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Application of Demerit Chart and Fuzzy Demerit Chart for Monitoring Paper Production

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ABSTRACT

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Statistical Process Control (SPC) is an important method in quality control to monitor and improve production processes. Control charts are one of the SPC tools that are often used to quickly detect the causes of process variation so that improvements can be made before more nonconforming products are produced. The u chart is commonly used to monitor the number of defects in a production unit. However, this control chart has limitations in handling variations in defect severity, so demerit and fuzzy demerit control charts were developed to assign weights to defects based on their severity. Demerit and fuzzy demerit control charts have been applied in various production cases, but the study of the application of demerit and fuzzy demerit control charts in the industrial field, especially the paper industry, has never been done. The purpose of this study is to apply demerit and fuzzy demerit control charts to monitor and evaluate the quality of the paper production process at PT. Bosowa Media Grafika (Tribun Timur). The data used in this study are secondary data obtained from research conducted by Ilham (2012). The results obtained that the demerit chart and the fuzzy demerit chart show that the paper production process at PT Bosowa Media Grafika (Tribun Timur) is still in a stable condition (incontrol) in each observation. This shows that demerit and fuzzy demerit control charts have the same performance in monitoring the paper production process.

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1. Introduction

Statistical process control (SPC) is a method for quality control that can provide an overview of the ongoing process by taking samples to be analyzed using statistical techniques. One of the most popular tools in SPC is the control chart. Control Chart can be used to quickly detect the presence of causal cases of a process shift. So that improvements can be made to the process, before too many processes that do not conform to the standard are running [1] .

There are two types of process parameters that are monitored on a control chart, namely the average and variance. To monitor the average means to monitor the accuracy of a process, while to monitor the variance means to monitor the variability of a process [2]. Based on the type of quality characteristics, control charts can be divided into two types, namely variable control charts and attribute control charts. Variable control charts are control charts for monitoring characteristics that can be measured (interval or ratio scale). While attribute control charts are used to monitor characteristics that cannot be measured (categories). Based on the number of quality characteristics monitored, both variable and attribute control charts can be categorized into two types, namely univariate control charts and multivariate control charts[3][4].

The results of measuring quality characteristics are not always in the form of numerical values, in the field of services/manufacturing, often obtained categorical data (conforming or not conforming and defective or not defective). One of the control chart that is suitable for these conditions is to use a u chart. The control chart is a control chart used to monitor the number of defects in an observation unit of different sizes. However, in its implementation, this control chart has a weakness because it only calculates the number of defects without considering the severity or impact of defects. Therefore, to overcome this, a demerit control chart was developed that allocates weight to each defect according to its severity[5].

Meanwhile, according to Montgomery [3] Demerit chart is a control chart that is suitable in conditions if the types of defects in each product have different hierarchies (levels). The advantage of the demerit chart is that it can provide a more accurate assessment of product quality by considering weights based on the severity of defects, so as to prioritize repairs on more serious defects [3][6]. Some research on demerit control charts that are applied to various fields include Ramadhani *et al* [7], applying demerit control charts to the case of 240 ml bottled drinking water production. Then Aksioma [8], applied a demerit chart for quality control of cement bags and so on. In addition to demerit control charts, there are also demerit fuzzy control charts. The fuzzy demerit chart is a statistical tool that combines quality control concepts with fuzzy logic to measure and monitor defective products based on demerit weights adjusted for defect severity. In this chart, product quality is evaluated by accounting for uncertainty and variation in defect assessment, using fuzzy values to define defect severity more flexibly than traditional methods. This approach enables more precise and adaptive quality assessment in complex and varied production processes, and helps in better decision-making for quality improvement [2].

The advantage of fuzzy demerit control charts is to handle uncertainty and variability in product defect assessment by using fuzzy logic, which allows demerit weights to be determined more flexibly and realistically, depending on the severity of defects that cannot always be measured with certainty. This approach is particularly useful in situations where defect boundaries are unclear or highly subjective, such as in products of varying quality or complex production processes. Thus, fuzzy demerit allows for more accurate and adaptive quality control, and aids in better and more grounded decision making [9]. Some research on demerit fuzzy control charts, namely Pradini & Mashuri [10] applying demerit fuzzy control charts to control the quality of laminated type automotive glass products. Then Nanda [11], applied a demerit fuzzy chart for quality control of cement bags.

