

# Comparison of Single Net Premium of Unit Linked Endowment Life Insurance Using Annual Ratchet Method and Black Scholes Model

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## ABSTRACT

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### Keywords

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Annual Ratchet is an indexing method. Black Scholes is a model used to determine option. The purpose of this study is to compare the results of single net premium of unit-linked endowment life insurance using the Annual Ratchet method and the Black Scholes Model. The data used in this study are data on the daily closing share price of PT Telkom Indonesia (Persero) Tbk for the period December 20, 2021 to December 20, 2022, Bank Indonesia interest rates and the 2019 Mortality Table. In this study, a comparison is made between the Annual Ratchet method and the Black-Scholes model to calculate the net single premium of unit-linked endowment life insurance for a 30-year-old male insured. The results show that the premium calculated using the Annual Ratchet method is greater than the premium from the Black-Scholes model, which is Rp 8,725,000. This is due to the additional protection feature in the Annual Ratchet method, which provides a minimum guaranteed investment value, thus increasing the premium value to be paid.



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## 1. Introduction

Currently, all important aspects of human life are not always in a good and safe condition. This is because in real life, these important aspects will always be surrounded by various things that can threaten security, safety, and cause financial losses. One of the efforts to overcome this is by transferring the financial loss to another party, which then led to the existence of insurance. Insurance, or also referred to as coverage, is an agreement between two or more people in which an insurer gives money to the insured if they experience loss, damage, loss, or uncertain events that may occur [1]. Insurance is a contract made between an insurance provider (insurer) and an individual holding a policy (insured), in which the insured pays a fee to receive protection against potential losses, legal responsibilities to others that the insured might face, and receives payouts contingent on the insured's death or life and based on an agreed amount of benefits or the performance of managed funds [2].

Life insurance is divided into two types: traditional insurance and modern insurance. Traditional insurance consists of term life insurance, whole life insurance, and endowment insurance, while modern insurance consists of universal life insurance, unitized with profit, and unit-linked insurance. Generally, life insurance companies only provide protection benefits to policyholders. Based on that, a more innovative type of life insurance program emerged, namely unit-linked life insurance. A type of life insurance that combines insurance benefits and investment is unit-linked life insurance, but in a unit-linked life insurance contract, the policyholder's assets and the insurance company's assets remain separate from each other [3]. Endowment unit-linked life insurance is a combination of endowment life insurance and unit-linked life insurance. The determination of the net single premium for the dual benefit unit-linked life insurance is based on the existence of a minimum guaranteed value. This is because a minimum guarantee is required to involve the related insurance company in the risk-sharing of the investment in the insurance product [4].

When stock prices experience a drastic decline, the benefits received will also decrease, causing both the insured and the insurer to potentially incur losses. To address this, insurance companies usually set a minimum benefit (floor), which guarantees that the insured will receive benefits no less than the value agreed upon in the contract. Conversely, if the stock price drastically rises, the benefits received will also increase, providing advantages for both the insured and the insurer. In this case, the insurance company sets a maximum benefit limit (cap) that the insured can receive, so the insured or their heirs will receive the maximum benefit according to the cap value at that time and a minimum benefit equal to the agreed floor value.

Unit-linked insurance is the most popular product among the Indonesian public [5]. This product combines elements of protection, savings, and investment in one package. The calculation of benefits obtained by the insured from unit-linked insurance can be done using the indexing method. There are three indexing methods that are often applied, namely point-to-point, annual ratchet, and high water mark. Insurance has a relationship with stocks, especially in terms of investment, risk protection, and financial stability.

Annual ratchet is a method used to calculate the premium value and benefit value of unit-linked life insurance. The annual ratchet method is an indexing method based on the participation rate evaluated year by year (annual) [6]. The annual ratchet method consists of two types of designs, namely simple ratchet and compound ratchet. The investment gain in a simple ratchet is only calculated once for one period against a fixed principal amount, whereas in a compound ratchet, the investment gain from the previous period is included to calculate the gain for the next period. The advantages of the annual ratchet method compared to other methods; namely, this method considers the value of premiums and benefits based on the existence of a minimum interest rate (floor) and a maximum interest rate (cap), so that both the company and the policyholders are protected from the risk of loss [7].

Black-Scholes is a method that can be used to calculate the premium and benefit value of unit-linked life insurance. The steps in calculating the minimum guarantee (floor) in unit-linked life insurance are analogous to the calculation of financial options. An option is an agreement between the option seller (seller or writer) and the option buyer, where the seller guarantees the buyer's right to buy or sell certain stocks at a predetermined time and price. In terms of determining the option price, one of the methods that can be used is the Black-Scholes model. The advantage of the Black-Scholes model is that it is a widely accepted model for valuing call options in the financial community because the option values generated by the Black-Scholes model are considered "fair" or accurate [8].

