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Value At Risk Prediction For The GJR-GARCH Aggregation Model

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ABSTRACT

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Volatility is the level of risk faced due to price fluctuations. The greater the volatility brings, the greater the risk. We need a measure such as Value at Risk (VaR) and volatility modeling to overcome this. The most frequently used volatility model in the financial sector is GARCH. However, this model is still unable to accommodate the asymmetric nature, so the GJR-GARCH model was developed. In addition, this study also used aggregation returns with two assets in them. This study aimed to determine the VaR prediction for the GJR-GARCH(1.1) aggregation model and its comparison with the GARCH(1.1) aggregation model. The results obtained indicate that the prediction of volatility using the GJR-GARCH(1.1) aggregation model is more accurate than the GACRH(1.1) aggregation model because it has a correct VaR value that is close to the given confidence level.

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1. Introduction

Return modeling in finance is more widely used than price. It is based on two reasons. First, the return has a more complete and scale-free investment summary, and the return is more stationery [1]. However, information on return behavior alone is not sufficient to analyze financial time series data, so another measure is needed, namely volatility. Volatility is a measure that describes changes in the return value and is expressed as a conditional variance. In general, volatility expresses the level of risk faced due to price fluctuations. The greater the volatility brings, the greater the risk.

Volatility becomes interesting to observe because volatility is not directly observed. It is because many things affect the amount of volatility [2]. Therefore, volatility modeling is needed to determine an asset's future value. One of the well-known volatility models is the GARCH model introduced by [3]. The GARCH model has undergone many modifications to cover its weaknesses, including the GJR-GARCH model. The GJR-GARCH model was introduced by [4]. It can predict responses to positive shocks or negative shocks. Responses to positive or negative shocks are changes in positive or negative information [5][6]. It is also in line with the opinion expressed by [7] that a good volatility model is a model that can accommodate the empirical nature of returns and volatility.

Volatility modeling can calculate the maximum loss of a return associated with a measure of risk. Value at Risk (VaR) is a risk measure that can predict future losses. Prediction of risk measures, in general, is not limited to one type of loss data. In this study, we will discuss risk measures for loss aggregation, where the components of this aggregation are built from two types of loss data. In addition, the GJR-GARCH(1,1) and GARCH(1,1) models will also be used, with their innovations being normally distributed and student-t. The purpose of this study was to determine the VaR prediction for the GJR-GARCH(1.1) aggregation model and its comparison with the GARCH(1.1) aggregation modell.

2. Research Methods

2.1 Aggregation Components

The aggregation component in this study is loss data which is expressed in the form of a negative return from an asset and is defined as follows:

$$\mathrm{X}_{\mathrm{t,i}} = -\left(\frac{P_{t,i}-P_{t-1,i}}{P_{t-1,i}}\right) \tag{1}$$

where $P_{t,i}$ is the price of asset i at time t and i = 1, 2. This aggregation with two assets is an interesting study. Its simplicity makes the analysis more broader and clearer. Especially in terms of the dependence between the two constituent assets. However, this study will not examine its dependence because the components of the assets that make up the aggregation this time are assumed to be independent. For instance, $X_{t,1}$ and $X_{t,2}$ return from assets 1 and 2, respectively, with the proportion a for asset 1 and (1 - a) for asset 2 so that the aggregation for ${X^+}_t$ which is compiled from the return on assets is formulated as follows:

$${X^+}_t = aX_{t,1} + (1-a)X_{t,2} \tag{2}$$

2.2 Aggregation in Two Processes GJR-GARCH(1,1)

If ${X^+}_t$ namely the random variable that represents the aggregation return value to t with the error in the GJR-GARCH(1,1) model for the normal distribution (GJR-GARCH N) and the Student's-t distribution (GJR-GARCH T) as follows:

$${X^+}_t = \sigma_{\mathrm{t,p}}\,\varepsilon_{t,p}\,, \quad {\sigma^{2+}}_t = \alpha_0 + (\alpha_1 + \gamma_1\, I_{t-1}){X^2}_{t-1,p} + \beta_1 {\sigma^2}_{t-1}, \quad \varepsilon_{t,p} \sim N(0,1), \varepsilon_{t,p} \sim t_d \tag{3}$$

