behavior concerning detrimental personal habits. Specifically, the significance of the awareness parameter in modifying individual behavior has been incorporated into a mathematical model, for example: a model for electoral list [7], Alcohol model [8], Tuberculosis model [9], Malaria model [11], HIV/AIDS model [12], Mpox model [13], and others [14, 15, 16].

2. METHOD

2.1 Type Of Research

The methodology adopted was a comprehensive literature review, entailing the systematic collection, critical examination, and thorough analysis of diverse scholarly sources, including scientific journals, research papers, and pertinent publications,

focusing on the stability, sensitivity, and awareness aspects concerning the impact of e-cigarettes.

2.2 Research Tools And Materials

For the purpose of preparing the report, the resources utilized consisted of a laptop for accessing digital sources and MATLAB for data processing. The resources employed comprised a compilation of pertinent academic writings, including a research journal authored by Jung et al. [10].

2.3 Research Procedure

The study utilized stability and sensitivity analysis to gauge parameter stability and identify the most influential parameters affecting e-cigarette awareness. The process for conducting the research is as follows.

1. Collect data
2. Develop a model of the impact of e-cigarettes incorporating awareness factors
3. Determine the equilibrium point and the basic reproduction number
4. Conduct stability and sensitivity analyses on the model of the impact of e-cigarettes incorporating awareness factors
5. Draw conclusions.

3. RESULTS AND DISCUSSION

3.1 A Model of the Impact of E-Cigarettes on Awareness

This study conceptualizes the awareness factor as an individual's degree of insight into the health hazards pertaining to smoking, which in turn influences modifications in smoking conduct. The influence of this factor is captured by a parameters model, particularly those related to e-cigarette adoption and smoking cessation. Regarding the impact of e-cigarettes using this awareness factor, it is assumed that the total population (N) will be divided among individuals and grouped into five variable populations or compartments. These five sub-populations consist of the population of new potential smokers (P_1), the population of regular potential smokers (P_2), the population of active conventional smokers (S), the population of e-cigarette users (E), and the population of smokers who have quit smoking (Q). The following is a compartment diagram for constructing a model of the impact of e-cigarettes with an awareness factor (see **Figure 1**).

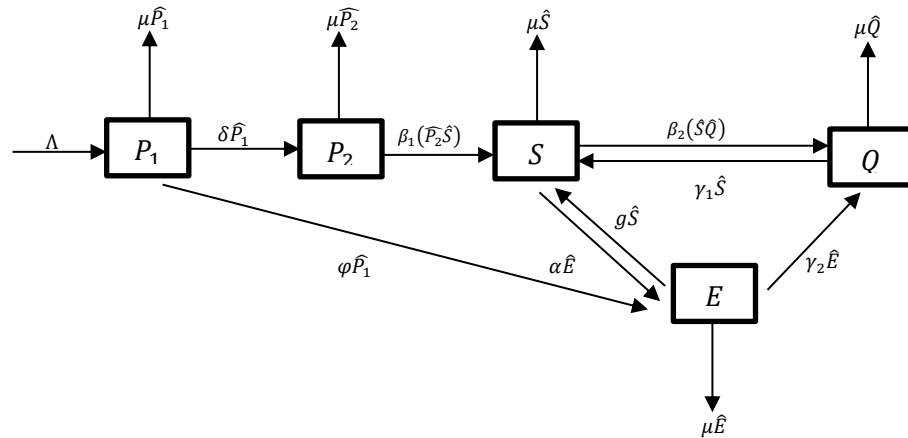


Figure 1. The scheme of smoking

A system of nonlinear differential equations, as presented below, can mathematically model the conventional and e-cigarette smoking demographics, referencing the preceding graph.