## 2. Research Methods

In this study, the net single premium of endowment unit-linked life insurance is discussed using the annual ratchet method and the Black-Scholes model. Unit-linked dual-benefit life insurance is a combination of dual-benefit life insurance and unit-linked life insurance. The determination of the net single premium for the dual benefit unit-linked life insurance for Annual Ratchet is based on the 2019 Mortality Table, the age of the policyholder, the insurance term, gender, risk-free interest rate, premium return rate (guarantee), participation rate, risk-free interest rate, cap, floor, volatility, return, constructing the contract value from the benefits to be obtained with a minimum guarantee, and then forming the net single premium for the dual benefit unit-linked life insurance. Meanwhile, the determination of the net

single premium for the dual benefit unit-linked life insurance for Black Scholes is based on the age of the policyholder, gender, insurance term, risk-free interest rate, volatility, and the probability of someone dying according to the 2019 Indonesian Mortality Table.

## 2.1. Unit Linked Life Insurance

Endowment unit-linked life insurance is insurance that combines traditional endowment insurance with modern unit-linked insurance. Endowment life insurance provides certain benefits whether the insured lives or dies during the coverage period [9]. Each endowment life insurance policy has a maturity date. The maturity date is the date on which the insurance company will pay the policyholder the sum assured if the insured is still alive. The maturity date will arrive at the end of the specified period or when the insured reaches the specified age. The calculation of the net single premium for an endowment unit-linked life insurance is based on the existence of a minimum guaranteed value (floor). This is because a minimum guarantee is required to involve the related insurance company in the risk-sharing of the investment in the insurance product.

## 2.2. Premium

Premium refers to a sum of money established by an insurance provider or a reinsurance firm and accepted upon by the individual who has taken out the policy to receive payment according to the insurance contract or reinsurance contract [10]. Therefore, the insurance premium is an amount of money that must be paid by the insurance participant to the insurance provider in accordance with the agreement in the policy. Life insurance premium is an amount of money that the policyholder must pay to the insurance company within a certain period, with the nominal amount determined according to the agreement. With the payment of the insurance premium, the policyholder will receive insurance benefits or risk transfer from the customer to the insurance company when a risk occurs in the future. In Indonesia, there are three types of life insurance, namely term life insurance, whole life insurance, and endowment policy. Endowment life insurance is a type of insurance that provides two benefits, where the company will pay a benefit if the insured survives until the end of the policy term or dies within the policy term [11]. There are several factors that can affect life insurance premiums, such as the possibility that a person of a certain age will die within a certain period (mortality), interest rates are interest rates obtained from invested funds, and marketing costs and other administrative costs associated with policy management [12].

## 2.3. Stock Return Volatility

The volatility of stock returns, denoted by  $\sigma$ , is the standard deviation of stock returns over an annual period. This volatility is used to measure the level of risk of a stock. One method for estimating the volatility of stock returns is historical volatility, which is the volatility calculated based on past prices. Here are the steps to estimate the historical volatility of stock prices:

1. The closing stock prices are denoted as  $S(1), S(2), \dots, S(n)$ . The data used is daily data, so

$$\Delta t = \frac{1}{n} \text{year}$$

2. Calculate the daily stock return value [13].

$$R(t) = \ln \ln \left( \frac{S(t)}{S(t-1)} \right) \quad (1)$$

3. Calculate the estimated mean and variance of daily stock returns,

$$\begin{aligned} \overline{R(t)} &= \frac{\sum_{t=1}^n R(t)}{n} \\ \sigma^2 &= \frac{\sum_{t=1}^n (R(t) - \overline{R(t)})^2}{n-1} \end{aligned} \quad (2)$$

4. Calculate the estimated variance of annual stock returns.

$$\sigma^2 = \frac{1}{\Delta t} \left[ \frac{\sum_{t=1}^n (R(t) - \overline{R(t)})^2}{n-1} \right] = n \left[ \frac{\sum_{t=1}^n (R(t) - \overline{R(t)})^2}{n-1} \right]$$

5. The volatility approach is the standard deviation of annual stock returns.

$$\sigma = n \left[ \frac{\sum_{t=1}^n (R(t) - \overline{R(t)})^2}{n-1} \right] \quad (3)$$

## 2.4. Normality Test

One technique to test whether data is normally distributed is the Kolmogorov-Smirnov (K-S) test. The basic concept of the Kolmogorov-Smirnov normality test is to compare the theoretical (observation) and empirical (observation) distributions based on cumulative frequencies.

For example,  $X_1, X_2 \dots X_n$  is a random sample of size  $n$  from a population with distribution function  $F(x)$ . Suppose  $F_0(x)$  is a certain distribution and will be tested

$$H_0: F(x) = F_0(x) \text{ for all } x \quad (\text{certain distributed data})$$

$$H_1: F(x) \neq F_0(x) \text{ for all } x \quad (\text{data is not distributed})$$

Kolmogorov-Smirnov test using test statistic

$$T = \sup_{-\infty < x < \infty} |F_n(x) - F_0(x)|$$

$F_n(x)$  is the empirical distribution function based on random samples  $X_1, X_2 \dots X_n$ . The conclusion drawn is to reject  $H_0$  if the value of  $T > T_{n,\alpha}$ . Where  $T > T_{n,\alpha}$  the critical value of the statistic.