From the explanation that has been described, until now there has been no research on the application of demerit and fuzzy demerit control charts applied to monitor the quality of the paper production process. Therefore, this research aims to apply demerit and fuzzy demerit control charts to monitor the quality of the paper production process at PT. Bosowa Media Grafika (Tribun Timur).

PT. Bosowa Media Grafika is a company that publishes Tribun Timur newspaper, one of the regional newspapers with high popularity among the people of Makassar. To maintain readers' trust and remain competitive in the market, product quality is one of the important aspects that must be maintained by the company. However, production data still shows a number of products with defects. Through data analysis published by [16], it is known that product defects still occur during the production process. Based on these findings, the authors were inspired to conduct research with the title application of demerit chart and fuzzy demerit chart for monitoring paper production.

2. Research Methods

2.1 Data

The data in this study is secondary data, obtained from [11] publication on damage/defects in the production process produced by PT. Bosowa Media Grafika.

2.2 Method

a. Demerit Chart

The demerit chart is a control chart developed from the u control chart by giving weight to the types of defects that occur. There are several classes of defects or damage, which are as follows [12].

1. Class A – Critical : A very serious defect in a product.
2. Class B – Major : A serious defect in a product.
3. Class C – Minor : Defects that do not affect the usability of the product.

The data structure used in the Demerit chart in Table 1.

Table 1. Data structure of demerit chart

Sub group	Unit	Type of defect			Number of defects (d)	U
		Critical	Major	Minor		
1	n_1	c_{1a}	c_{1b}	c_{1c}	d_1	$d_{1/n}$
2	n_2	c_{2a}	c_{2b}	c_{2c}	d_2	$d_{2/n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
i	n_i	c_{ia}	c_{ib}	c_{ic}	d_i	$d_{i/n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
m	n_m	c_{ma}	c_{mb}	c_{mc}	d_m	$d_{m/n}$
Total	$\sum_{i=1}^{m} n_i$	$\sum_{i=1}^{m} c_{ia}$	$\sum_{i=1}^{m} c_{ib}$	$\sum_{i=1}^{m} c_{ic}$		
Mean		$\bar{u}_a$	$\bar{u}_b$	$\bar{u}_c$		

The weighting will increase according to the defect level (1 minor defect and 10 critical defects). The weighting can be used with the following Equation (1)[13].

$$d_i = w_a c_{ia} + w_b c_{ib} + w_c c_{ic} \quad (1)$$

with

$i = 1,2, \dots, n$

n = sample size of the *i-th* subgroup.

The center line on the $\bar{u}$ demerit chart is shown in Equation (2) below.

$$\bar{u} = w_a \bar{u}_a + w_b \bar{u}_b + w_c \bar{u}_c \quad (2)$$

Where $\bar{u}_A$, $\bar{u}_B$, $\bar{u}_C$ indicates the average per unit defect in class A, class B, and Class C.

$$\sigma = \sqrt{\frac{w_a \bar{u}_a + w_b \bar{u}_b + w_c \bar{u}_c}{n_i}} \quad (3)$$

So the control limits of the demerit chart are shown in Equations (4)-(6) below [14].

$$\text{CL} = \bar{u} \quad (4)$$

$$\text{UCL} = \bar{u} + 3\sigma \quad (5)$$

$$\text{UCL} = \bar{u} - 3\sigma \quad (6)$$

b. Fuzzy Demerit Chart

The fuzzy demerit chart is a control chart that uses *fuzzy* concepts . The use of *fuzzy* weights can better describe the classification of product defects [14]. Each linguistic weight can be described as a *fuzzy number* (FN), so that the *fuzzy* demerit chart can be built from *fuzzy set theory* (FST)[9].