where $\alpha_0 > 0, \alpha_1 > 0, \beta_1 \geq 0, \alpha_1 + \gamma_1 \geq 0$ and I_{t-1} is an indicator function that has a value of 1 for ${X^+}_{t-1} < 0$ and is 0 for ${X^+}_{t-1} > 0$. The indicator function in the GJR-GARCH model can be used to predict an event that occurs suddenly (asymmetric response). This is what makes the weaknesses in the GARCH(1,1) model overcome.

In addition, the variance in the GJR-GARCH(1,1) model depends on the parameter α_1, γ_1, β_1 so that to fulfill the assumption of stationarity, the conditions for the GJR-GARCH(1,1) model parameters are $\alpha_0 > 0, \alpha_1 > 0, \beta_1 \geq 0, \alpha_1 + \gamma_1 \geq 0, \alpha_1 + \gamma_1 + \beta_1 \geq 0$. The following is the result of the stationary simulation on the GJR-GARCH(1,1) model with the parameters α_0 = 0.1, α_1= 0.2, β_1 = 0.1, and γ_1 = 0.3. From the simulation results, it can be seen that the mean and variance converge to zero.

The method used to estimate the parameters of the GJR-GARCH(1,1) model is the maximum likelihood method. If $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_\mathrm{n}$ a random variable whose independent and identically distributed (iid) with pdf $\mathrm{f}(\mathrm{x};\theta)$, the estimation will be determined by the parameters θ. The first step is to determine the log-likelihood function as follows:

$$L = (\theta; x) = \prod_{i=1}^{n} f(x_i; \theta) \tag{4}$$

The next step is to find the log-likelihood function by

$$l(\theta) = \log(L(\theta)) = \sum_{i=1}^{n} log\ f(x_i; \theta) \tag{5}$$

Furthermore, parameter estimates are obtained by maximizing the likelihood function of θ, where $\frac{\partial l(\theta)}{\partial \theta} = 0$.

2.3 Risk Measures in the GJR-GARCH Aggregation Model(1,1)

The most popular risk measure in finance is Value-at-risk (VaR). VaR is the greatest risk value that can occur at a certain confidence level. If $X^{+}{}_{t}$ is the value of aggregation loss at time t and is normally distributed and student-t with model parameter vector θ. To predict losses at time t+1 with a confidence level is as follows:

$$P(X^{+}{}_{t+1,p} \leq VaR_{\alpha}(X^{+}{}_{t}|X^{+}{}_{t-1,p}); \vec{\theta}) = \alpha$$

$$F_{X^{+}{}_{t+1,p}}(VaR_{\alpha}(X^{+}{}_{t}); \vec{\theta}) = \alpha$$

$$VaR_{\alpha}(X^{+}{}_{t}) = F^{-1}{}_{X^{+}{}_{t+1,p}|X^{+}{}_{t}}(\alpha;\ \vec{\theta})$$

If the error follows a normal distribution and the t distribution with each parameter is $\vec{\theta} = (\mu, \sigma)$ and $\vec{\theta} = (\mu, \sigma, \upsilon)$ so that VaR on the level of trust α can be stated as follows:

$$\widehat{VaR}_{\alpha}^{N}(X^{+}{}_{t}) = \hat{\mu} + \Phi^{-1}(\alpha)\hat{\sigma}^{2}{}_{t,p} = \Phi^{-1}(\alpha)\sqrt{\hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\gamma}_1\ {}_{I_{t-1}})X^{2}{}_{t-1,p} + \hat{\beta}_1\sigma^{2}{}_{t-1,p}}$$

$$\widehat{VaR}_{\alpha}^{T}(X^{+}{}_{t}) = \hat{\mu} + t_{c,\upsilon}\hat{\sigma}^{2}{}_{t,p} = t_{c,\upsilon}\sqrt{\hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\gamma}_1\ {}_{I_{t-1}})X^{2}{}_{t-1,p} + \hat{\beta}_1\sigma^{2}{}_{t-1,p}}$$

3. Results And Discussion

The data used in this aggregation component was data on losses from shares of PT. Astra International Tbk. (ASII) and PT. Astra Agro Lestari Tbk. (AALI) (source: www.yahoofinance.com) for January 2, 2020 to May 31, 2022. The following are descriptive statistics of the two stock price returns.

