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Application of Max-Plus Algebra for Time Optimization of Wooden Furniture Production System "Berkah Usaha" Jepara Regency

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ABSTRACT

This research aims to provide a periodic schedule on the Berkah Usaha wood furniture production system in Jepara using Max-Plus Algebra. The data required in the research is data on the flow and processing time of each project work unit. In this research, the main focuses on the Berkah Usaha wooden furniture dining table production system whose processing stages consist of raw material preparation, measurement, wood cutting, wood smoothing, measurement of puruses and holes, table floor manufacturing, puruses and holes manufacturing, table floor smoothing, assembling, sanding, and finishing. The techniques of data collection used in this research are interviews and field observations. Then, a directed graph and model of the Berkah Usaha wood furniture production system were compiled based on the flow data, processing time for each project work unit, and the rules for applying Max-Plus Algebra to the scheduling system. Analysis of the Max-Plus Algebra model of the Berkah Usaha wooden furniture production system using the Power Algorithm with Scilab software assisted. The results of the analysis obtained the production time period of the Berkah Usaha wooden furniture for 253 minutes and the optimal time to start production on each processing unit are 0, 11, 46, 92, 115, 117, 127, 137, 155, 195, 223 time units (in minutes).

1. INTRODUCTION

Wooden furniture production activities in Jepara are widely known in Indonesia. Its role as a region that produces furniture is very large because this area is able to produce furniture on a very large scale. The activity of making furniture has become a daily activity and is an economic driver for the residents of this area. This can be seen from the large number of furniture industries scattered from along the gate to Jepara Regency to the middle of the city. This industry is engaged in the manufacture of various furniture, furniture or furniture made of wood, such as cupboards, beds, bookshelves, tables, chairs, and so on.

Wooden furniture in Jepara has advantages over other regions. This city, which is synonymous with craftsmanship, already has a long history in furniture with superior quality. The activity or activity carried out by the wooden furniture industry in Jepara is assembling wood into various furniture for homes, offices, schools, and so on. This activity in the process of work is carried out following a series of activities from one activity to another to the finishing and packing process, through several stages or sequences of work where each activity is known for sure and has a different grace period. Because through several stages of work, the process of making a furniture furniture takes a long time to be relevant. This of course indirectly harms the furniture party, both in terms of time and energy.

For optimization, it is necessary to model the wooden furniture production system for scheduling in order to get a periodic wooden furniture production schedule at each stage of the work. Modeling is an effort to represent a phenomenon in the form of a mathematical formula so that it is easy to understand and calculate, while scheduling or making a schedule is an important procedure in the production system to control, regulate, and optimize production.

Max-Plus Algebra can be used to model and analyze networks, such as production systems, project scheduling, queue networks, and so on. Modeling and analyzing a network with this approach can provide a periodic schedule at each stage of work so that it can be time-efficient.

Based on the description above, the author examines an application of Algebra Max-Plus in modeling a wooden furniture production system in Jepara. From the results of processing using Algebra Max-Plus, it is hoped that later it will provide a periodic schedule at each stage of work so that it does not spend much time in the process of making a set of furniture in Jepara.

2. RESEARCH METHODS

This research consists of several stages, namely literature study, data collection, data analysis, and conclusion drawn.

2.1 Literature Studies

At this stage, a search for references related to research is carried out, namely about Max-Plus Algebra, Scilab, Matrices in Max-Plus Algebra, Graphs in Max-Plus Algebra, Eigen Values and Eigen-Vectors in Max-Plus Algebra and production systems in relation to Max-Plus Algebra.

2.2 Data Collection

At this stage, data was collected used in the research. The data needed in the research is data regarding the flow and time of work in each project work unit. Data collection is carried out by direct observation and interviews.

2.3 Data Analysis

The stages of data analysis in this study are as follows.

1. Compile a directional graph of the Berkah Usaha wooden furniture production system based on flow data and work time on each project work unit.
2. Compile a production system model based on the rules of applying Max-Plus Algebra to the scheduling system which has the form of the equation $x(k + 1) = A \otimes x(k)$. This equation is a reference in compiling the model and forming a matrix from the model equations obtained.
3. Searching for eigen values and eigen vectors from Max-Plus Algebra matrices with the help of Scilab and Max-Plus Algebra Toolbox software to obtain periodic furniture production process schedules.