1. Representation Number of Units

The linguistic weight on an *i*-th category*is expressed* as FN, with a positive continuous membership function $\mu_{\bar{w}_i}$. The number of demerits on the *j*-th inspection unit can be defined by the following Equation (7) [15].

$$\tilde{D}_j = \sum_{i=1}^{k} \bar{w}_i C_{ji} \quad (7)$$

2. Control Limit Decrease

The *fuzzy* sample *mean* and *fuzzy* sample variance of the number of demerits in each inspection unit are defined by **Equations (8)-(9)** below.

$$\tilde{\tilde{D}} = \frac{1}{m}\sum_{j=1}^{m} \tilde{D}_j \tag{8}$$

$$\tilde{S}^2 = Var(\tilde{D}) = \sum_{i=1}^{k} \tilde{w}_i{}^2 \tilde{c}_i \tag{9}$$

Demerit *fuzzy* control limits with *K*-sigma can be formulated with the following **Equations (10)-(12)**.

$$\tilde{C}L = \tilde{\tilde{D}} \tag{10}$$

$$U\tilde{C}L = \tilde{\tilde{D}} + K\sqrt{\sum_{i=1}^{k} \tilde{w}_i{}^2 \tilde{c}_i} \tag{11}$$

$$L\tilde{C}L = max\left(\tilde{\tilde{D}} - K\sqrt{\sum_{i=1}^{k} \tilde{w}_i{}^2 \tilde{c}_i}\, , 0\right) \tag{12}$$

3. Decision Rule

If *monitoring the* process, it is necessary to compare the *fuzzy* demerit statistic $\tilde{D}_j$ with its control limits, so as to determine the condition of the production process. The *fuzzy ranking* method using *α-cuts* (*αc*) can be introduced in process control statistics **[8]**.

To compare two FN, namely $\tilde{A}$ dan $\tilde{B}$ at a value of a_c, with $0 \leq a_c \leq 1$, and determine the corresponding ${}^{a_c}\tilde{A} = [a, b]$ and ${}^{a_c}\tilde{B} = [c, d]$. Thus the relationship of $\tilde{A}$ and $\tilde{B}$ can be expressed by the following **Equation (13)**.

$$\tilde{A} \leq \tilde{B} \text{ jika } {}^{a_c}_{R}\tilde{A} = b \leq {}^{a_c}_{R}\tilde{B} = d \tag{13}$$

The relationship between two FNs can be determined by comparing the *right endpoints* of the closed intervals generated by the *α-cuts* depending on the selected *α-cuts* value, with *α-cuts* > 0.5. Therefore, to assess a process, one can compare ${}^{a_c}_{R}\tilde{D}_j$ with ${}^{a_c}_{R}U\tilde{C}L$ and ${}^{a_c}_{R}L\tilde{C}L$ in order.

2.3 Research Procedure

a. Demerit Chart

1. Characteristic description of defect data on products.
2. Create a demerit control chart.
   a. Identify defect inspection data into 3 categories of defect classes: blurred (minor), unregistered (major) and truncated (critical).
   b. Assigning weights to each defect class category.
   c. Obtaining the weighted number of defects for each class.
   d. Obtain the weighted defects in each subgroup using **Equation (1)**.
   e. Obtained the average defects per inspection unit for each subgroup.
   $$u_i = \frac{d_i}{n}$$
   f. Calculate the average number of defects per unit for the overall weighted defect type using **Equation (2)**.
   g. Calculate CL, UCL and LCL using **Equations (4)-(6)**.

b. Fuzzy Demerit Chart

1. Provide lingiacoustic weighting for each defect type.
2. Calculating fuzzy ${}^{a_c}\tilde{w}_j$ weights and *fuzzy* ${}^{a_c}\tilde{D}_j$ statistics using the following Equation.
   $${}^{a_c}\tilde{w}_i = [w_{i,1} + \alpha_c(w_{i,2} - w_{i,1}); w_{i,3} - \alpha_c(w_{i,3} - w_{i,2})] \text{ and}$$
   $${}^{a_c}\tilde{D}_i = \left[\sum_{i=1}^{k} c_{ji}\left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right); \sum_{i=1}^{k} c_{ji}\left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)\right]$$
3. Calculating the fuzzy control limits of ${}^{a_c}\tilde{C}L$, ${}^{a_c}U\tilde{C}L$ and ${}^{a_c}L\tilde{C}L$ using the following Equation **[11]**.