## 2.5. Endowment Life Insurance

An endowment life insurance policy for  $n$  years can be viewed as a combination of a term life insurance policy for  $n$  years and a pure endowment life insurance policy for  $n$  years [14]. Therefore, in endowment insurance, the death benefit is provided if the insured dies before  $n$  years; conversely, if the insured is still alive until  $n$  years, they will receive pure endowment insurance. Based on the timing of benefit payments, dual-benefit life insurance is divided into two types. endowment life insurance whose benefits are paid immediately after death, or in other words, uses continuous time. Then, there is endowment life insurance whose benefits are paid at the end of the year of death or at discrete times.

Defined as  $T$  is the remaining life span (future life time) of someone who participates in a life insurance product. In practice, life insurance companies obtain information about the probability distribution of  $T$  from discrete-time mortality tables. If  $K$  represents the complete remaining age of the insured on the policy, then the probability distribution of  $K$  is a function of  $T$  (Bowers, 1996). The benefit function ( $b_{k+1}$ ), which is the amount of benefit payments where the index indicates the remaining age of the policyholder. Discount function ( $v_{k+1}$ ), which is the discount factor of the interest rate set for the period from the time of payment return to the time the policy is issued. The present value at the time the policy is issued from the payment of insurance benefits is denoted by ( $z_{k+1}$ ), namely:

$$z_{k+1} = b_{k+1} v_{k+1} \quad (4)$$

## 2.6. $n$ Years Term Insurance

$n$  years Term Insurance benefit a provides of one unit at the end of the death year.

$$b_{k+1} = \begin{cases} 1 & , k = 0, 1, 2, \dots, n-1 \\ 0 & , k = n, n+1, \dots \end{cases} \quad (5)$$

$$v_{k+1} = \begin{cases} v^{k+1} & , k = 0, 1, 2, \dots, n-1 \\ 0 & , k = n, n+1, \dots \end{cases}$$

The actuarial present value for this insurance using the equivalence premium principle is given as follows.

$$A_{x:\overline{n}|}^1 = E[Z] = \sum_{k=0}^{n-1} v^{k+1} ({}_k p_x) (q_{x+k})$$

With  ${}_k p_x$  being probability that someone currently aged  $x$  years will live for  $k$  more years, and  $q_{x+k}$  being the probability that someone currently aged  $(x+k)$  years will die one year later.

## 2.7. $n$ Years Pure Endowment Insurance

$n$  years pure endowment insurance, which provides benefits at the end of the death year or benefits provides to the insured who is still alive at that time, is

$$b_{k+1} = \begin{cases} 0 & , k = 0, 1, 2, \dots, n-1 \\ 1 & , k = n, n+1, \dots \end{cases}$$

$$v_{k+1} = \begin{cases} 0 & , k = 0, 1, 2, \dots, n-1 \\ v^n & , k = n, n+1, \dots \end{cases}$$

Actuarial present value for this insurance using equivalence premiums principle is as follows.

$$A_{\frac{1}{x:\overline{n}|}} = I(v^n) {}_n P_x$$

Based on the descriptions (4) and (5), the actuarial present value for endowment life insurance is obtained as follows:

$$\begin{aligned} A_{x:\overline{n}|} &= A_{1_{x:\overline{n}|}} + A_{\frac{1}{x:\overline{n}|}} \\ &= \left( \sum_{k=0}^{n-1} v^{k+1} ({}_k P_x)(q_{x+k}) \right) + v^n {}_n P_x \end{aligned} \tag{6}$$

### 2.8. Benefit Structure with the Annual Ratchet Method

One of these methods used in unit-linked life insurance contracts is the method indexing with a participation rate. The method indexing used is the annual ratchet. The Annual Ratchet method indexing is divided into two types, namely Simple Ratchet and Compound Ratchet. In Simple Ratchet, the profit is only calculated once for one period against a fixed principal amount. Whereas in the Compound Ratchet, previous gains will be included in calculating the next gains. Thus, from period to period, the profits become larger, which is an accumulation of the previous profit additions. The benefit structure in an investment using the Compound Ratchet method is mathematically defined as follows.

$$\begin{aligned} B_{CR}(t) &= \max(\beta(1+g)^t, CR) = CR + \max(\beta(1+g)^t - CR, 0) \\ CR &= \prod_{t=1}^n (1 + \min(\max(\alpha(R(t)-1), f), c)) \end{aligned} \tag{7}$$