3. RESULTS AND DISCUSSION

3.1 Production Process of Wooden Furniture Blessing Business

In this case study, the production of the Berkah Usaha Wooden Furniture dining table is described. The production process of the dining table Berkah Usaha consists of preparation of raw material, measurement, wood cutting, wood smoothing, measurement of puruses and holes, table floor manufacturing, puruses and holes manufacturing, table floor smoothing, assembling, sanding, and finishing. After knowing the production flow of the dining table, then a diagram of the production process of the dining table along with the time needed can be described as follows.

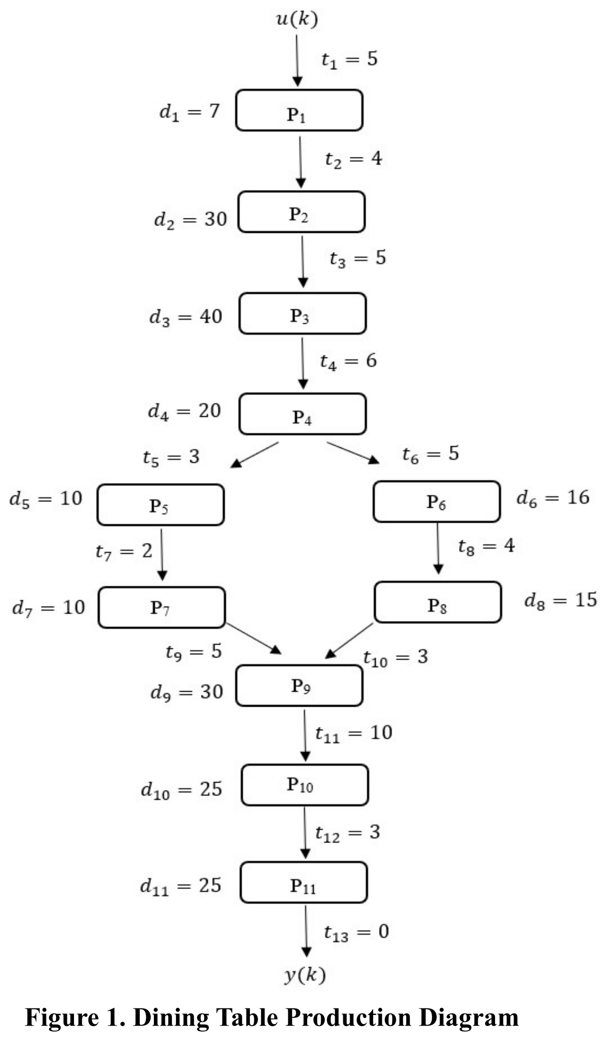

Figure 1. Dining Table Production Diagram

Information:
t_i : time of transfer to the next process, $i = 1,2,\dots,13$,
d_1 : time when raw material preparation,
d_2 : time at the time of measurement,
d_3 : time at the time of cutting,
d_4 : time when the wood is smoothed,
d_5 : time when measuring purus and hole,
d_6 : time when the table floor is made,
d_7 : time when the purus and hole are made,
d_8 : time when the table floor is scraped,
d_9 : time at assembly,
d_{10} : time during sanding,
d_{11} : time at *finishing,*
P_1 : raw material preparation process,
P_2 : measurement process,
P_3 : cutting process,
P_4 : wood grinding process,
P_5 : purus and hole measurement process,
P_6 : table floor manufacturing process,
P_7 : purus and hole making process,
P_8 : table floor scraping process,
P_9 : assembly process,
P_{10} : sanding process,
P_{11} : proses *finishing.*

3.2 Modeling of the Wooden Funiture Production System "Berkat Usaha"

The production system of Berkah Usaha Wooden Furniture consists of 11 processes or stages of production as shown in Figure 4.1. The process or stage of the production of wooden furniture "Berkah Usaha" is given the symbols P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11. The definition of the wooden furniture production process is as follows:

1. $u(k)$, the time when the raw material is fed into the system for processing to $(k + 1)$,
2. $x_i(k)$, the time when the ith processing unit starts actively working on the kth processing,
3. $y(k)$, the time when the finished wooden furniture product leaves the system for k-processing, for $i = 1,2,3,\dots,11$ and $k \in N_0$.