$$
\begin{aligned}
{}^{a_c}\widetilde{\mathrm{CL}} &= \left[\frac{1}{m}\sum_{j=1}^{m}\sum_{i=1}^{k} c_{ji} \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right); \frac{1}{m}\sum_{j=1}^{m}\sum_{i=1}^{k} c_{ji} \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)\right] \\
&= \left[\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right);\ \sum_{i=1}^{k} \bar{c}_i \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)\right].
\end{aligned}
$$

$$
\begin{aligned}
{}^{a_c}\widetilde{\mathrm{UCL}} = {}& \sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right) + K\sqrt{\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right)^2}; \\
& \sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right) + K\sqrt{\sum_{i=1}^{k} \bar{c}_i \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)^2}.
\end{aligned}
$$

$$
\begin{aligned}
{}^{a_c}\widetilde{\mathrm{LCL}} = {}& \max\left(\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right) - K\sqrt{\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right)^2}\right); \\
& \max\left(\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right) - K\sqrt{\sum_{i=1}^{k} \bar{c}_i \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)^2}\right).
\end{aligned}
$$

So based on the *right endpoint*, it can be summarized in **Table 2** below.

Table 2. TFN Linguistic Right Endpoint Notation and Equation

Notation	Equation
${}^{a_c}_{R}\widetilde{\mathrm{D}}_j$	$\sum_{i=1}^{k} c_{ji} \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)$
${}^{a_c}_{R}\widetilde{\mathrm{UCL}}$	$\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right) + K\sqrt{\sum_{i=1}^{k} \bar{c}_i \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)^2}$
${}^{a_c}_{R}\widetilde{\mathrm{LCL}}$	$\sum_{i=1}^{k} \bar{c}_i \left(w_{i,1} + a_c(w_{i,2} - w_{i,1})\right) - K\sqrt{\sum_{i=1}^{k} \bar{c}_i \left(w_{i,3} - a_c(w_{i,3} - w_{i,2})\right)^2}$

c. Create a demerit *fuzzy* chart.

2.4 Paper Production

Paper production is the process of processing cellulose fibers from raw materials such as wood, bamboo, or recycled materials into ready-to-use paper sheets. To ensure the efficiency, quality and sustainability of this process, integrated monitoring is required at every stage of production. This monitoring serves to maintain paper quality, such as thickness and other characteristics, while optimizing the use of raw materials and energy and reducing negative impacts on the environment. Advanced technologies, such as IoT sensors, supervisory control and data acquisition (SCADA) systems, and AI-based analytics, are increasingly being applied to improve accuracy and detect potential problems early, such as quality deviations or inefficient energy consumption. With an effective monitoring system, the paper industry can run optimized production processes, meet high quality standards, and support overall environmental sustainability.

3. Results And Discussion

3.1 Descriptive Statistics

This study uses daily data during December 2011 obtained from the publication of [16] on defects in paper production produced by PT. Bosowa Media Grafika Daily Tribun Timur. The following is the percentage of daily paper production during December 2011.