If  $\alpha$  represents the participation rate, which is the profit that policyholders will obtain if the invested stocks experience a significant increase. And  $(R(t) - 1)$  is the profit obtained at time  $t$ .  $\beta(1+g)^t$  is the lower bound to protect against losses from market declines. The percentage of the return is expressed by  $\beta$ , while  $(g)$  is the guaranteed rate interest. The guaranteed interest rate is the minimum profit guaranteed by the insurance companies, regardless of market changes. The cap interest rate ( $c$ ) is the maximum benefit limited by a price level called the cap price. If a claim occurs, the beneficiary or insured will receive the maximum benefit equal to the cap value at that time, which was agreed upon at the beginning of the contract. If a claim occurs when the stock price skyrockets, the insurance company will not be at a loss because there is a maximum benefit limit, which is the cap. The floor interest rate ( $f$ ) is the minimum benefit limited by a price level called the floor price. If a claim occurs when the stock price plummets, the heirs or the insured are not disadvantaged because there is a minimum benefit limit, which is the floor price. Therefore, it can be said that the benefit of the cap and floor benefits is advantageous for both parties.

### 2.9. Annual Ratchet

Annual Ratchet is a method used to calculate the premium value and the benefit value of unit-linked life insurance. The Annual Ratchet method is an indexing method based on the participation rate evaluated year by year (annual). The annual ratchet this method consists of two types of designs, namely simple ratchet and compound ratchet [6]. The investment return on a simple ratchet is only calculated once for one period against a fixed principal amount, whereas in a compound ratchet, the investment return from the previous period is included to calculate the return for the next period. [7] Mentions the advantages of the Annual Ratchet method compared to other methods, namely, the method considers this value of premiums and benefits based on the existence a minimum interest rate (floor) and a maximum interest rate (cap), thereby protecting both the company and the policyholders from the risk of loss.

The benefit structure in an investment contract at time  $n$  with a Compound Ratchet is mathematically defined as:

$$\begin{aligned} B_{CR} &= \max(\beta(1+g)^t, CR) \\ &= CR + \max(\beta(1+g)^t - CR, 0) \end{aligned} \tag{8}$$

where,

$$CR = \prod_{t=1}^n (1 + \min(\max(\alpha(R(t) - 1), f), c))$$

with,

- ( $R(t)-1$ ) = The profit obtained at time  $t$
- $\alpha$  = Participation rate
- $c$  = Upper limit interest rate (cap)
- $f$  = Lower bound interest rate (floor)
- $\beta$  = Minimum guarantee
- $g$  = Guaranteed interest rate
- $n$  = Investment contract term

The net single premium for for a unit-linked endowment life insurance contract using the Annual Ratchet method is as follows:

$$P(A_{x:\overline{n}|}) = \left[ \sum_{k=0}^{n-1} V(k, k+1)({}_k p_x)(q_{x+k}) \right] + V(n-1, n) {}_n p_x$$

where:

$P(A_{x:\overline{n}|})$  = The premium of endowment life insurance for a person aged  $x$  years with a period  $n$ -years

${}_k p_x$  = The chance of someone aged  $x$  living for another  $k$  years

${}_n p_x$  = The probability of a person aged  $x$  dying within the next  $n$  years

$q_{x+t}$  = The probability of someone aged  $x$  to  $t$  years dying within the next year

## 2.10. Black Scholes

Black Scholes is a method that can be used to calculate the premiums and benefits of unit-linked life insurance. The advantage of this model is that it is a well-accepted call option valuation model that is well-received in the financial community because the option price which is obtained from this model's calculations is a "fair" or accurate value. The price of the call option  $C(S, t)$  at the strike price  $K$  and expiration date  $T$ . If at the time of expiration  $S > K$ , then the call option will have value, and the call option buyer can exercise their right to buy the stock at the difference of  $S - K$ . On the other hand, if at the expiration date  $S < K$ , the buyer does not need to exercise their right; in this situation, the option buyer only loses the amount paid to purchase the option contract (premium) [15]. The value of the option price is defined as follows:

$$C(S, t) = \text{maks}(S - K, 0) \quad (11)$$

From Equation (11) shows that the stock price does not move  $S = 0$ , then Equation (11) becomes  $C(0, T) = 0$ , but as  $S \rightarrow \infty$ , the option will be executed more. the Black-Scholes equation to calculate the value of a call option is as follows:

$$\begin{aligned} C &= Sx\Phi(d_1) + \frac{K}{e^{rt}}\Phi(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + (r + 0,05(\sigma^2))t}{(\sigma)(\sqrt{t})} \\ d_2 &= d_1 - (\sigma)(\sqrt{t}) \\ d_2 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \end{aligned}$$

The Black-Scholes model assumes that stock used as the underlying asset does not pay dividends. Because stock prices tend to rise, the chances of experiencing a price increase are greater than a decrease. The expected price increase causes the stock price at maturity to follow a lognormal distribution. In the Black-Scholes formula, there is a factor  $\ln\left(\frac{S}{K}\right)$  which indicates that  $d$  follows a normal distribution.  $\ln\left(\frac{S}{K}\right)$  is normally distributed, which means that  $S$  is lognormally distributed. Lognormal means that the natural logarithm of the underlying asset price follows a normal distribution.