It then determines the time when P1 starts actively working for the $(k + 1)$ process. If the raw material is fed into the system for the $(k + 1)$ process, then the raw material is available as input P1 at the time u(k) + t_1. However, P1 can only start actively working to process raw materials when the previous process is completed. Since the processing time of P1 is 7 minutes, the product that has been processed in P1 leaves P1 at the $t = x_1(k) + 7$, so the latest or longest time is the time that P1 begins to actively work on the $(k + 1)$ processing. From the statement, it can be written as follows.

$$\begin{aligned} x_1(k+1) &= \max(u(k) + t_1, x_1(k) + d_1) \\ &= \max(u(k) + 5, x_1(k) + 7). \end{aligned}$$

The above equation means that when $k = 0$, then in process 1, namely $x_1(0)$, the raw materials have entered in process 1, so that process 1 starts working.

Using the same approach as in the P1 processing unit, then the P2, P3 and so on. From the equations obtained, it can be written as follows:

$$\begin{aligned} x_1(k+1) &= u(k) \otimes 5 \oplus x_1(k) \otimes 7 \\ x_2(k+1) &= u(k) \otimes 16 \oplus x_1(k) + 18 \oplus x_2(k) + 30 \end{aligned}$$

and so on until $x_{11}(k + 1)$ and obtained $y(k) = x11(k) \otimes 25$.

From the modeling of the wooden furniture production system using the Max-Plus Algebra above, it can be written in the form of a matrix equation as follows.

$$(k+1) = \begin{bmatrix} 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 18 & 30 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 53 & 65 & 40 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 99 & 111 & 86 & 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 122 & 134 & 109 & 43 & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 124 & 136 & 111 & 45 & \varepsilon & 16 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 134 & 146 & 121 & 55 & 22 & \varepsilon & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 144 & 156 & 131 & 65 & \varepsilon & 36 & \varepsilon & 15 & \varepsilon & \varepsilon & \varepsilon \\ 162 & 174 & 149 & 83 & 37 & 54 & 25 & 33 & 30 & \varepsilon & \varepsilon \\ 202 & 214 & 189 & 123 & 77 & 94 & 65 & 73 & 70 & 25 & \varepsilon \\ 230 & 242 & 217 & 151 & 105 & 122 & 93 & 101 & 98 & 53 & 25 \end{bmatrix} \otimes \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \\ x_8(k) \\ x_9(k) \\ x_{10}(k) \\ x_{11}(k) \end{bmatrix} \oplus \begin{bmatrix} 5 \\ 16 \\ 51 \\ 97 \\ 120 \\ 122 \\ 132 \\ 142 \\ 160 \\ 200 \\ 228 \end{bmatrix} \otimes \mathrm{u(k)}$$

and $y(k) = [\varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad \varepsilon \quad 235] \otimes \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \\ x_8(k) \\ x_9(k) \\ x_{10}(k) \\ x_{11}(k) \end{bmatrix}$

Furthermore, it is assumed that $u(k) = y(k)$ because the time at the time of production has been completed at the kth time indicates the time to restart production at the kth time so that there is no delay in production. So that

$$x(k+1) = A \otimes x(k) \oplus B \otimes C \otimes x(k) = \bar{A} \otimes x(k).$$