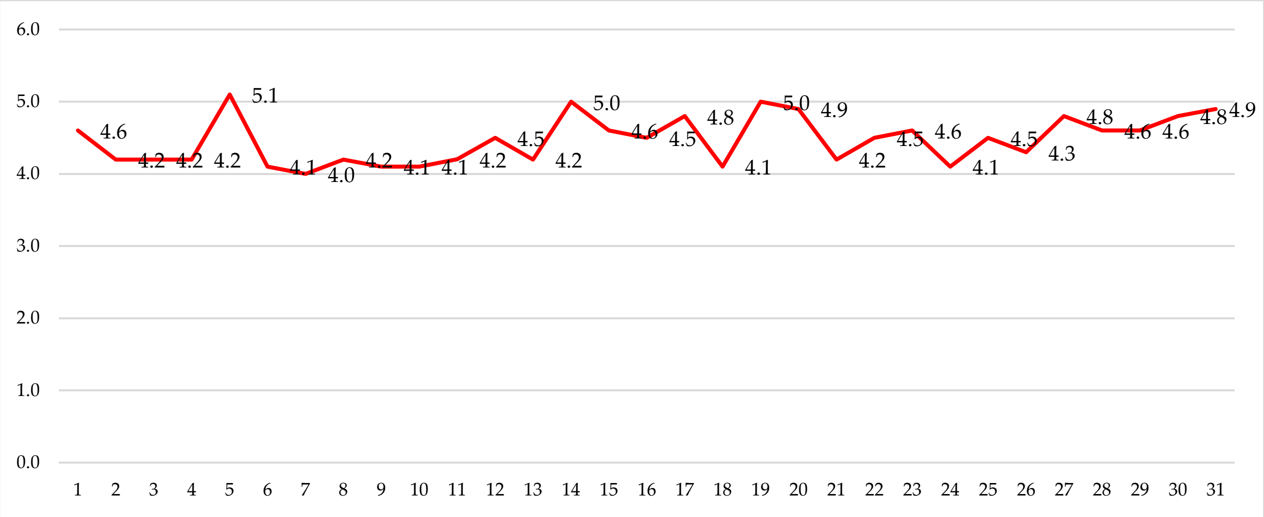

Figure 1. Percentage of Paper Production as of December 2011

Based on **Figure 1**, it is known that the percentage of paper production at PT. Bosowa Media Grafika Daily Tribun Timur, from time to time fluctuates.

3.2 Classification of Defect Types

Based on the data obtained, the types of defects in this study are classified into 3 types of defect classes, namely blurred classes, unregistered classes and truncated classes shown in **Table 2** below.

Table 2. Defective Product Report at PT. Bosowa Media Grafika Daily Tribun Timur

No	Classification of Defective Products	Total	Maximum	Minimum	Mean
1.	Flee	57555	2218	1657	1857
2.	No Register	8855	341	255	286
3.	Truncated	7381	284	212	238

Based on **Table 2**, it shows that the defective product report at PT. Bosowa Media Grafika Daily Tribun Timur is classified into 3 defect classes, including the categories of blurred (minor), unregistered (major) and cut (critical) classes. The blurred product class category was 57555 during December 2011, with an average defect of 1857 per day. Then the unregistered class category was 8855 during December 2011, with an average defect of 286 per day. Then the truncated class category was 7381 during December 2011, with an average defect of 238 per day. A visualization of the class classification of defective products can be seen in **Figure 2** below.

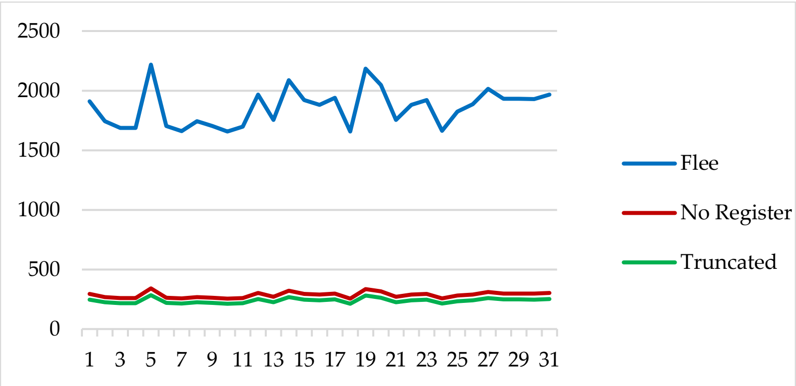

Figure 2. Blurred, unregistered, and truncated products survived December 2011.