Table 1. ASII and AALI Descriptive Statistics of Return

Statistics	Sample	Mean	Variance	Skewness	Kurtosis
Return ASII	585	0.000115	0.000623	0.027881	5.615186
Return AALI	585	-0.000267	0.000819	-0.000535	9.681617

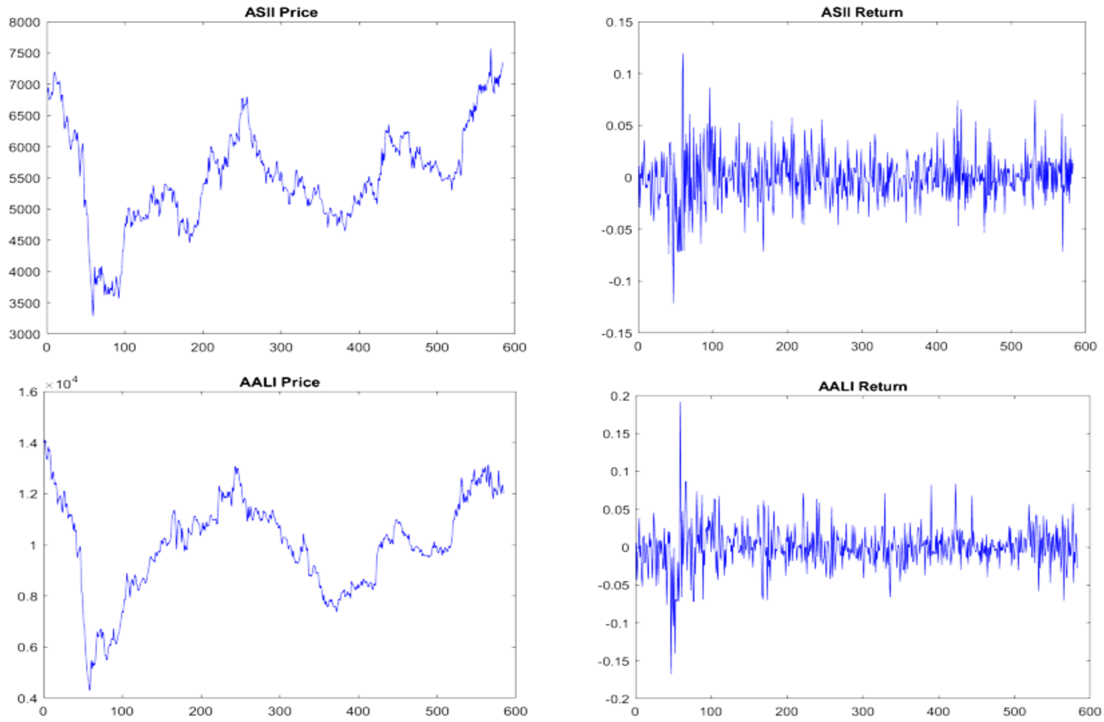

Figure 1. ASII and AALI prices and returns

Based on Table 1, it can be concluded that the distribution of the two returns is not normal. The kurtosis value shows values above 3, namely 5. 615186 and 9. 681617. It means that this return distribution has a thick tail, or it can also be said that the tail of this distribution is slower to zero compared to the normal distribution. Likewise, when viewed from the skewness value of the data. The value is greater than 0 and is negative. It shows that the distribution of the return data is skewed to the right and skewed to the left. In addition, based on Figure 1, it can be seen that the value of the variance is not constant. It shows that both assets have fluctuating volatility values.