According to Stampfli, the final payment for a dividend-free call option maturing at time  $T$  with  $K$ =strike price, stock price  $S_0$ , risk-free interest rate  $r$ , and volatility  $\sigma$ . Then the call price  $C$  is as follows:

$$C = e^{-rt} E[\text{maks}(S_T - K, 0)] \quad (12)$$

Unit-linked life insurance with a minimum guarantee provides benefits to policyholders equal to the value of the investment at the time of the claim or at least the minimum guarantee agreed upon initially. Assume that the amount of the guarantee at time  $T$ , denoted as  $G_T$ , is formulated with

$$G_T = S_0 e^{rT},$$

with  $r = \ln(1 + i)$

Equation (10) can be seen as the payoff of European call options with  $G_T$  as the strike price. Thus, the option price can be determined using risk-neutral valuation, where the utility value at time zero is 0.

$$\begin{aligned} U(T) &= E[v_T[S_T, G_T]] \\ &= \{E^{-rT} G_T + e^{-rT} [[S_T, G_T]]\} \end{aligned} \quad (13)$$

Based on the equation for European call options, then:

$$U(T) = E[v_T[S_T, G_T]]$$

$$= \{e^{-rT} G_T + e^{-rT} [[S_T, G_T]]\}$$

$$= S_0 + S_0 N(d_1) - G_T e^{-rT} N(d_2)$$

Thus, the equation for the net single premium of a Endowment unit-linked life insurance with a minimum guarantee is as follows:

$$P(A_{x:\overline{T}|}) = \sum_{t=1}^T \pi(t)(S_0 + S_0 \Phi(d_1) - G_T e^{-rT} \Phi(d_2)) \tag{14}$$

With  $\Phi$  being the standard cumulative distribution function.

$$\pi(t) = \begin{cases} {}_{t-1}p_x q_{x+t-1} = {}_{t-1}q_x, & t = 1, 2, \dots, T - 1 \\ {}_{T-1}p_x q_{x+T-1} + {}_T p_x = {}_{T-1}p_x, & t = T' \end{cases}$$

where:

- $P$  = The amount of the single net premium
- $t - 1p_x$  = The probability of a person aged  $x$  years dying within the next  $t$  years.
- $q_{x+t-1}$  = The probability of someone aged  $x$  to  $t$  years dying within the next year
- $S_0$  = Initial stock price
- $\Phi$  = Standard cumulative distribution function
- $G_T$  = Minimum Warranty
- $T$  = Due date
- $\sigma$  = Volatility of stock prices

### 3. Results and Discussion

The data used in this study consists of two types of data, namely historical data and simulation data. The historical data used consists of the daily closing stock prices of PT. Telkom Indonesia (Persero) Tbk for the period from December 20, 2021, to December 20, 2022. The daily closing stock price data of PT. Telkom Indonesia (Persero) Tbk used 247 over the course of 1 year.

The simulation data description is a man (the insured) who is ( $x$ ) 30 years old and will purchase an Endowment unit-linked life insurance contract. The duration is set for ( $n$ ) 5 years with the chosen investment type being Telkom Indonesia shares. The shares were purchased during the period from December 20, 2021, to December 20, 2022, at an initial share price of Rp 3,490 per share. The shares to be purchased amount to 2,500 per share.

The man wants to know the comparison of the net single premium value of a Endowment life insurance k-year unit link using the Annual Ratchet and Black Scholes methods with a cap interest rate (upper limit) of 15% and a floor interest rate (lower limit) of 0%. In the insurance policy contract, it is known that the participation rate is 79,57% with a risk-free interest rate equal to 8%. The minimum guarantee set by the company is 90%, with a guaranteed interest rate of 5%.

Life insurance companies calculate all their life insurance premiums based on the mortality table. Therefore, the calculation of the premium that the man will receive using the Annual Ratchet and Black Scholes methods will also be assisted by the Mortality Table. In this study, the table used is the Indonesian Mortality Table of 2019.

#### 3.1. Determining the Premium for Dual Benefit Life Insurance

Based on Equation (6), the calculation of the single premium value for traditional life insurance is as follows:

$$A_{30:5} = \sum_{k=0}^{5-1} v^{k+1} k p_{30} q_{30+k} + v^5 P_{30}$$

$$= 1.00293$$

Meanwhile, the annual premium value is as follows:

$$\ddot{a}_{30:5} = \sum_{k=0}^{5-1} v^k k p_{30}$$

$$= 4.30200$$

The value of the Endowment life insurance premium can be calculated using Equation (6), with  $x$  as the insured's age, which is 30 years, and  $k$  as the coverage period, which is 5 years. The probability of someone dying according to the Indonesian Mortality Table 2019, the value of the Endowment life insurance premium is 1.00293 and the annual premium is 4.30200.