3.2 Demerit Chart

Demerit charts are used when defect types are categorized into classes based on the severity of the defect. In the production of paper at PT. Bosowa Media Grafika Daily Tribun Timur, 3 types of defects were obtained, including the categories of blurred class (minor), not registering (major) and cut (critical). Based on the weighting values that have been explained, the calculation of the weighted number of defects for each subgroup can be done. The calculation of the weighted number of defects is shown in **Table 3** below.

Table 3. Weight on each type of defect

	Truncated (C)	**No Register (B)**	**Flee (A)**	**Total**
AQL	6.5	1	0.065	7.565
Weight	1.164	7.565	116.385	125.113

Based on **Table 3**, the weight for each defect class is obtained, including the critical class (truncated class) of 116.385%, the major class (unregistered class) of 7.565% and the minor class (blurred class) of 1.164%, so that it can be formed into.

$$d_1 = 1.164(1911) + 7.565(294) + 116.385(245) = 32962.84$$
$$d_2 = 1.164(1744) + 7.565(268) + 116.385(224) = 30127.68$$
$$\vdots$$
$$d_{31} = 1.164(1968) + 7.565(303) + 116.385(252) = 33911.97$$

Then the average defect per unit is obtained as follows.

$$u_1 = \frac{32962.84}{2450} = 13.45$$

$$u_2 = \frac{30127.68}{2236} = 13.47$$

$$\vdots$$

$$u_{31} = \frac{33911.97}{2523} = 13.44$$

Next calculate the average value of the number of defects in each class, which is obtained from the number of defects in each class divided by the total number of samples.

$$\bar{u}_1 = \frac{57554}{1650650} = 0.034 = 3.5\%$$

The value of $\bar{u}_1$ is obtained from the number of defects in the fuzzy class divided by the total sample. From the calculation results, the average number of blurred class defects for each unit of product is 3.5%. This indicates that in every 1000 units of paper produced, the number of defects that occur in the minor category (fuzzy class) is 35 defects.

$$\bar{u}_2 = \frac{8855}{1650650} = 0.005 = 0.5\%$$

The value of $\bar{u}_2$ is obtained from the number of defects in the non-registered class divided by the total sample. From the calculation results, the average number of non-registered grade defects for each unit of product is 0.5%. This indicates that in every 1000 units of paper produced, the number of defects that occur in the major category (non-registered grade) is 5 defects.

$$\bar{u}_3 = \frac{7381}{1650650} = 0.004 = 0.4\%$$

The value of $\bar{u}_3$ is obtained from the number of defects in the truncated class divided by the total sample. From the calculation results, the average number of cut grade defects for each unit of product is 0.4%. This indicates that in every 1000 units of paper produced, the number of defects that occur in the critical category (cut class) is 4 times defective. Furthermore, to get the center line (CL), it is necessary to calculate the value according to **Equation (4)**.

$$\begin{aligned} \bar{u} &= w_1\bar{u}_1 + w_2\bar{u}_2 + w_3\bar{u}_3 \\ &= 0.041 + 0.041 + 0.520 \\ &= 0.602 \end{aligned}$$

and standard deviation $(\hat{\sigma}_u)$ value ie,

$$\hat{\sigma}_u = \sqrt{\frac{w_1^2\bar{u}_1 + w_2^2\bar{u}_2 + w_3^2\bar{u}_3}{n_i}}$$
$$= \sqrt{\frac{(1.164^2 \times 0.035) + (7.565^2 \times 0.005) + (116.385^2 \times 0.004)}{53250}}$$
$$\hat{\sigma}_u = 0.033$$

So that the UCL and LCL are obtained according to **Equations (5)-(6)**.

$$\text{UCL} = \bar{u} + 3\sigma = 0.602 + (3 \times 0.033) = 0.703$$
$$\text{LCL} = \bar{u} - 3\sigma = 0.602 - (3 \times 0.033) = 0.501$$

So that a demerit chart is obtained as shown in **Figure 3** below.