$$\frac{d\widehat{P}_1}{dt} = \Lambda - \delta\widehat{P}_1 - \varphi\widehat{P}_1 - \mu\widehat{P}_1 \quad (1)$$

$$\frac{d\widehat{P}_2}{dt} = \delta\widehat{P}_1 - \beta_1(\widehat{P}_2\widehat{S}) - \mu\widehat{P}_2 \quad (2)$$

$$\frac{d\widehat{S}}{dt} = \beta_1(\widehat{P}_2\widehat{S}) + \beta_2(\widehat{S}\widehat{Q}) + \alpha\widehat{E} - \gamma_1\widehat{S} - g\widehat{S} - \mu\widehat{S} \quad (3)$$

$$\frac{d\widehat{E}}{dt} = g\widehat{S} + \varphi\widehat{P}_1 - \alpha\widehat{E} - \gamma_2\widehat{E} - \mu\widehat{E} \quad (4)$$

$$\frac{d\widehat{Q}}{dt} = \gamma_1\widehat{S} + \gamma_2\widehat{E} - \beta_2(\widehat{S}\widehat{Q}) - \mu\widehat{Q} \quad (5)$$

with the following parameter descriptions (see **Table 1**).

Table 1. Description of the Parameters and the Parameter Values

Parameter	Description
μ	Natural mortality rate
Λ	Recruitment pace
δ	The rate at which new potential smokers become regular potential smokers
φ	The rate at which new users transition from potential smokers to e-cigarette users
β_1	The rate at which potential smokers transition into conventional smokers
β_2	The rate at which former smokers relapse and become active conventional smokers
γ_1	The quit rate among conventional smokers
γ_2	The quit rate among e-cigarette users
g	The rate at which e-cigarette users revert to being active conventional smokers
α	The rate at which conventional smokers switch to e-cigarettes

3.2 Equilibrium Point

The equilibrium point according to the system in **(1)-(5)** are determined by the condition $\frac{dP_1}{dt} = \frac{dP_2}{dt} = \frac{dS}{dt} = \frac{dE}{dt} = \frac{dQ}{dt} = 0$, resulting in two equilibrium points: the smoke-free equilibrium point and the endemic equilibrium point.

3.2.1 Smoke-Free Equilibrium Point

The smoke-free equilibrium state signifies a population where no individuals engage in smoking. The condition of a smoke-free state, wherein there are no active smokers, no e-cigarette consumption, and no former smokers who have never smoked, permits the initialization of $S(t)$ to 0. The smoke-free equilibrium point is denoted by E_0 , where the smoke-free equilibrium point.

$$E_0(P_1, P_2, S, E, Q) = \left(\frac{\Lambda}{\delta + \varphi + \mu}, \frac{\delta\Lambda}{\delta\mu + \varphi\mu + \mu^2}, 0, 0, 0 \right). \quad (6)$$

3.2.2 Smokers' Equilibrium Point

At time t , the endemic equilibrium point represents the prevailing rate of smoking in a population. This study construes the smoking behavior with awareness functioning

as a relevant determinant. If $S \neq 0$, there will be active users of conventional cigarettes, which may lead to an epidemic. If $E \neq 0$, there will be users of electronic cigarettes, who may influence others and lead to the smokers equilibrium point is exists. Thus, the smoker equilibrium point is

$$E^* = (P_1^*, P_2^*, S^*, E^*, Q^*), \quad (7)$$

with:

$$\begin{aligned} P_1^* &= \frac{\Lambda}{\delta + \varphi + \mu}, \\ P_2^* &= \frac{\Lambda\delta}{(\delta + \varphi + \mu)(\beta_1 S^* + \mu)}, \\ E^* &= \frac{(\delta + \varphi + \mu)gS^* - \Lambda\varphi}{(\delta + \varphi + \mu)(\alpha + \gamma_2 + \mu)}, \\ Q^* &= \frac{\gamma_1 S^* + \gamma_2 \left(\frac{(\delta + \varphi + \mu)gS^* - \Lambda\varphi}{(\delta + \varphi + \mu)(\alpha + \gamma_2 + \mu)} \right)}{(\beta_2 S^* + \mu)}, \end{aligned}$$

and next S^* is possible positive roots of cubic equation.