### 3.2. Value of Stock Return Volatility

To calculate the daily stock price return using Equation (1)

For  $t = 1$ ,

$$\begin{aligned} R(t) &= \ln \ln \frac{S(t)}{S(t-1)} \\ &= \ln \ln \frac{4160}{4080} \\ &= 0.01942 \end{aligned}$$

Next, the average return value is calculated. The calculation of the average daily stock return is performed using Equation (2) as follows.

$$\begin{aligned} \overline{R(t)} &= \frac{1}{n} \sum_{t=1}^n R(t) \\ &= -0.00038 \end{aligned}$$

Thus, the estimated average return value is 0.0008. With  $(R_t)$  as the average daily stock price return,  $(\overline{R_t})$  as the daily stock price return, and  $n$  as the number of return data. To calculate volatility value, the standard deviation of the annual data return stock uses Equation (3) as follows.

$$\begin{aligned} \sigma &= \sqrt{247 \times \frac{\sum_{t=1}^{247} 0,00038}{247 - 1}} \\ &= 0.01609 \end{aligned}$$

Based on Telkom stock data for the year 2021, the annual stock return volatility is 0.01609.

### 3.3. Stock Return Normality Test

Return is normally distributed Normal is an assumption of net single premium model for a two-benefit life insurance unit-linked  $k$ -year. Therefore, it is necessary to test the normality of the daily stock data return of Telkom Indonesia (Persero) Tbk from December 20, 2021, to December 2022. The Normality Test on the stock return data used in this study is the Kolmogorov-Smirnov test using SPSS software. Here are the results of the normality test on the TLKM stock return data.

1. Hypothesis

$H_0$  : The return data of Telkom Indonesia (Persero) Tbk is normally distributed

$H_1$  : The return data of Telkom Indonesia (Persero) Tbk is not normally distributed

2. Significance level  $\alpha = 0.05$

3. Test Criteria:

If the Sig value  $> \alpha$ , then the  $H_0$  hypothesis is accepted.

4. Decision: from the Kolmogorov-Smirnov test result calculation on Telkom Indonesia (Persero) Tbk stock return data, as seen from the SPSS output, the value Sig  $> \alpha$  is obtained, which is (0.053  $>$  0.05), therefore  $H_0$  is accepted. Which means that the data return of Telkom Indonesia (Persero) Tbk shares are normally distributed.

### 3.4. Determining the Net Single Premium for Unit-Linked Endowment Life Insurance

Before obtaining the benefit structure value, the net single premium of unit-linked Endowment life insurance with a minimum guarantee, upper benefit limit (floor), and lower benefit limit (cap) is first determined using the Annual Ratchet method.

**Table 1.** Calculation of Net Single Premium of Unit Linked Endowment Life Insurance with Minimum Guarantee, Lower Benefit Limit (floor), and Upper Benefit Limit (cap) using Metode Annual Ratchet

$x$	$k$	$q_x$	$p_x$	${}_k p_x$	${}_k p_x q_{x+k}$	$v_{CR}$	$B_{CR}$	Compound
30	0	0.00075	0.99925	0.99925	0.00075	1.00000	1.06628	0.00080
31	1	0.00081	0.99919	0.99844	0.00081	0.92312	1.13694	0.00085
32	2	0.00087	0.99913	0.99757	0.00087	0.85214	1.21230	0.00090
33	3	0.00093	0.99907	0.99664	0.00093	0.78663	1.29264	0.00094
34	4	0.00099	0.99901	0.99566	0.99566	0.72615	1.37831	0.99651

Thus, the premium index value is obtained as  $0.00080 + 0.00085 + 0.00090 + 0.00094 + 0.99651 = 1.00000$ . Next, we find the minimum guarantee values based on Equation (7) Thus, the result of the minimum warranty value for time  $t$  is 0.9000.

Next, the values in the simulation data description are entered into the net single premium equation of unit-linked endowment life insurance with minimum guarantee using the Annual Ratchet method. The premium index value for this insurance is 1, so the participation rate is 79.57%. The amount of net single premium that payable by the insured is as follows:

$$\begin{aligned}
 \text{premium} &= \text{Rp } 3,490 \times 2,500 \text{ per share} \times 1.00 \\
 &= \text{Rp } 8,725,000
 \end{aligned}$$

### 3.5. Determining the Benefit Structure with Participation Rate Using Compound Ratchet

Next, the benefits that will be received by the insured are stated in Equation (10). this insurance contract for a period of 5 years starting from December 20, 2021, to December 2022.

Here is a recap of the stock prices and the benefits that the insured will receive if the insured passes away within the insurance contract period.