Then specify the value of $\bar{\mathrm{A}}$ using Scilab with the command *A_bar=maxplusoplus(A,maxpluspotimes(B,C))* is generated

$$\bar{A} = \begin{bmatrix} 7 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 30 \\ 18 & 30 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 41 \\ 53 & 65 & 40 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 76 \\ 99 & 111 & 86 & 20 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 122 \\ 122 & 134 & 109 & 43 & 10 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 145 \\ 124 & 136 & 111 & 45 & \varepsilon & 16 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 147 \\ 134 & 146 & 121 & 55 & 22 & \varepsilon & 10 & \varepsilon & \varepsilon & \varepsilon & 157 \\ 144 & 156 & 131 & 65 & \varepsilon & 36 & \varepsilon & 15 & \varepsilon & \varepsilon & 167 \\ 162 & 174 & 149 & 83 & 37 & 54 & 25 & 33 & 30 & \varepsilon & 185 \\ 202 & 214 & 189 & 123 & 77 & 94 & 65 & 73 & 70 & 25 & 225 \\ 230 & 242 & 217 & 151 & 105 & 122 & 93 & 101 & 98 & 53 & 253 \end{bmatrix}$$

Then determine the eigenvalue of the matrix to know the length of the production time of the dining table and the eigenvector of the matrix to know the good time to start working on each processing unit. Using Scilab with the command [l,v]=maxplusmaxalgol(A_bar), the eigenvalues and eigenvectors corresponding to the matrix are $\lambda = 253$ and

$$v = [30 \quad 41 \quad 76 \quad 122 \quad 145 \quad 147 \quad 157 \quad 167 \quad 185 \quad 225 \quad 253]^T$$

However, in order for the available materials to be processed immediately and not to wait 30 minutes to start the first process of table production, the time when the system starts working is determined by selecting an eigenvector that has a minimum of non-negative elements. After obtaining an eigenvector with a minimum non-negative value, then the element of the smallest eigenvector is converted to zero [4].

Therefore, by selecting $\alpha = -30$, a new eigenvector is obtained to determine the time when the system starts working as follows.

$$v' = \alpha \otimes \mathrm{v} = -30 \otimes \begin{bmatrix} 30 \\ 41 \\ 76 \\ 122 \\ 145 \\ 147 \\ 157 \\ 167 \\ 185 \\ 225 \\ 253 \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \\ 46 \\ 92 \\ 115 \\ 117 \\ 127 \\ 137 \\ 155 \\ 195 \\ 223 \end{bmatrix}$$

Based on v' above, then the processing units P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11 can work for the first consecutive production process at 0, 11, 46, 92, 115, 117, 127, 137, 155, 195, 223 minutes and can work periodically with a period = 253 minutes. Based on the results of the field study, the production process of dining tables in the wooden furniture industry "Berkah Usaha" began at 08.00 WIB, so that at minute 0 in Table 1 it can be represented that P1 started working at 08.00 WIB and so on. The production schedule that refers to working hours is as follows.

Table 1. Schedule of the "Berkah Usaha" Wooden Furniture Production System According to Working Hours

Production Process	Production Time (WIB)
Raw Material Preparation	08.00
Measurement	08.11
Cutting	08.46
Wood Smoothing	09.32
Measurement of Grooves and Holes	09.55
Making Table Floors	09.57
Making Grooves and Holes	10.07
Sharpening Table Floors	10.17
Assembly	10.35
Sanding	11.15
Finishing	11.43

Based on the model and the results of the scheduling that has been carried out, the production time period of Berkah Usaha wooden furniture dining table is obtained for 253 minutes, with processing time for raw material preparation for 11 minutes, measurement for 35 minutes, cutting for 46 minutes, wood smoothing for 23 minutes, measurement of purus and holes for 2 minutes, making table floor for 10 minutes, 10 minutes of purus and hole making, 18 minutes of table floor scraping, 40 minutes of assembly, 28 minutes of sanding, and 30 minutes of finishing.

4. CONCLUSIONS

From this study, Max-Plus Algebra can be applied to the wood furniture production process system and it can be concluded that the equation $x(k+1) = A \otimes x(k) \oplus B \otimes u(k)$ and $y(k) = C \otimes x(k)$to be used to model the production process of wooden furniture. In addition, the dynamic behavior is studied using Scilab and Max-Plus Algebra Toolbox version 1.01, so a periodic schedule of wooden furniture production is formed so that it is expected to be a reference in determining the start time of production and the completion time of wooden furniture production.

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