$$(S^*)^3 + 3\sigma_2(S^*)^2 + 3\sigma_1(S^*) + \sigma_0 = 0. \quad (8)$$

where:

$$\begin{aligned} \sigma_0 &= \frac{z_0}{z_3}, \\ \sigma_1 &= \frac{z_1}{3z_3}, \\ \sigma_2 &= \frac{z_2}{3z_3}, \\ z_0 &= -\alpha\mu^2\Lambda\varphi, \\ z_1 &= \beta_1\beta_2\Lambda\delta l\mu + \alpha\mu^2kg - \beta_2\mu\gamma_2\Lambda\varphi - \alpha\mu\beta_2\Lambda\varphi - \alpha\beta_1\mu\Lambda\varphi - mkl\mu^2, \\ z_2 &= \beta_1\beta_2\Lambda\delta l - \beta_1\beta_2\gamma_2\Lambda\varphi + \beta_2\mu\gamma_1kl + \beta_2\mu\gamma_2kg + \alpha\mu\beta_2kg + \alpha\beta_1\mu kg - \alpha\beta_1\beta_2\Lambda\varphi \\ &\quad - mkl\beta_1\mu - mkl\mu\beta_2, \\ z_3 &= \beta_1\beta_2\gamma_1kl + \beta_1\beta_2\gamma_2kg + \alpha\beta_1\beta_2kg - mkl\beta_1\beta_2. \end{aligned}$$

Equation (8) can be written as,

$$z^3 + 3\phi_1z + \phi_0 = 0. \quad (9)$$

with:

$$\begin{aligned} \phi_1 &= \sigma_1 - \sigma_2^2, \\ \phi_0 &= \sigma_0 - 3\sigma_1\sigma_2 + 2\sigma_2^3. \end{aligned}$$

All possible positive roots of equation (9) can be evaluated by Cardan's method as stated in **Lemma 1**. Next, the existence of the positive root in equation (9) can be investigated using **Lemma 1** as follows [17].

Lemma 1. *The existence of positive root of equation (9) can be described as follows.*

- 1) If $\phi_0 < 0$, then (9) has a positive root,
- 2) If $\sigma_0 > 0$ and $\sigma_1 < 0$, then :
 - a) If $\phi_0^2 + 4\phi_1^3 = 0$, then (9) has a positive root of multiplicity two,
 - b) If $\phi_0^2 + 4\phi_1^3 < 0$, then (9) has a two positive roots.
- 3) If $\phi_0 = 0$ and $\phi_1 < 0$, then (9) has a single positive root.

3.3 Basic Reproduction Number

In this model, the classes that influence the impact of e-cigarettes on awareness are conventional smokers (S), e-cigarette users (E), and smokers who have successfully quit smoking (Q). Let $x = (S, E, Q)^T$, so that system (1)-(5) can be written as follows.

$$\frac{dx_i}{dt} = F_i - V_i \quad \text{with } i = 1, 2, 3$$

F_i denotes the rate at which conventional smokers enter population i , namely:

$$F_i = \begin{bmatrix} \beta_1(P_2S) + \beta_2(SQ) \\ 0 \\ 0 \end{bmatrix}, \quad (10)$$

and V_i denotes the rate at which individuals leave and enter population i from other populations, namely:

$$V_i = \begin{bmatrix} -\alpha E + (\gamma_1 + gS + \mu S)S \\ -gS - \varphi P_1 + (-\alpha - \gamma_2 - \mu)E \\ -\gamma_1 S - \gamma_2 E + \beta_2(SQ) + \mu Q \end{bmatrix}. \quad (11)$$

Next, we will find the linearization of matrices F and V using the Jacobian matrix as follows:

$$F = \begin{bmatrix} \beta_1(P_2) + \beta_2(Q) & 0 & \beta_2(S) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$V = \begin{bmatrix} \gamma_1 + g + \mu & -\alpha & 0 \\ -g & -\alpha - \gamma_2 - \mu & 0 \\ -\gamma_1 + \beta_2(Q) & -\gamma_2 & \beta_2(S) + \mu \end{bmatrix} \quad (13)$$

Next, the substitution of the smoke-free equilibrium point.