**Table 2.** Results of the Diwa Endowment Unit Linked Insurance Benefits with Compound Ratchet and Minimum Guarantee for Women Aged 30 Years

Year	Stock Price	Return	Compound Ratchet ( $\alpha = 79,57\%$ )	Minimum warranty	Compound Ratchet
0	3,490	0.0000	1.0000	0.9000	8,725,000
1	4,170	0.1948	1.1500	0.9450	10,033,750
2	3,820	-0.0839	1.1500	0.9923	10,033,750
3	3,970	0.0393	1.1859	1.0419	10,347,271
4	2,690	-0.3224	1.1859	1.0940	10,347,271

According to Table 2 above, it can be seen that in the first, second, third, and fourth years, the compound value obtained exceeds the guaranteed value, so the benefit that the insured will receive is a proportion of the profit value from the initial stock price and the number of shares. Thus, the profit obtained is  $\text{Rp } 10,347,271 - \text{Rp } 8,725,000 = \text{Rp } 1,622,271$ , the proportion of profit obtained by the customer is  $79.57\% \times \text{Rp } 1,622,271 = \text{Rp } 1,290,919$  and the proportion of profit obtained by the company is  $1 - 79.57\% = 20.43\%$  or  $\text{Rp } 331,352$ .

**Table 3.** Comparison of the Benefits of Dual Benefit Unit Linked Life Insurance with Compound Ratchet and Minimum Guarantee for Men and Women Aged 30, 35, 40 Years

Age	Man			Woman		
	Profit	Customer	Broker	Profit	Nasabah	Broker
30	1,622,271	1,290,919	331,352	1,622,053	1,289,847	332,206
35	1,622,778	1,293,413	329,366	1,622,353	1,291,323	331,030
40	1,623,817	1,298,518	325,298	1,622,900	1,294,001	328,889

According to Table 3 above, it can be seen that the profits obtained by men aged 30 and 35 are higher than the profits obtained by women aged 30 and 35, which are  $\text{Rp } 1,622,271$  and  $\text{Rp } 1,622,778$  respectively. However, for men aged 40, the profits obtained are lower than the profits obtained by women aged 40, which are  $\text{Rp } 1,623,817$ . Next, the profits obtained by male clients at ages 30, 35, and 40 are higher than the profits obtained by female clients at ages 30,

35, and 40, which are Rp 1,290,919, Rp 1,293,413, and Rp 1,298,518 respectively. On the other hand, the profits obtained by brokers/companies from male clients at ages 30, 35, and 40 are lower than the profits obtained by brokers/companies from female clients at ages 30, 35, and 40, which are Rp 331,352, Rp 329,366, and Rp 325,298 respectively.

### 3.6. Determining the Value of Unit Link Endowment Life Insurance Using the Black Scholes Model

First, the minimum guarantee amount provided by the insured party will be determined using Equation (13).

**Table 4.** Minimum Warranty

T	$G_T$
1	3,780.6719
2	4,095.5529
3	4,436.6595
4	4,806.1759
5	5,206.4682

Next, the value of the standard cumulative distribution normal function will be calculated with risk-free a interest rate ( $r$ ) of 8%, and the minimum guarantee ( $G_T$ ) as the strike price. The value of the standard normal cumulative distribution function can be calculated using software Microsoft Excel. Before determining the value of the standard normal cumulative distribution function, the values of  $d_1$  and  $d_2$  are first calculated using Equation (12).

Based on the values of  $d_1$  and  $d_2$  obtained at  $T = 1$ , the values of  $\Phi(d_1)$  and  $\Phi(d_2)$  can be calculated, and the subsequent calculation values can be seen in Table 5.

**Table 5.** Value of the Standard Normal Distribution Cumulative Function

$d_1$	$d_2$	$\Phi(d_1)$	$\Phi(d_2)$
0.0080	-0.0080	0.5032	0.4968
0.0228	-0.0147	0.5091	0.4941
0.0418	-0.0198	0.5167	0.4921
0.0644	-0.0241	0.5257	0.4904
0.0899	-0.0279	0.5358	0.4889

In Table 5, the values of the normal distribution function obtained from Equation (12) over a period of 5 years are known, namely 0.5032 for the distribution normal function value of  $d_1$  and 0.4968 for the distribution normal function value of  $d_2$  in the first year, continuing until the fifth year with values of 0.5358 for the normal distribution function of  $d_1$  and 0.4889 for the normal distribution function of  $d_2$ .

### 3.7. Determining the Value of Dual Benefit Unit Link Life Insurance Using the Black Scholes Model

Based on Equation (14), the price of the Black Scholes option can be determined, where in this case it represents the value at time 1 as follows:

**Table 6.** The Value of Unit Link Endowment Life Insurance Using the Black-Scholes Model

T	$\Phi(d_1)$	$\Phi(d_2)$	$U(T)$
1	0.5032	0.4968	3,548.6975
2	0.5091	0.4941	3,691.4260
3	0.5167	0.4921	3,904.8014
4	0.5257	0.4904	4,190.5361
5	0.5358	0.4889	4,554.5749

Thus, based on Table 6, the benefit value over a period of T years, specifically during the 5-year coverage, is 3,548.6975 at the end of the first year and continues to 4,554.5749 at the end of the fifth year.

### 3.8. Determining the Net Single Premium for Unit Link Endowment Life Insurance Using the Black Scholes Model

Next, based on the Indonesian Mortality Table 2019, the values of  ${}_{k-1}p_x q_{x+k-1}$  from Table 7 can be determined, resulting in the following values for the Endowment unit-linked life insurance premium.