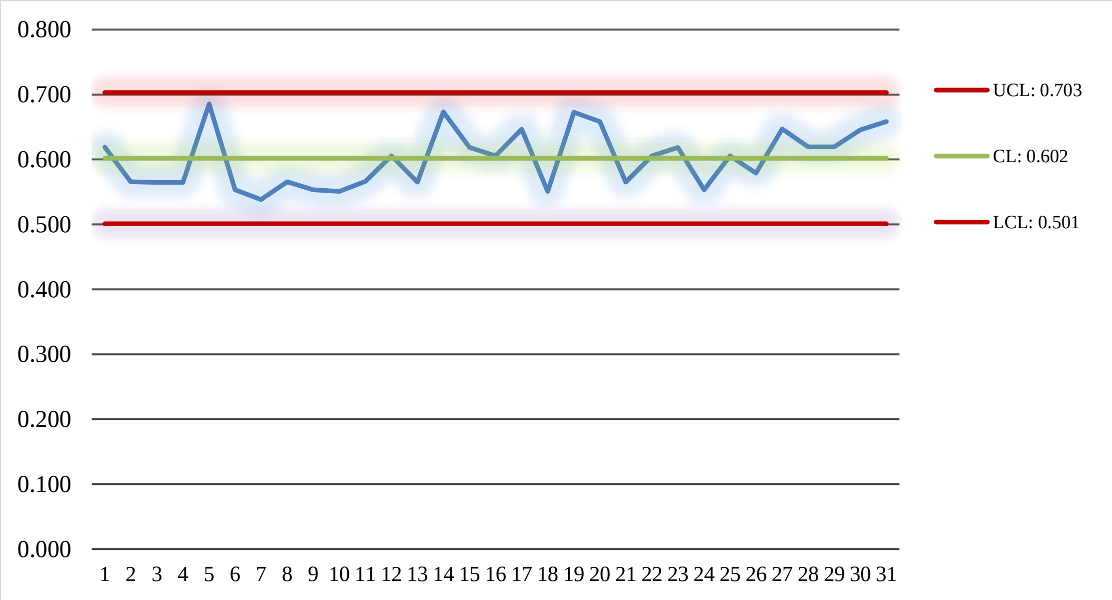

Figure 3. Demerit Control Chart

Based on **Figure 3**, it is shown that the demerit chart that has been formed in monitoring the paper production process is in a statistically controlled condition. This is indicated by the absence of events in conditions outside the control limits (out of control) on the demerit chart.

3.3 Fuzzy Demerit Chart

1. Defect Linguistic Weight Selection

The defect events that occur in the investigation results will be classified into different defect class categories, based on the weight level of the product defect, namely.

Flee : $\widetilde{w}_1$ = (0.1/0.2/0.4)
No Register : $\widetilde{w}_2$ = (0.3/0.5/0.7)
Truncated : $\widetilde{w}_3$ = (0.6/0.8/1)

Each linguistic weight is described by a positive *triangular fuzzy number* (TFN) with the membership function of the linguistic weight to $\widetilde{w}_i$ which is denoted by the coordinate vector $(w_{i,1}/w_{i,2}/w_{i,3})$..

2. Calculation $^{\alpha_c}\widetilde{\mathrm{D}}_j$, $^{\alpha_c}\mathrm{U\tilde{C}L}$ and $^{\alpha_c}\mathrm{L\tilde{C}L}$.

This research uses an α-cuts value of 0.8, hence the following $^{\alpha_c}\widetilde{w}_j$,.
$^{\alpha_c}\widetilde{w}_1 = [0.10; 0.20]$
$^{\alpha_c}\widetilde{w}_2 = [0.40; 0.55]$
$^{\alpha_c}\widetilde{w}_3 = [0.75; 0.90]$

Based on arithmetic operations on closed intervals $^{\alpha_c}\widetilde{D}_j$, is obtained,