$$F = \begin{bmatrix} \frac{\beta_1 \Lambda \delta}{\delta \mu + \varphi \mu + \mu^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (14)$$

$$V = \begin{bmatrix} \gamma_1 + g + \mu & -\alpha & 0 \\ -g & -\alpha - \gamma_2 - \mu & 0 \\ -\gamma_1 & -\gamma_2 & \mu \end{bmatrix}. \quad (15)$$

From matrix in (15), we obtain,

$$V^{-1} = \begin{bmatrix} \frac{(-\alpha - \gamma_2 - \mu)}{((\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - \alpha g)} & \frac{\alpha}{((\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - \alpha g)} & 0 \\ \frac{g}{((\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - \alpha g)} & \frac{(\gamma_1 + g + \mu)}{((\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - \alpha g)} & 0 \\ \frac{g\gamma_2 + \gamma_1(-\alpha - \gamma_2 - \mu)}{\mu((\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - \alpha g)} & \frac{\gamma_2(\gamma_1 + g + \mu) + (\alpha\gamma_1)}{\mu((\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - \alpha g)} & \frac{1}{\mu} \end{bmatrix}. \quad (16)$$

Futhermore, the basic reproduction number (R_0) is the spectral radius of the next-generation matrix FV^{-1} [7,18], with,

$$R_0 = \rho(FV^{-1}) = \frac{(\beta_1 \Lambda \delta)(-\alpha - \gamma_2 - \mu)}{(\delta \mu + \varphi \mu + \mu^2)(\gamma_1 + g + \mu)(-\alpha - \gamma_2 - \mu) - (\delta \mu + \varphi \mu + \mu^2)\alpha g}. \quad (17)$$

3.4 Local Stability of the Smoke-Free Equilibrium Point

The stability at the smoke-free equilibrium point (E_0) will be determined; by linearizing equation (1)-(5), we obtain

$$J(E_0) = \begin{bmatrix} a_1 & 0 & 0 & 0 & 0 \\ \delta & -\mu & a_2 & 0 & 0 \\ 0 & 0 & a_3 & \alpha & 0 \\ \varphi & 0 & g & a_4 & 0 \\ 0 & 0 & \gamma_1 & \gamma_2 & -\mu \end{bmatrix}, \quad (18)$$

where:

$$a_1 = -(\delta + \varphi + \mu),$$

$$a_2 = \left(\frac{\beta_1 \delta \Lambda}{\delta \mu + \varphi \mu + \mu^2} \right),$$

$$a_3 = \left(\frac{\beta_1 \delta \Lambda}{\delta \mu + \varphi \mu + \mu^2} \right) - (\gamma_1 + g + \mu),$$

$$a_4 = -(\alpha + \gamma_2 + \mu).$$

From the matrix in equation (18), we get a polynomial,

$$\lambda^5 + b_1 \lambda^4 + b_2 \lambda^3 + b_3 \lambda^2 + b_4 \lambda + b_5 = 0 \quad (19)$$

where:

$$b_1 = 2\mu - (a_1 + a_3 + a_4),$$

$$b_2 = \mu^2 + a_1 a_3 + a_1 a_4 + a_3 a_4 - (\alpha g + 2\mu(a_1 + a_3 + a_4)),$$

$$b_3 = 2\mu(a_1 a_3 + a_1 a_4 + a_3 a_4) + \alpha g a_1 - \mu^2(a_1 + a_3 + a_4) - 2\alpha g \mu,$$

$$b_4 = 2\alpha g \mu a_1 + \mu^2 a_1 a_3 + \mu^2 a_1 a_4 + \mu^2 a_3 a_4 - (2\mu a_1 a_3 a_4 + \alpha g \mu^2),$$

$$b_5 = \mu^2 a_1 (\alpha g - a_3 a_4).$$

Next, using the Routh-Hurwitz criterion to determine the necessary and sufficient condition to fulfill the $\text{Re}(\lambda) < 0$ for the characteristic polynomial in (19) [18,20].