**Table 7.** Premium Value of Unit Link Endowment Life Insurance Using the Black scholes Model

$x$	$U(T)$	${}_{t-1}p_x q_{x+t-1}$	${}_{t-1}p_x q_{x+t-1} U(T)$
30	3,548.6795	0.00075	2.6615
31	3,691.4260	0.00081	2.9901
32	3,904.8014	0.00087	3.3972
33	4,190.5361	0.00093	3.8972
34	4,554.5749		

Next, based on Table 7, the premium value of endowment unit-linked life insurance is Rp 4,551.1465.

The net single premium value of the endowment unit-linked life insurance is as follows:

$$\begin{aligned} \text{Premium} &= (4,551.1465) \times (2,500) \text{ per share} \\ &= \text{Rp } 11,377.866 \end{aligned}$$

Thus, the net single premium value of endowment unit-linked life insurance for a 30-year-old male insured, calculated using the Black-Scholes option pricing model, is Rp 11,377.866 over a 5-year insurance period.

**Table 8.** The Value of Dual Benefit Unit Link Life Insurance Premiums with Varying Ages (30, 35, 40 years)

Insurance Period $x$	$A_{x:T }$	Premium Value
30	4,551.1465	Rp 11,377.866
35	4,549.0290	Rp 11,372.572
40	4,544.6137	Rp 11,361.534

It can be seen in Table 8 that as the insured person's age increases, the premium value decreases (becomes cheaper). This happens because it is influenced by the increasing probability of a person's death (Mortality Table) with age.

### 3.9. Comparison of Net Single Premium Results of Endowment Unit Life Insurance Using the Annual Ratchet Method

The known information is then incorporated into the net single premium of the Endowment unit-linked life insurance using the Annual Ratchet method and the Black Scholes model. The net single premium for the endowment unit-linked life insurance was obtained as shown in Table 9. The known information is then incorporated into the net single premium of the endowment unit-linked life insurance using the Annual Ratchet method and the Black Scholes model. The net single premium for the endowment unit-linked life insurance was obtained as shown in Table 9.

**Table 9.** Net Single Premium Results of Endowment Unit Linked Life Insurance with Different Ages and Varying Interest Rates (6%, 7%, 8%)

$x$	Net Singles Premium	
	Annual Ratchet	Black Scholes
30	8,725.000	11,377.866
35	8,724.863	11,171.230
40	8,724.837	11,257.432

As shown in Table 9, the net single premium for an endowment unit-linked life insurance in the Annual Ratchet method is higher, at Rp 8,725.000, compared to the premium in Black Scholes method, which is Rp 11,377.866. Furthermore, if the insured's age increases and risk-free interest rates decrease, the premium value becomes lower (cheaper). This happens because it is influenced by a person's mortality risk (Mortality Table) and risk-free interest rate. The risk-free interest rate is a variable of which is a discount function. The smaller the risk-free interest rate, the higher the value of the discount function, and thus the higher the contract value of the investment. Conversely, the higher the risk-free interest rate, the smaller the value of the discount function, and thus the lower the value of the investment contract.

In summary, the difference between the Annual Ratchet and Black-Scholes premiums arises from their contrasting economic and actuarial foundations. The Annual Ratchet approach applies a risk-limiting mechanism that stabilizes returns through caps and floors, resulting in lower exposure to market volatility and, consequently, lower premiums. In contrast, the Black-Scholes model adopts a fully market-consistent valuation by treating the insurance benefit as a financial option sensitive to stock price fluctuations, the risk-free rate, and the time value of money, leading to higher premiums. Therefore, the choice between these models reflects an insurer's strategic orientation—whether prioritizing capital protection and stability, as in the Annual Ratchet, or embracing a more dynamic, market-based assessment of investment risk, as represented by the Black-Scholes framework.

#### 4. Conclusions

Based on the analysis and discussion conducted, this study concludes that the difference in premium values between the Annual Ratchet method and the Black-Scholes model stems from the fundamental differences in their underlying economic and actuarial frameworks. The Annual Ratchet method produces lower premiums because it incorporates a built-in risk control mechanism through the limitation of investment gains and losses (cap and floor), thereby reducing exposure to market volatility. In contrast, the Black-Scholes model results in higher premiums as it values insurance benefits as a full financial option influenced by stock price volatility, the risk-free interest rate, and the time value of money. These findings indicate that the Annual Ratchet approach reflects a conservative and stability-oriented pricing strategy, while the Black-Scholes model represents a market-consistent valuation approach that takes into account greater financial uncertainty. Therefore, the selection of a premium valuation model should be aligned with the insurer's risk profile and product objectives. Companies that prioritize stability and value protection may prefer the Annual Ratchet method, whereas those aiming to align pricing with market dynamics may adopt the Black-Scholes model. Overall, this study reinforces the importance of selecting an appropriate premium valuation model to maintain a balance between profitability, competitiveness, and financial sustainability within the insurance industry.

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