Next, we will determine the parameter estimates from the GARCH(1,1) and GJR-GARCH(1,1) models using aggregation return data from ASII and AALI. By using the Matlab program, the estimation results are obtained as follows:

Table 2. Aggregation Model Parameter Estimation

Model	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\hat{\gamma}_1$
GARCH (N)	0.000057	0.112582	0.828179	-
GARCH (T)	0.000039	0.092127	0.865562	-
GJR-GARCH (N)	0.000059	0.056319	0.836278	0.089101
GJR-GARCH (T)	0.000037	0.004760	0.880418	0.062939

Based on Table 2, it is found that the GARCH (T) and GJR-GARCH (T) models have a value $\hat{\alpha}_0$ which is small when compared to GARCH (N) and GJR-GARCH (T). The value $\hat{\alpha}_0$ the model shows the tendency of returns to approach the average value from time to time. The value $\widehat{\alpha_1}$ shows the amount of the influence of the return value of the previous time on the value of today's return. The values of $\hat{\beta}_1$ shows persistence on volatility, and the value of $\hat{\gamma}_1$ shows an asymmetrical component. In addition, it can also be seen that the value of $\widehat{\alpha_1}$ and $\hat{\beta}_1$ each of which ranges from 0.004760 to 0.112582 and 0.28179 to 0.880418. This shows that the effect of previous time volatility is significantly affects on the current volatility value. Meanwhile, the GJR-GARCH model has additional parameters in the model so that the model can capture the asymmetric nature so that it will be better in modeling volatility [8].

Furthermore, the VaR prediction will be made from the GARCH(1,1) and GJR-GARCHa(1,1) aggregation model returns with α= 99%, α = 95% and α = 90% as in the following Table 3.

Table 3. The Prediction of Aggregation Model VaR

Model	$VaR_{99\%}^{t+1}$	$VaR_{95\%}^{t+1}$	$VaR_{90\%}^{t+1}$
GARCH (N)	0.0176	0.0124	0.0097
GARCH (T)	0.2727	0.1751	0.1315
GJR-GARCH (N)	0.0223	0.0143	0.0107
GJR-GARCH (T)	0.3176	0.2113	0.1585

Based on Table 3, it can be seen that the VaR prediction with α= 99%, α = 95%, and α = 90% for the GJR-GARCH (N) and GJR-GARCH (T) models are higher than the GARCH (N) and GARCH (T) models. Furthermore, the *t* distribution for GARCH and GJR-GARCH has a higher predictive value of VaR. It is because the level of trust given can capture extreme values. The accuracy of VaR prediction can be known through the proportion of correct VaR. Correct VaR proportion is a method of determining VaR accuracy by looking at the actual loss proportion that is less than or equal to the VaR prediction. The correct VaR can be seen in the following Table 4:

Table 4. Correct VaR Agregation Model

Model	$\boldsymbol{Correct_{99\%}}$	$\boldsymbol{Correct_{95\%}}$	$\boldsymbol{Correct_{90\%}}$
GARCH (N)	82.1612	75.1286	70.6689
GARCH (T)	97.6132	89.0331	77.8566
GJR-GARCH (N)	98.1355	77.1869	72.2126
GJR-GARCH (T)	97.1904	94.2419	88.4391

In Table 4, it can be seen that the GJR-GARCH (N) and GJR-GARCH (t) models have the smallest difference between the correct VaR value and the level of confidence given, especially at = 99%. The same thing is also seen in the GJR-GARCH (T) model at = 95% and = 90%. It means that the GJR-GARCH (N) and GJR-GARCH (t) models are the most suitable for modeling aggregation stock returns.

4. Conclusions

The conclusion from the discussion is that the GARCH(1,1) and GJR-GARCH(1,1) models can be used to determine the VaR prediction result of an aggregation return. The accuracy of the VaR prediction can be determined by calculating the correct VaR proportion. The results of prediction and correct VaR obtained that the GJR-GARCH model is the most suitable for modeling aggregated return data.

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