3.5 Local Stability of the Smoker's Equilibrium Point

Derive each of the functions from (1)-(5), then substitute the endemic equilibrium point E^* into the Jacobian matrix (J) as follows.

$$J(E^*) = \begin{bmatrix} c_1 & 0 & 0 & 0 & 0 \\ \delta & c_2 & c_3 & 0 & 0 \\ 0 & c_4 & c_5 & \alpha & c_6 \\ \varphi & 0 & g & c_7 & 0 \\ 0 & 0 & c_8 & \gamma_2 & c_9 \end{bmatrix}, \quad (20)$$

with:

$$c_1 = -(\delta + \varphi + \mu),$$

$$c_2 = -\beta_1(S^*) - \mu,$$

$$c_3 = -\beta_1(P_2^*),$$

$$c_4 = c_6 = \beta_1(S^*),$$

$$c_5 = \beta_1(P_2^*) + \beta_2(Q^*) - (\gamma_1 + g + \mu),$$

$$c_7 = -(\alpha + \gamma_2 + \mu),$$

$$c_8 = \gamma_1 - \beta_2(Q^*),$$

$$c_9 = -\beta_2(S^*) - \mu.$$

From the matrix in Equation (20), we get a polynomial

$$\lambda^5 + d_1 \lambda^4 + d_2 \lambda^3 + d_3 \lambda^2 + d_4 \lambda + d_5 = 0 \quad (21)$$

where:

$$d_1 = -(c_1 + c_2 + c_5 + c_7 + c_9),$$

$$d_2 = c_1 c_7 + c_1 c_9 + c_2 c_7 + c_2 c_9 + c_5 c_7 + c_5 c_9 + c_7 c_9 - (\alpha g + c_3 c_4 + c_6 c_8),$$

$$d_3 = \alpha g(c_1 + c_2 + c_9) + c_1 c_3 c_4 + c_1 c_6 c_8 + c_2 c_6 c_8 + c_3 c_4 c_7 + c_3 c_4 c_9 + c_6 c_7 c_8$$

$$- (g c_6 \gamma_2 + c_1 c_2 c_5 + c_1 c_2 c_7 + c_1 c_2 c_9 + c_1 c_5 c_7 + c_1 c_5 c_9 + c_1 c_7 c_9$$

$$+ c_2 c_5 c_7 + c_2 c_5 c_9 + c_2 c_7 c_9 + c_5 c_7 c_9),$$

$$d_4 = g c_1 c_6 \gamma_2 + g c_2 c_6 \gamma_2 + c_1 c_2 c_5 c_7 + c_1 c_2 c_5 c_9 + c_1 c_2 c_7 c_9 + c_1 c_5 c_7 c_9 + c_2 c_5 c_7 c_9$$

$$- (\alpha g c_1 c_2 + \alpha g c_1 c_9 + \alpha g c_2 c_9 + c_1 c_2 c_6 c_8 + c_1 c_3 c_4 c_7 + c_1 c_3 c_4 c_9)$$

$$- (c_1 c_6 c_7 c_8 + c_2 c_6 c_7 c_8 + c_3 c_4 c_7 c_9),$$

$$d_5 = \alpha g c_1 c_2 c_9 + c_1 c_2 c_6 c_7 c_8 + c_1 c_3 c_4 c_7 c_9 - (g c_1 c_2 c_6 \gamma_2 + c_1 c_2 c_5 c_7 c_9).$$

We use the same method as section 3.4. By using the Routh-Hurwitz criterion to determine the necessary and sufficient condition to fulfill the $\text{Re}(\lambda) < 0$ for the characteristic polynomial in (21) [18,20].

3.6 Smoke-Free Numerical Simulation ($R_0 < 1$)

Based on the assumptions used in the journal article by Jung et al. in [10], a smoke-free numerical simulation was conducted using the initial values of P_1, P_2, S, E , and Q in Table 3, as well as the parameter values listed in Table 2.