D_1 = $[0.10(1911) + 0.40(294) + 0.75(245); 0.20\,(1911) + 0.55(294) + 0.90(245)]$ = $[492.5;\ 764.4]$
D_2 = $[0.10(1744) + 0.40(268) + 0.75(224); 0.20\,(1744) + 0.55(268) + 0.90(224)]$ = $[449.6;\ 697.8]$
$\vdots$
D_{31} = $[0.10(1968) + 0.40(303) + 0.75(252); 0.20\,(1968) + 0.55(303) + 0.90(252)]$ = $[507;\ 787]$

With K=3, ${}^{\alpha_c}\mathrm{U\tilde{C}L}$ dan ${}^{\alpha_c}\mathrm{L\tilde{C}L}$ is obtained from the demerit fuzzy chart formed as follows.

$$ {}^{\alpha_c}\mathrm{U\tilde{C}L} = [2529.63; 2601.69] $$
$$ {}^{\alpha_c}\mathrm{L\tilde{C}L} = [2119.77; 2240.30] $$

So that the fuzzy demerit chart is obtained in **Figure 4** below.

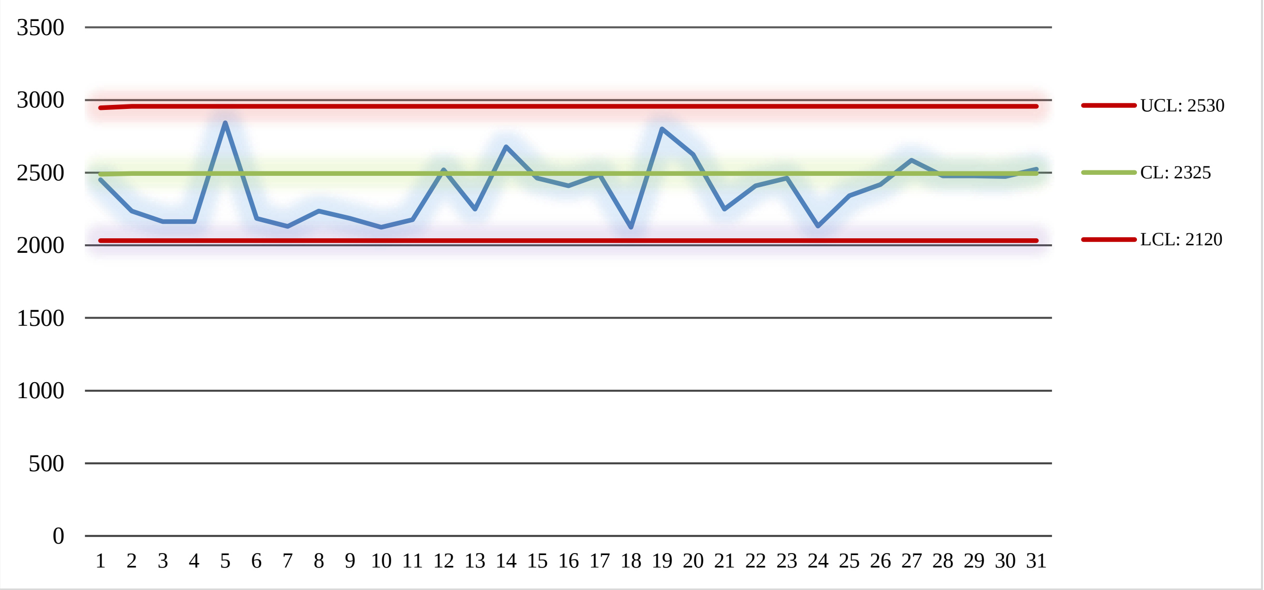

Figure 4. Fuzzy Demerit Control Chart

Based on **Figure 4**, it is intended that the fuzzy demerit chart that has been formed in monitoring the paper production process is in a statistically controlled condition. This is indicated by the absence of events in conditions outside the control limits (out of control) on the fuzzy demerit chart.

4. Conclusions

Based on the results and discussion of the application of demerit chart and fuzzy demerit chart, it shows that the paper production process at PT. Bosowa Media Grafika (Tribun Timur) is still in a stable condition (incontrol) in each observation. This also shows that demerit and fuzzy demerit chart have the same performance in monitoring the paper production process.

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