Table 2. Parameter Values for Smoke-Free Numerical Simulation ($R_0 < 1$)

Parameter	Value	Source
μ	1/79.8	[21]
Λ	0.01253	Assumed
δ	0.03	Assumed
φ	0.07	Assumed
β_1	0.1-1.6	Assumed
β_2	0.0101	[10]
γ_1	0.0772	[10]
γ_2	0.1008	[1]
α	0.0822	[1]
g	0.1245	[1]

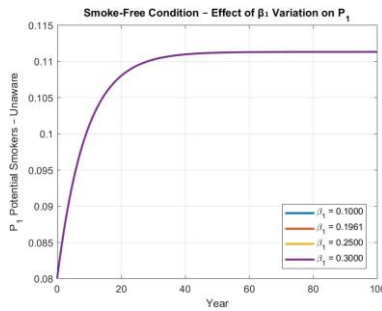
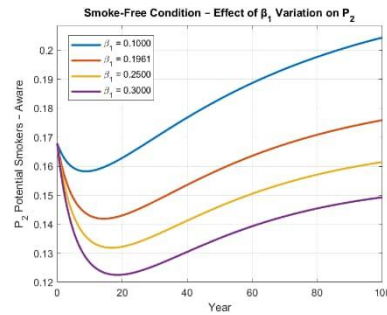
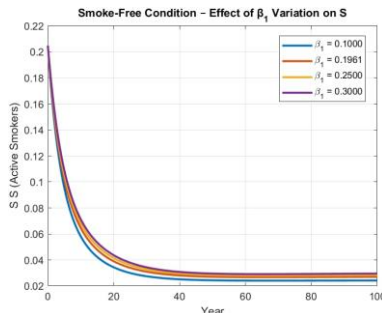
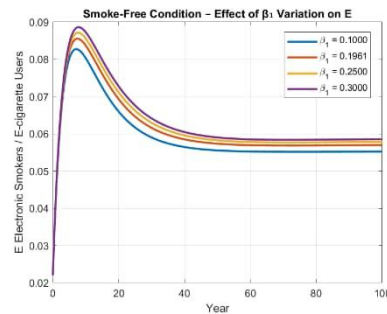
Table 3. Initial Value of the Smoke-Free Compartment ($R_0 < 1$)

Variable	Value	Source
P_1	0.080	Assumed
P_2	0.168	Assumed
S	0.205	[1]
E	0.022	[1]
Q	0.525	[1]

The Basic Reproduction Number R_0 was obtained,

$$R_0 = 0.1961 < 1 \quad (22)$$

To simulate a smoke-free population model using MATLAB software based on the equation in (1)-(5), the following graphs were obtained: a graph of the potential population of new smokers, a graph of the potential population of regular smokers, a graph of the population of conventional smokers, a graph of the population of e-cigarette users, and a graph of the population of smokers who have successfully quit smoking.

**Figure 2.** Potential New Smokers Population**Figure 3.** Potential Regular Smokers Population**Figure 4.** Active Conventional Smokers Population**Figure 5.** Electronic Smokers Population

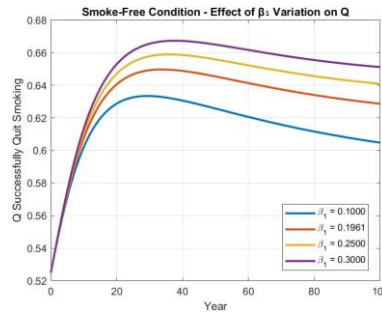


Figure 6. Quit Smoking Population

As depicted in **Figure 2**, the population of prospective new smokers experiences an initial rise before swiftly attaining equilibrium. Changes in β_1 do not appear to substantially alter the curve's pattern, suggesting that the population of potential new smokers is contained and does not experience significant long-term expansion. In **Figure 3**, the population initially decreases, then gradually increases until it reaches a stable condition. Higher values of β_1 result in a lower final population level, indicating that the transition of individuals into regular smokers is not dominant under smoke-free conditions. **Figure 4** exhibits a precipitous initial decrease, subsequently transitioning to a slow stabilization at an exceedingly minimal level. While minor fluctuations may influence the ultimate outcome, there is a pronounced decline in the number of individuals who smoke. This trend suggests a substantial decrease in traditional smoking habits. **Figure 5** illustrated the prevalence of electronic smokers initially rises, plateaus, and subsequently experiences a gradual decrease followed by stabilization. Increased values of β_1 correlate with marginally elevated peak and steady-state measurements, indicating a transient transition towards electronic nicotine delivery systems prior to eventual stabilization at a regulated threshold. **Figure 6** illustrates an initial sharp ascent to a maximum point, followed by a marginal decline and subsequent stabilization at a considerably elevated level. The effectiveness of smoking cessation in smoke-free conditions is evidenced by the greater number of individuals who cease smoking as β_1 increases.

3.7 Numerical simulation of a smoker ($R_0 > 1$)

Based on the assumptions used in the journal article by Jung et al. in [10], a smoke-free numerical simulation was conducted using the initial values of P_1, P_2, S, E, Q , as well as the parameter values listed in **Table 2** and **Table 3**. The basic reproduction number R_0 was obtained

$$R_0 = 1.98 > 1 \quad (23)$$

A simulation of cigarette distribution under conditions of endemic smoking will be displayed in a graph using MATLAB software.

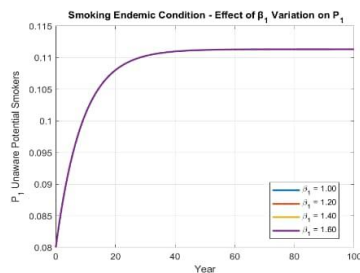


Figure 7. The potential new cigarette users

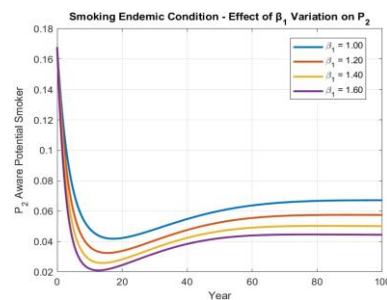


Figure 8. The potential cigarette-smoking population

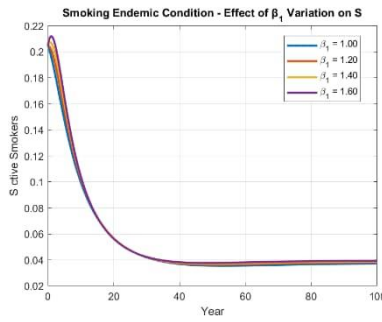


Figure 9. Population of active conventional smokers

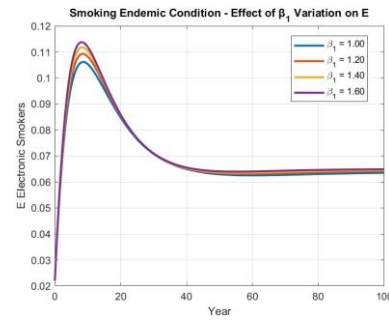


Figure 10. Population of electronic cigarette users

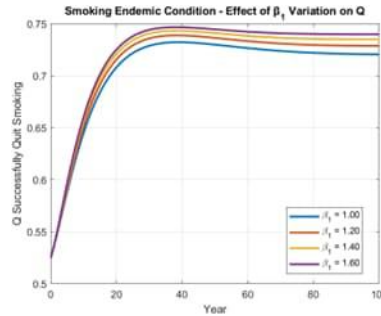


Figure 11. Population of smokers who have quit smoking

Figure 7 provided indicates a rapid initial increase in the potential new smoker population, which subsequently stabilizes promptly. Changes in β_1 do not substantially alter the curve's pattern, suggesting that the quantity of individuals who may initiate smoking remains regulated and does not exhibit considerable long-term growth. The population depicted in **Figure 8** undergoes an initial rapid decrease, subsequently experiencing a slow increase until achieving stability. A greater β_1 leads to a reduced final population, demonstrating that the progression to habitual smoking is less pronounced with an increased contact rate. A significant initial drop is depicted in **Figure 9**, succeeded by a gradual stabilization towards a constant equilibrium. Alterations in β_1 do not substantially differentiate the curves, indicating that the active conventional smoker population achieves a comparable low steady state irrespective of the contact rate. **Figure 10** shows an early ascent to a crest, which then experiences a measured decline until it reaches a constant value. An increase in β_1 yields a modest elevation in the peak, yet all curves approach a comparable equilibrium over time, illustrating the shift from conventional to electronic smoking before attaining a prolonged stable condition. The graphical in **Figure 11** representation indicates a gradual and unwavering upward trend from its starting point, progressing towards a steady state throughout the simulation's duration. Diminished values of β_1 correlate with a marginally elevated final equilibrium. Conversely, amplified values expedite convergence, albeit to a slightly reduced level, demonstrating that the contact rate impacts the temporal progression of successful smoking cessation.

3.8 Sensitivity Analysis

The sensitivity parameter with respect to the basic reproduction number R_0 is calculated using the approach described in **Definition 1** that is $\beta_1, \Lambda, \delta, \alpha, \gamma_1, \gamma_2, \varphi, \mu, g$ toward R_0 .

Definition 1

The normalization of the sensitivity index is defined as follows:

$$C_p^V = \frac{\partial V}{\partial p} \times \frac{p}{V} \quad (24)$$

where p is a parameter and V is the R_0 variable to be normalized or analyzed [10].

From the Basic Reproduction Number in (17), we get the diagram which illustrates the effect of each parameter.

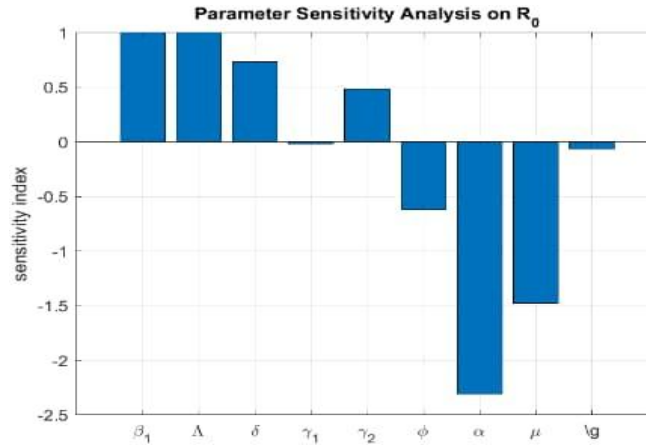


Figure 12. The parameter sensitivity plot

The results of the sensitivity analysis reveal that parameter α , defined as the rate of adoption of e-cigarettes by conventional smokers, is the most influential factor affecting the basic reproduction number (R_0), as indicated by the largest negative value of its sensitivity index. These findings suggest that heightened awareness of e-cigarette effects alters smoking behavior dynamics in the model, leading to a reduction in the R_0 value. It is important to note that this is not an endorsement of e-cigarette usage, but an indication of evolving smoking behaviors that affect the control of smoking prevalence in the population. This behaviour is despited in **Figure 12**.

4. CONCLUSION

An integrated model was developed to assess the influence of e-cigarettes, incorporating awareness factors, by adapting the e-cigarette sensitivity model proposed by Jung et al. in [10]. This model divides the population into five compartments: potential new users (P_1), potential regular users (P_2), active conventional smokers (S), e-cigarette users (E), and individuals who have quit smoking (Q). The model demonstrates two separate stable states: the smoke-free equilibrium, indicating a complete absence of smoking, and the smoking equilibrium, characterized by widespread smoking. The analysis results shows that the smoke-free equilibrium point is locally asymptotically stable if $R_0 < 1$, and the smoking equilibrium point is stable if $R_0 > 1$. Additionally, the results of the sensitivity analysis demonstrate that parameter α , which quantifies the rate at which traditional smokers adopt e-cigarettes, is the most impactful factor on the basic reproduction number R_0 . This is indicated by the largest negative sensitivity index value. This particular value signifies the most substantial detrimental impact on the outcome being measured, given a unit change in the input parameter α . It has been ascertained that a greater awareness regarding the ramifications of e-cigarettes correlates with shifts in smoking behavior trajectories as modeled, resulting in a significant decrement to the value of R_0 .

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